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b-CHROMATIC NUMBER OF CORONA PRODUCT OF CROWN GRAPH AND COMPLETE BIPARTITE GRAPH WITH PATH GRAPH

Vijayalakshmi, D. and G. Mohanappriya

Department of Mathematics, Kongunadu Arts & Science College, Coimbatore-641 029.
E.mail: vijikasc@gmail.com; mohanappriyag25@gmail.com

ABSTRACT

A b-coloring of a graph is a proper coloring where each color admits at least one node (called dominating node) adjacent to every other used color. The maximum number of colors needed to b-color a graph G is called the b-chromatic number and is denoted by φ(G). In this paper, we find the b-chromatic number and some of the structural properties of corona product of crown graph and complete bipartite graph with path graph.

Keywords: Corona product, crown graph, complete bipartite graph, path graph.

1. INTRODUCTION

A b-coloring by k-colors is a proper coloring of the vertices of graph G such that in each color classes there exists a vertex that has neighbors in all the other k-1 color classes. The b-chromatic number φ(G) is the largest number k for which G admits a b-coloring with k-colors (Irving and Manlove, 1999). The corona G1 ∘ G2 of two graphs G1 and G2 is defined as a graph obtained by taking one copy of G1 (which has p1 vertices) and p1 copies of G2 and attach one copy of G2 at every vertex of G1 (Harary, 1972).

In this paper we find for which the largest number k for which corona product of crown graph and complete bipartite graph with path graph admits a b-coloring with k-colors. And also we find some of its structural properties (Venkatachalam and Vernold Vivin, 2010; Vernold Vivin and Venkatachalam, 2012; Vijayalakshmi and Thilagavathi, 2012)

2. Definition

2.1. Crown Graph

A crown graph on 2n vertices is an undirected graph with two sets of vertices u_i and v_i and with an edge from u_i to v_j whenever $i \neq j$. The crown graph can be viewed as a complete bipartite graph from which the edges of a perfect matching have been removed (Wikipedia).

2.2. Complete Bipartite Graph

A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset, and every possible edge that could connect vertices in different subsets is part of the graph. That is, it is a bipartite graph (V_1, V_2, E) such that for every two vertices $v_1 \in V_1$ and $v_2 \in V_2$, v_1v_2 is an edge in E. A complete bipartite graph with partitions of size $|V_1|=m$ and $|V_2|=n$, is denoted $K_{m,n}$ (Balakrishnan, 2004; Balakrishnan and Ranganathan, 2012).

2.3. Fan Graph

and A Fan graph $F_{m,n}$ is defined as the graph join K_m, where K_m the empty graph on nodes is and P_n is the path on n nodes (Wikipedia).

2.4. Path Graph

The path graph P_n is a tree with two nodes of vertex degree 1, and the other n-2 nodes of vertex degree (Harary, 1972).

2.5. Corona Product

Corona product or simply corona of any graph G1 and graph G2, defined as the graph which is the disjoint union of one copy of G1 and |V1| copies of G2 (|V1| is the number of vertices of G1) in which each vertex of the copy of G1 is connected to all vertices of a separate copy of G2 (Harary, 1972).

2.6. b-coloring

A b-coloring of a graph is a proper coloring such that every color class contains a vertex that is adjacent to all other color classes. The b-chromatic number of a graph G, denoted by φ (G), is the maximum number t such that G admits a b-coloring with t colors (Irving and Manlove, 1999).

3. CORONA PRODUCT OF CROWN GRAPH WITH PATH GRAPH

3.1. b-chromatic number of corona product of Crown Graph with Path Graph

3.1.1. Theorem

For any n ≥ 3, $\varphi[S_n^0 \circ P_n] = 2n$.

Proof: Let S^0 be any Crown graph with vertices, V = $\{v_1, v_2, \ldots, v_n\}$ and $V' = \{v'_1, v'_2, \ldots, v'_n\}$ i.e. $V(S_n^0) = V \cup V'$. Let the edges of L_n be $E(S_n^0) = \{e_j : 1 \le j \le n^2 - n\}$ where e_j is the edge connecting v_i and v'_j for every $i \neq j$.

Let P_n be ant path graph of length n-1 with n-vertices. $V(p^l_n) = \{u_{ij} : 1 \le i \le 2n, 1 \le l \le n, 1 \le j \le n\}$ and and $E P_n$ be $\{e_{pi} : 2n - 1 \le i \le n-1\}$.

