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NANO bT CLOSED SET IN NANO TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of set namely nano bT -closed sets in nano topological space. We also discussed some properties of nano bT closed set.

Keywords: Nano T closed and nano bT-closed.

1. INTRODUCTION

Nano topological space was introduced (Lellis Thivagar and Camel Richard, 2013a, b) with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He also established certain weak forms of nano open sets such as nano α – open sets, nano semi- open sets and nano pre open sets. b- open sets in topological spaces was introduced and studied (Andrijevic,1996). Several properties of a new type of sets called supra T-closed set and supra T-continuous maps was studied (Arockiarani and Trintia Pricilla, 2011). Also a new class of bT – closed set in supra topological spaces was introduced and studied (Krishnaveni and Vigneshwaran, 2013). In this paper, we introduced a new class of set called nano bT - closed sets and study its basic properties.

2. PRELIMINARIES

2.1. Definition

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$.That is $L_R(X) = {}_{x\epsilon U}\, R\, x : R(x) \subseteq X$,where R(x) denotes the equivalence class determined by $x \epsilon U$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.

That is $U_R(X) = {}_{x\epsilon U}\, R\, x : R\, x \cap X \neq \phi$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.

That is, $B_R(X) = U_R(X) - L_R(X)$.

2.2. Property

If (U,R) is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) = L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

2.3. Definition

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \epsilon \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets.

2.4. Definition

If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A).

That is, Ncl(A) is the smallest nano closed set containing A.

2.5. Definition

A subset A of a topological space (X,τ) is said to be b-open

if $A \subseteq cl(int(A)) \cup int(cl(A))$. The complement of b- open set is called a b – closed set.

2.6. Definition

A set A of X is called generalized b-closed set (simply gb−closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The complement of generalized b-closed set is generalized b-open set.

2.7. Definition

Let (X,μ) is a supra topological spaces. A subset A of (X,μ) is called T^{μ} -closed set if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^{\mu}b$ - open in (X,μ). The complement of T^{μ} - closed set is called T^{μ} -open set.

2.8. Definition

A subset A of a topological space (X,τ) is called regular open if $A = cl(int(A))$. The complement of regular open set is called regular closed set.

2.9. Definition

A subset A of a topological space (X,τ) is called generalized b- regular closed set if $bcl(A) \subseteq U$ and whenever $A \subseteq U$ and U is regular open of (X,τ). The complement of generalized b- regular closed set is called generalized b- regular open set.

2.10. Definition

A subset A of a supra topological space (X, μ) is called bT^{μ} -closed set (Krishnaveni and Vigneshwaran, 2013) if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is T^{μ} - open in (X, μ).

3. NANO bT-CLOSED SET

3.1. Definition

Let $(U, \tau R(X))$ be a nano topological space. A subset A of $(U, \tau R(X))$ is called nano T - closed set if $Nbcl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano gb- open in $(U, \tau R(X))$.

3.2. Example

Let U = {a,b,c,d} with U/R = {{a},{d},{b,c}} and X = {a,c}. Then the nano topology $\tau_R(X) = \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. The nano T closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,c,d}, {a,c,d} and{a,b,d}.

3.3. Definition

Let $(U, \tau R(X))$ be a nano topological space. A subset A of $(U, \tau R(X))$ is called nano bT - closed set if $Nbcl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano T - open in $(U, \tau R(X))$. The complement of nano bT - closed set is called nano bT -open set

3.4. Example

Let U = {a,b,c,d} with U/R = {{a},{d},{b,c}} and X = {a,c}. Then the nano topology $\tau_R(X) = \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,c,d}, {a,c,d},{a,b,d} and {a,b,c}.

3.5. Theorem

Every nano closed set is nano bT closed.

Proof Let $A \subseteq U$ and U is nano T- open set.Since A is nano closed then $Ncl(A) = A \subseteq U$. We know that $Nbcl(A) \subseteq N cl(A) \subseteq U$, implies $Nbcl(A) \subseteq U$. Therefore A is nano bT - closed.

