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COMPLETELY SUPRA N-CONTINUOUS FUNCTION

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ABSTRACT

In this paper, we introduce a new concept called completely supra N-continuous function and investigated its relationship with other functions.

Keywords: Completely supra N-continuous function.

1. INTRODUCTION

The notion of supra topological spaces, s-continuous functions and s* -continuous functions was introduced (Mashhour *et al*, 1983). Supra N-closed set was introduced and supra N-continuity and supra N-irresoluteness investigated (Vidyarani and Vigneshwaran, 2013a).

In this paper, we introduce the concept of completely supra N-continuous function and investigated its relationship with other functions in supra topological space.

2. PRELIMINARIES

2.1. Definition

A subfamily μ of X is said to be supra topology on X if

i) $X, \phi \in \mu$

ii) If $A_i \in \mu \; \forall i \in j$ then $\cup A_i \in \mu$. (X,μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

2.2. Definition

The supra closure of a set A is denoted by cl^{μ} (A), and is defined as supra cl(A) = ∩ {B : B is supra closed and A ⊆ B}.

The supra interior of a set A is denoted by int^{μ}(A), and is defined as supra int(A) = ∪ {B : B is supra open and A ⊇ B}.

2.3. Definition

Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

2.4. Definition

Let (X, μ) be a supra topological space. A set A of X is called

(i) supra semi- open set (Levine, 1991), if A ⊆ $cl^{\mu}(int^{\mu}(A))$.

(ii) supra α -open set (Devi *et al.,* 2008), if A ⊆ $int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.

(iii) supra Ω closed set (Noiri and Sayed, 2005), if $scl^{\mu}(A) \subseteq int^{\mu}(U)$, whenever A ⊆ U , U is supra open set.

(iv) supra N-closed set (Vidyarani and Vigneshwaran, 2013a), if $\Omega cl^{\mu}(A) \subseteq U$, whenever A ⊆ U, U is supra α open set.

The complement of above supra closed set is supra open and vice versa.

2.5. Definition

A map f:(X, τ) → (Y, σ) is said to be

(i) supra N-continuous (Vidyarani and Vigneshwaran, 2013b), if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

(ii) Supra N-irresolute (Vidyarani and Vigneshwaran, 2013a), if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N - closed set V of (Y, σ).

(iii) strongly supra N-continuous (Vidyarani and Vigneshwaran, 2013b), if $f^{-1}(V)$ is supra closed in (X, τ) for every supra N-closed set V of (Y, σ).

3. COMPLETELY SUPRA N-CONTINUOUS FUNCTIONS

3.1. Definition

A map f:(X, τ) → (Y, σ) is called completely supra continuous function, if f^{-1}(V) is supra Regular closed in (X, τ) for every supra closed set V of (Y, σ).

3.2. Definition

A map f:(X, τ) → (Y, σ) is called completely supra N-continuous function, if f^{-1}(V) is supra Regular closed in (X, τ) for every supra N - closed set V of (Y, σ).

3.3. Theorem

Every completely supra N-continuous function is completely supra continuous function.

Proof Let f:(X, τ) → (Y, σ) be a completely supra N-continuous function. Let V be supra closed set in (Y, σ). Then V is supra N-closed set in (Y, σ), since every supra closed set is supra N-closed set. Since f is completely supra N-continuous function, then f^{-1}(V) is supra regular closed in (X, τ). Therefore f is completely supra continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

3.4. Example

Let X=Y={a, b, c} and τ = {X, φ, {a},{b, c}} , σ = {Y, φ, {a} }. Let f:(X, τ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is completely supra continuous but not completely supra N-continuous, since V={a,c} is supra N - closed in (Y, σ) but f^{-1}({a,c}) = {a,b} is not supra regular closed set in (X, τ).

3.5. Theorem

Every completely supra N-continuous function is supra N-continuous function.

Proof Let f:(X, τ) → (Y, σ) be a completely supra N-continuous function. Let V be supra closed set in (Y, σ). Then V is supra N-closed set in (Y, σ), since every supra closed set is supra N-closed set. Since f is completely supra N-continuous function, then f^{-1}(V) is supra regular closed in (X, τ). Since every supra regular closed set is supra closed set and every supra closed set is supra N-closed set, then f^{-1}(V) is supra N-closed in (X, τ). Therefore f is supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

3.6. Example

Let X=Y={a, b, c} and τ = {X, φ, {a},{b, c}} , σ = {Y, φ, {a} }. Let f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N-continuous but not completely supra N-continuous, since V={a,c} is supra N - closed in (Y, σ) but f^{-1}({a,c}) = {a,b} is not supraregular closed set in (X, τ).

3.7. Theorem

Every completely supra N-continuous function is supra N-irresolute function.