By the definition of corona graph each vertex in S_n^0 is adjacent to every vertex copy of P_n, i.e. vertices of $V(L_n \circ P_n) = V(S_n^0) \cup V(P_n)$ .Let $E[S_n^0 \circ P_n]$ be $E(S_n^0) \cup E P_n \cup \{e_i : n^2 - n + 1 \le i \le 5n^2 - 3n\}$.

Consider the color class $C = \{c_1, c_2, c_3, \ldots, c_n, c_{n+1}, c_{n+2}, \ldots, c_{2n}\}$ to color the vertices of $(S_n^0 \circ P_n)$. Assign the colors $c_1, c_2, c_3, \ldots, c_n$ to $v_1, v_2, \ldots, v_n$, i.e. v_i's and $c_{n+1}, c_{n+2}, \ldots, c_{2n}$ to v'_js , j = 1,2,3,…,n respectively for every i≠ j, i,j= 1,2,…,n.

From the figure we see that, each v'_is are adjacent to every v'_js for every i not equal to j and vice versa. Hence both v_i's and v'_js earns its adjacent color for every i≠ j. To make the above coloring to be b-chromatic proper coloring of $V(p^l_n)$ by corresponding non-adjacent vertices of its v_i's or v''s respectively. Thus each color has the neighbour in the every other color class. Thus, $\varphi[(S_n^0 \circ P_n] = 2n$.

Let us assume that $\varphi[S_n^0 \circ P_n] > 2n$, let it be $\varphi[S_n^0 \circ P_n] = 2n+1$. The graph $[S_n^0 \circ P_n]$ must requires 2n+2 vertices of degree 2n+1, all with distinct color and each must have adjacent with all of the other color class, but at least one color class which does not have a color dominating vertex in $[S_n^0 \circ P_n]$, which invalidates the definition of b-coloring. Hence, $\varphi[S_n^0 \circ P_n]$ not equal to 2n+1, it must be less than 2n+1 i.e. $\varphi[S_n^0 \circ P_n] = 2n$. Thus, for any n ≥3, the b-chromatic number of corona graph of crown graph with path graph is 2n.

3.2. Illustration: b-coloring of corona product of Crown Graph with Path Graph

3.2.1. Theorem

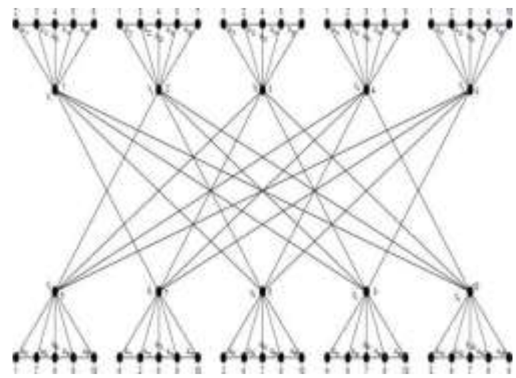
For any n ≥ 3, $q[S_n^0 \circ P_n] = 5n^2 - 3n$

Proof: $q[(S_n^0 \circ P_n] =$ Number of edges in $S^0{}_n + 2n$ x Number of edges in P_n

$= n^2 - n + 2n\,(2n-1)$

$= n^2 - n + 4n^2 - 2n - 1.$

$= 5n^2 - 3n.$



$\varphi[S_5^0 \circ P_5] = 10$

3.2.1. Theorem

For any n ≥ 3, the vertex polynomial of $(S_n^0 \circ P_n)$ be $4nx^2 + 2n^2 - 4n\, x^3 + 4x^{n+2}$

Proof: $V((S_n^0 \circ P_n ; x) = \sum_{k=1}^{\Delta(G)} V_k x^k$

= No of vertices having degree 2× x^2 +

No of vertices having degree 3× x^3 +

No of vertices having degree n+2× x^{n+2}

$= 4nx^2 + 2n^2 - 4n\, x^3 + 2n\, x^{n+2}$.