The converse of the above theorem need not be true as seen from the following example.

3.6 . Example

Let U = {a,b,c,d} with U/R = {{a},{d},{b,c}} and X = {a,c}. Then the nano topology $\tau_R(X) = \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. The nano closed sets are U,ϕ,{b,c,d},{a,d} and {d}.

The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d} and {a,b,c}. Here the sets {b}, {a}, {c}, {d}, {b,c}, {c,d}, {a,c,d}, {a,b,d} and {a,b,c} are nano bT closed sets but not in nano closed sets.

3.7. Theorem

Every nano b closed set is nano bT closed.

Proof Let A $\subseteq$ U and U is nano T- open set. Since A is nano b closed then Nbcl (A) $\subseteq$ U . Therefore A is nano bT - closed.

The converse of the above theorem need not be true as seen from the following example.

3.8. Example

Let U = {a,b,c,d} with U/R = {{a},{d},{b,c}} and X = {a,c}. Then the nano topology τ_R (X) = {U,ϕ,{a},{b,c},{a,b,c}}. The nano b closed sets are U,ϕ,{b,c,d}, {a,c,d}, {a,b,d}, {c,d}, {b,d}, {a,d}, {b,c},{a},{b},{c}and{d}.The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,c,d}, {a,c,d},{a,b,d}and{a,b,c}. Here the set {a,b,c} is nano bT closed sets but not in nano b closed set.

3.9. Theorem

Every nano bT - closed set is nano gb - closed set.

Proof Let A $\subseteq$ U and U is nano open set. We know that every nano open set is nano T - open set , then U is nano T- open set. Since A is nano bT -closed set, we have Nbcl (A) $\subseteq$ U. Therefore A is nano gb-closed set.

3.10. Example

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {b,d}. Then the nano topology τ_R (X) = {U,ϕ,{b,d}}. The nano gb closed sets are U, ϕ,{b,c,d}, {a,c,d}, {a,b,d}, {a,b,c}, {c,d}, {a,b},{a,c},{a,d}, {b,c}, {a},{b}, {c} and {d}.The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d},{a,b},{a,c},{a,c,d} and {a,b,c}. Here the set {b,c,d} and {a,b,d} is nano gb closed sets but not nano bT closed set.

3.11. Theorem

Every N a n o bT- closed set is nano gbr - closed set.

Proof Let A $\subseteq$ U and U is nano regular open set. We know that every nano regular open set is nano T-open set, then U is nano T-open set. Since A is nano bT -closed set, we have Nbcl(A) $\subseteq$U . Therefore A is nano gbr- closed set.

3.12. Example

Let U = {a,b,c,d} with U/R = {{a},{d},{b,c}} and X = {a,c}. Then the nano topology τ_R (X) = {U,ϕ,{a},{b,c},{a,b,c}}. The nano gbr closed sets are U,ϕ, {a,b,c},{b,c,d}, {a,c,d}, {a,b,d},{a,b}, {a,c} {c,d}, {b,d}, {a,d}, {b,c},{a},{b},{c}and{d}. The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,c,d}, {a,c,d},{a,b,d} and {a,b,c}. Here the sets {a,b}, {a,c} and {b,d} is nano gbr closed sets but not in nano bT closed set.

3.13. Theorem

The union of two nano bT - closed set is nano bT- closed set.

Proof Let A and B two nano bT - closed set. Let A$\cup$ B $\subseteq$ G, where G is nano T - open. Since A and B are nano bT-closed sets. Therefore Nbcl(A) $\cup$ Nbcl (B) $\subseteq$ G. Thus N bcl (A$\cup$ B) $\subseteq$ G. Hence A$\cup$B is Nano bT- closed set.

3.14. Theorem

Let A be n a n o bT -closed set of (U,X). Then Nbcl (A) - A does not contain any non empty nano T- closed set.