Proof Let f:(X, τ) → (Y, σ) be a completely supra N-continuous function. Let V be supra N-closed set in (Y, σ). Since f is completely supra N-continuous function, then f^{-1}(V) is supra regular closed in (X, τ). Since every supra regular closed set is supra closed set and every supra closed set is supra N-closed set, then f^{-1}(V) is supra N-closed in (X, τ). Therefore f is supra N-irresolute function.

The converse of the above theorem need not be true. It is shown by the following example.

3.8. Example

Let X=Y={a, b, c} and τ = {X, φ, {a},{b, c}} , σ = {Y, φ, {a} }. Let f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N-irresolute but not completely supra N-continuous, since V={a,c} is supra N - closed in (Y, σ) but f^{-1}({a,c}) = {a,b} is not supra regular closed set in (X, τ).

3.9. Theorem

Every completely supra N-continuous function is strongly supra N-continuous function.

Proof Let f:(X, τ) → (Y, σ) be a completely supra N-continuous function. Let V be supra N-closed set in (Y, σ). Since f is completely supra N-continuous function, then f^{-1}(V) is supra regular closed in (X, τ). Since every supra regular closed set is supra closed set, then f^{-1}(V) is supra closed in (X, τ). Therefore f is strongly supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

3.10. Example

Let X=Y={a, b, c} and τ = {X, φ, {a},{b},{a,b},{b,c}} , σ = {Y, φ, {a,b},{b,c}}. Let f:(X, τ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is strongly supra N-continuous but not completely supra N-continuous, since V={a,c} is supra N-closed in (Y, σ) but $f^{-1}(\{a,c\}) = \{a,b\}$ is not supra regular closed set in (X, τ).

3.11. Remark

Composition of two completely supra N-continuous function is completely supra N-continuous

3.12. Theorem

If f:(X, τ) → (Y, σ) is supra N-continuous and g: (Y, σ) →(Z,η) is completely supra N-continuous then gof: (X, τ) →(Z,η) is supra N-irresolute.

Proof Let V be supra N-closed set in Z. Since g is completely supra N-continuous, then $g^{-1}(V)$ is supra regular closed set in Y. Since every supra regular closed set is supra closed set, $g^{-1}(V)$ is supra closed set in Y. Since f is supra N-continuous, then $f^{-1}\, g^{-1}(V)$ is supra N-closed in X. Hence gof is supra N-irresolute.

3.13. Theorem

If f:(X, τ) → (Y, σ) is completely supra N-continuous and g: (Y, σ) →(Z,η) is supra N-continuous then gof: (X, τ) →(Z,η) is completely supra continuous.

Proof Let V be supra closed set in Z. Since g is supra N-continuous, then $g^{-1}(V)$ is supra N-closed set in Y. Since f is completely supra N-continuous, then $f^{-1}g^{-1}(V)$ is supra regular closed set in X. Hence gof is completely supra N-continuous.

3.14. Theorem

If f:(X, τ) → (Y, σ) is strongly supra N-continuous and g: (Y, σ) →(Z,η) is completely supra N-continuous then gof: (X, τ) →(Z,η) is strongly supra continuous.

Proof Let V be supra N-closed set in Z. Since g is completely supra N-continuous, then $g^{-1}(V)$ is supra regular closed set in Y. Since every supra regular closed set is supra closed set and every supra closed set is supra N-closed set, $g^{-1}(V)$ is supra N-closed set in Y. Since f is strongly supra N-continuous, then $f^{-1}g^{-1}(V)$ is supra closed set in X. Hence gof is strongly supra N-continuous.

3.15. Remark

The following implications is obtained from the above theorems

Completely supra N-continuous → strongly supra N-continuous→ supra N-irresolute → supra N-continuous

REFERENCES

Devi, R., S. Sampathkumar and M. Caldas, (2008). On supra α open sets and sα-continuous maps. *General Math.,* **16**(2): 77-84.

Levine, N. (1991). Semi-open sets and Semi-continuity in topological spaces. *Amer. Math.*, **12**: 5-13.

Mashhour, A.S., A.A. Allam, F.S. Mahmoud and F.H. Khedr, (1983). On supra topological spaces. *Indian J. Pure Appl. Math.,***14**(A): 502-510.

Noiri, T. and O.R. Sayed, (2005). On Ω closed sets and Ωs closed sets in topological spaces. *Acta. Math.,* **4**: 307-318.

Vidyarani, L. and M. Vigneshwaran, (2013a). N-Homeomorphism and N*-Homeomorphism in supra topological spaces. *Int. J. Math. Stat. Inven.,* **1**(2): 79-83.

Vidyarani, L. and M. Vigneshwaran, (2013b). On supra N-closed and sN-closed sets in supra Topological spaces. *Int. J. Math. Arch.,* **4**(2): 255-259