3.2.3. Some Structural Properties of $(S_n^0 \circ P_n)$n ≥ 3.

Properties Graphs	No. of Vertex	No. of Edges	Maximum Degree	Minimum Degree	Vertex Polynomial
Path Graph	n	n-1	2	1	$2x + (n-2)x^2$
Crown Graph	2n	n^2-n	n-1	n-1	$(2n)x^{n-1}$
$(S_n^0 \circ P_n)$	2n(n+1)	$5n^2 - 3n$	2n-1	2	$4nx^2 + (2n^2 - 4n)x^3 + (2n)x^{n+2}$

4. CORONA PRODUCT OF COMPLETE BIPARTITE GRAPH WITH PATH GRAPH

4.1. b-chromatic number on Corona Product of Complete Bipartite Graph with Path Graph

4.1.1. Theorem

For any n ≥ 3, φ $[K_{m,n} \circ P_n]$ = 2n, m = n; 2n+1, m > n; 2n-1, m < n

Proof: Let $K_{m,n}$ be any complete bipartite graph with vertices, V = {$v_1, v_2, \ldots, v_n$} and V' = {$v'_1, v'_2, \ldots, v'_n$} i.e. V($K_{m,n}$) = V U V'. Let the edges $K_{m,n}$ of be E($K_{m,n}$)={e_j: $1 \le j \le n^2$} where e_j is the edge connecting v_i *and* v'.

Let P_n be an path graph of length n-1 with n-vertices. V(p^l_n) = {u_{ij}: $1 \le i \le 2n$, $1 \le l \le n$, $1 \le j \le n$} and $E\,P_n$ be {e_{pi} : 2n − 1 ≤ i ≤n-1}

By the definition of corona graph each vertex in $K_{m,n}$ is adjacent to every vertex copy of P_n, i.e. vertices of $V(K_{m,n} \circ P_n)$ = V($K_{m,n}$) $UV(P_n)$.

Let the edges of $K_{m,n} \circ P_n$ be E[$K_{m,n} \circ P_n$] =E($K_{m,n}$) U $E\,P_n$ U {e : $n^2 + 1 \le i \le 5n^2 - 2n$}, for m = n, E[$K_{m,n} \circ P_n$] =E($K_{m,n}$) U $E\,P_n$ U {e_i: $n^2 + 1 \le i \le 5n^2 - 5n + 1$ }, for m < n,

E[$K_{m,n} \circ P_n$] =E($K_{m,n}$) U $E\,P_n$ U {e_i: $n^2 + 1 \le i \le 5n^2 + n - 1$ }, for m > n.

Consider the color class C = {$c_1, c_2, c_3, \ldots, c_n, c_{n+1}, c_{n+2}, \ldots, c_{2n+1}$} to color the vertices of ($K_{m,n} \circ P_n$). The proof follows from the following cases.

Case (i) m=n

Consider the color class C'_1 = {$c_1, c_2, c_3, \ldots, c_n, c_{n+1}, c_{n+2}, \ldots, c_{2n}$} to color the vertices of $K(_{m,n} \circ P_n)$, $m = n$. Assign the colors $c_1, c_2, c_3, \ldots, c_n$ to $v_1, v_2, \ldots, v_n$, *i. e.* v_i *'s* and $c_{n+1}, c_{n+2}, \ldots, c_m$ *to* v''_j s , j = 1,2,3,…,m respectively.

From the figure we assure that, each v'_i*'s* are adjacent to every v''_j*s* for and vice versa. Hence both v'_i*'s* and v''_j*s* earns its adjacent color. To make the above coloring to be b-chromatic proper coloring of V(p^l_n) by corresponding non-adjacent vertices of its v_i 's or v'_j *'s* respectively,and the remaining vertices are colored properly by the colors in the color class. Thus each color has the neighbor in the every other color class. Thus, φ[$K_{m,n} \circ P_n$] = 2n.

Let us assume that φ[($K_{m,n} \circ P_n$]> 2n, say φ[$K_{m,n} \circ P_n$] = 2n+1. The graph [$K_{m,n} \circ P_n$] must requires 2n+2 vertices of degree 2n+1, all with distinct color and each must have adjacent with all of the other color class which is not possible, since maximum degree of $K_{m,n} \circ P_n$ is 2n, hence at least one color class does not have the color dominating vertex, which contradicts the definition of b-coloring. Hence, φ[$K_{m,n} \circ P_n$] not equal to 2n+1, must be less than 2n+1 i.e. φ[$K_{m,n} \circ P_n$] = 2n . Thus, for any n ≥3, the b-chromatic number of corona graph of complete bipartite graph with path graph is 2n for each m = n.

Case (ii) m>n

Consider the color class C_2 = {$c_1, c_2, c_3, \ldots, c_n, c_{n+1}, c_{n+2}, \ldots, c_{2n+1}$} to color the vertices of K($_{m,n} \circ P_n$), m < n. Assign the colors $c_1, c_2, c_3, \ldots, c_n$ to $v_1, v_2, \ldots, v_n$, i. e. v'_i s and $c_{n+1}, c_{n+2}, \ldots, c_{2n+1}$ to v'_j s , j = 1,2,3,…,m respectively.