Proof: Necessity Let A be n a n o bT - closed set. suppose F$\neq\phi$ is a n a n o T - closed set of Nbcl (A) - A. Then F$\subseteq$ N bcl (A) - A implies F$\subseteq$ Nbcl(A) and A^C. This implies A$\subseteq F^C$. Since A is nano bT- closed set, Nbcl(A) $\subseteq U^C$.Consequently, F$\subseteq$ [Nbcl $(A)]^C$.Hence F$\subseteq$Nbcl(A) $\cap$ [Nbcl(A)^C] = ϕ. Therefore F is empty , a contradition.

Sufficiency: Suppose A $\subseteq$ U and that U is nano T - open. If N bcl (A) $\not\subset$ U. Then Nbcl (A) $\cap U^C$ is a not empty n a n o T- closed subset of Nbcl(A) - A.

Hence Nbcl(A) $\cap U^C$ = ϕ and N bcl(A)$\subseteq$U.Therefore A is nano bT - closed.

3.15. Theorem

If A is nano bT - closed set in a supra topological space (U,X) and A$\subseteq$B $\subseteq$Nbcl (A) then B is also nano bT- closed set.

Proof Let U be nano T- open in set (U,X) such that B $\subseteq$ U. Since A$\subseteq$ B $\Rightarrow$ A$\subseteq$U and since A is nano bT -closed set in (U,X). Nbcl (A) $\subseteq$ U, since B $\subseteq$ Nbcl(A). Then N bcl(B) $\subseteq$U. Therefore B is also nano bT - closed set in (U,X)

3.16. Theorem

Let A be nano bT - closed set then A is nano b- closed if Nbcl(A)-A is nano T- closed.

Proof Let A be nan o bT- closed set. If A is nano b - closed, we have N bcl (A) - A =ϕ , which is nano T- closed. Conversely, let N bcl(A)-A is nano bT - closed. Then by the theorem 3.13, Nbcl (A) - A does not contain any non empty nano T- closed and N bcl (A)-A=ϕ. Hence A is nano b - closed.

3.17. Theorem

A subset A$\subseteq$X is nano bT - open iff F $\subseteq$ N bint(A) whenever F is nano T - closed and F$\subseteq$ A.

Proof Let A be n a n o bT - open set and suppose F$\subseteq$ A, where F is nano T- closed. Then X-A is nano bT - closed set contained in the n a n o T - open set X-F. Hence N bcl (X-A)$\subseteq$ X-F. Thus F$\subseteq$ N bint(A). Conversely, if F is nano T - closed set with F$\subseteq$ Nbint(A) and F$\subseteq$A , then X-Nbint (A) $\subseteq$ X − F . This implies that Nbcl(X-A) $\subseteq$ X-F. Hence X-A is nano bT - closed. Therefore A is nano bT - open set.

3.18. Theorem

If B is nano T- open and n a n o bT - closed set in X, then B is nano b- closed.

Proof Since B is nano T- open and n a n o bT - closed then Nbcl (B) $\subseteq$ B, but B$\subseteq$ N bcl(B). Therefore B=Nbcl(B).Hence B is nano b - closed.

3.19. Corollary

If B is nano open and n a n o bT - closed set in X. Then B is nano b-closed.

3.20. Theorem

Let A be nano g b-open and nano bT - closed set. Then A$\cap$ F is nanoT- closed whenever F is nano b- closed.

Proof Let A be nano g b-open and n a n o bT- closed set then Nbcl (A)$\subseteq$ A and also A$\subseteq$ Nbcl (A). Therefore N bcl (A) = A. Hence A is nano b-closed. Since F is nano b-closed. Therefore A$\cap$ F is nano b - closed in X. Hence A$\cap$ F is nano T - closed in X.

From the above theorem and example we have the following diagram

nano closed
$\downarrow$
nano b closed
$\downarrow$
nano bT-closed $\rightarrow$ nano gb-closed
$\downarrow$
nano gbr-closed

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