The remaining proof of the theorem follows immediately from case (i). Hence φ [$K_{m,n} \circ P_n$] = 2n+1,m > n.

Case (iii) m<n

Consider the color class C'_2 = {$c_1, c_2, c_3, \ldots, c_n, c_{n+1}, c_{n+2}, \ldots, c_{2n-1}$} to color the vertices of K($_{m,n} \circ P_n$), m < n. Assign the colors $c_1, c_2, c_3, \ldots, c_n$ to $v_1, v_2, \ldots, v_n$, i. e. v'_i s and $c_{n+1}, c_{n+2}, \ldots, c_{2n-1}$ to v''_j s , j = 1,2,3,…,m respectively.

The remaining proof of the theorem follows immediately from case (i). Hence φ [$K_{m,n} \circ P_n$] = 2n-1, m < n. Hence theproof.

4.1.1. Illustration Corona Product of Complete Bipartite Graph with Path Graph

4.1.2. Theorem

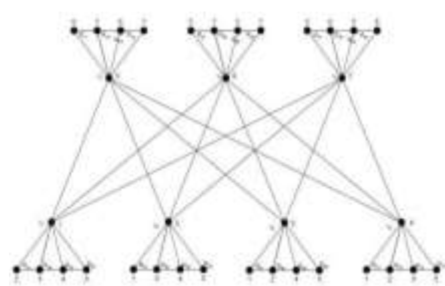
For any m, n ≥ 3, q[($K_{m,n} \circ P_n$)] = $2n^2$ +3mn-m-n.

Proof: q[($S^0_n \circ P_n$] = Number of edges in $K_{m,n}$ + 2n x Number of edges in Fan graph F_n

= mn+ (m+n) (2n-1)

= mn+2mn-m+$2n^2$ − n .

= $2n^2 + 3mn - m - n$.



φ [$K_{3,4} \circ P_4$] = 7

4.1.3. Theorem

For any m, n ≥ 3, the vertex polynomial of be$(K_{m,n} \circ P_n)$ $4nx^2 + (2n^2 - 4n)x^3 + 2nx^{2n}$, m=n.

Proof: $V(K_{m,n} \circ P_n ; x) = {}^{\Delta(G)}_{k=1} V_k x^k$

= No of vertices having degree 2× x^2 +

No of vertices having degree 3× x^3 +

No of vertices having degree 2n× x^{2n} +

= $4nx^2 + 2n^2 - 4n\, x^3 + 2n\, x^{n+2}$.

For any m, n ≥ 3, the vertex polynomial of be $(K_{m,n} \circ P_n)$ $4nx^2 + (2n^2 - 4n)x^3 + 2nx^{2n}$, m > n.

Proof: $V(K_{m,n} \circ P_n ; x) = {}^{\Delta(G)}_{k=1} V_k x^k$

= No of vertices having degree 2× x^2 +

No of vertices having degree 3× x^3 +

No of vertices having degree 2n-1× x^{2n+1}

$=(4n-2)x^2 + (2n^2 - 3n - 2)x^3 + (2n+1)x^{2n+1}$.

4.1.4. Theorem

4.4.5. Some Structural Properties of $(K_{m,n} \circ P_n)$, n ≥ 3.

Properties Graphs	Number of Vertex	Number of Edges	Maximum Degree	Minimum Degree	Vertex Polynomial
Path Graph	n	n-1	2	1	$2x+(n-2)x^2$
Complete Bipartite Graph	m+n	n^2	max{m,n}	min{m,n}	$(2n)x^n$
$(K_{m,n} \circ P_n)$ m = n	$2n^2 + 2n$	$5n^2 - 2n$	2n	2	$4nx^2 + (2n^2 - 4n)x^3 + 2nx^{2n}$
$(K_{m,n} \circ P_n)$ m < n	$2n^2$ +n-1	$5n^2$ −5n+1	2n	2	$(4n-2)x^2 + (2n^2 - 5n + 2)x^3 + nx^{2n-1} + (n-1)x^{2n}$
$(K_{m,n} \circ P_n)$ m > n	$2n^2$ +3n+1	$5n^2$ +n-1	2n+1	2	$(4n+2)x^2 + (2n^2 - 3n - 2)x^3 + (2n+1)x^{2n+1}$

5. CONCLUSION

In this paper we operated the graph operation corona product on crown graph and complete bipartite graph with path graph, we get corona product of crown graph with path graph and corona product of complete bipartite graph with path graph and also we find its b-chromatic number and some of its structural properties.

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