DOI:10.26524/krj118

Kong. Res. J. 3(1) : 15-16, 2016 ISSN 2349-2694
Kongunadu Arts and Science College, Coimbatore.

ON *gα-FUZZY CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

Devi, R. and M. Vigneshwaran*
Department of Mathematics, Kongunadu Arts And Science College (Autonomous), Coimbatore-29.
*E.mail: vignesh.mat@gmail.com

ABSTRACT

In this paper we introduce the concept of *gα- fuzzy closed sets in fuzzy topological spaces and study some of its properties.

Keywords: *gα- fuzzy closed sets.

1. INTRODUCTION

Levine introduced generalized closed sets (Levine, 1970) in topological spaces. The concept of fuzzy closed set (Chang, 1968) is an important role in fuzzy topological spaces. The concept of gα-closed sets (Maki *et al.,* 1993) in a topological space was introduced.

Throughout this paper X and Y are represents fuzzy topological spaces. For a fuzzy set A of a topological spaces X, the notations cl(A), Int(A) and 1-A will respectively stand for the fuzzy closure, fuzzy interior and fuzzy compliment of A.

2. *gα- FUZZY CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

DEFINITION 2.1 (Balasubramanian and Sundram, 1997)

Let X be a fuzzy topological space. A fuzzy set p in X is called fuzzy generalized -closed if cl(p)≤ q, whenever p≤ q and q is fuzzy open.

DEFINITION 2.2 (Devi and Bhuvaneswari, 2006)

Let X be a fuzzy topological space. A fuzzy set p in X is called fuzzy gα-closed if αcl(p)≤ q, whenever p≤ q and q is fuzzy α-open.

DEFINITION 2.3

A fuzzy set p in X is called *gα-fuzzy closed if cl(p)≤ q, whenever p≤ q and q is fuzzy gα-open.

THEOREM 2.4

Every *gα-fuzzy closed set is fuzzy g-closed.

PROOF

Let p≤ q and q is fuzzy open. But every fuzzy open set is fuzzy gα-open. Since p is *gα-fuzzy closed, cl(p)≤ q and q is fuzzy gα-open. Therefore p is fuzzy g-closed.

The converse of the above theorem need not be true by the following example.

EXAMPLE 2.5

Let X = {a, b, c}. Define the fuzzy sets A,B,C : X [0, 1] as follows.

A(a) = 0.2	B(a) = 0.6	C(a) = 0.3
A(b) = 0.3	B(b) = 0	C(b) = 0.2
A(c) = 0.7	B(c) = 1	C(c) = 1

Consider the fuzzy topology τ = {0, 1, C}. Here A and B are fuzzy g-closed set. but not *gα-fuzzy closed set.

THEOREM 2.6

If A and B are *gα-fuzzy closed set in X, then A∨ B is a *gα-fuzzy closed set in X.

PROOF

Assume that A and B are *gα-fuzzy closed set in X. Let q be a fuzzy gα -open set in X such that A ≤ q and B ≤ q . Then A∨ B ≤ q. Since A and B are *gα-fuzzy closed cl(A) ≤ q and cl(B) ≤ q . Therefore

cl(A∨ B) = cl(A) ∨ cl(B)

≤ q ∨ q

= q.

Implies cl(A∨ B) ≤ q. Hence A∨ B is *gα-fuzzy closed set in X.

THEOREM 2.7

Let A is *gα-fuzzy closed set in a fuzzy topological space X, and A ≤ B ≤ cl(A), then B is *gα-fuzzy closed set in X.

PROOF

Let q be a fuzzy gα-open set such that B≤ q. Then A ≤ q, since A is *gα-fuzzy closed set in X, cl(A)≤

q. Now B≤ cl(A) implies cl(B)≤ cl(cl(A)) = cl(A) ≤ q. Hence B is *gα-fuzzy closed set in X.

THEOREM 2.8

Let X be a fuzzy topological space. A fuzzy set A of X is *gα-fuzzy open if and only if B≤ Int(A), whenever B is fuzzy gα -closed set and B ≤ A.

PROOF

Let A be a *gα-fuzzy open set and B is fuzzy gα-closed such that B ≤ A implies 1-B ≥ 1-A is *gα-fuzzy closed. So cl(1-A) ≤ 1-B implies (1-cl(1-A)) ≥ (1-(1-B)) = B. But (1-cl(1-A)) = Int(A). Thus B≤ Int(A).

Conversely, suppose that A is fuzzy set such that B≤ Int(A), whenever B is fuzzy gα -closed set and B ≤ A. We show that 1-A is *gα-fuzzy closed set. Let 1-A ≤ B, where B is fuzzy gα-open. Since 1-A ≤ B implies that 1-B ≤ A. By assumption that we must have 1-B ≤ Int(A) or 1-Int(A) ≤ B. Now 1-Int(A) = cl(1-A) which implies that cl(1-A) ≤ B and 1-A is *gα-fuzzy closed set.

THEOREM 2.9

Let A is *gα-fuzzy open set in a fuzzy topological space X and Int(A) ≤ B ≤ A, then B is *gα-fuzzy open set in X.

PROOF

Given that Int(A) ≤ B ≤ A, we have 1-A≤ 1-B ≤ 1-Int(A). Since A is *gα-fuzzy open in X, 1-A is *gα-fuzzy closed in X and so by theorem 2.7, 1-B is *gα-fuzzy closed in X. Hence B is *gα-fuzzy open in X.

THEOREM 2.10

Let X be a fuzzy topological space and gα-f-open(X) stand for the family of all gα-fuzzy open set of X and gα-f-closed(X) stand for the family of all gα-fuzzy closed set of X. If every fuzzy subset of X is a *gα-fuzzy closed set then gα-f-open(X) = gα-f-closed(X).

PROOF

Let us assume that every fuzzy set p is *gα-fuzzy closed set in X. Let p∈ gα-f-open(X). Since p ≤ p and p is *gα-fuzzy closed set, we have cl(p) ≤ p, but p ≤ cl(p). Therefore cl(p) = p implies p is gα-f-closed(X). Therefore

gα-f-open(X) ⊆ gα-f-closed(X) **(1)**

Assume that p is gα-f-closed(X) then 1-p is gα-fuzzy open. By(1) gα-f-open(X) ⊆ gα-f-closed(X). Implies 1-p is gα-f-closed(X) implies p is gα-f-open(X). Hence

gα-f-closed(X) ⊆ gα-f-open(X) **(2)**

From **(1)** and **(2)** we get gα-f-open(X) = gα-f-closed(X).

REMARK 2.11

A and B are *gα-fuzzy closed set, but A ∧ B is not *gα-fuzzy closed set.

It can be seen by the following example.

EXAMPLE 2.12

Let X = {a, b, c}. Define the fuzzy sets A,B,C : X [0, 1] as follows.

A(a) = 0.7	B(a) = 0.3	C(a) = 0.3
A(b) = 0.8	B(b) = 1	C(b) = 0.2
A(c) = 1	B(c) = 1	C(c) = 1

Consider the fuzzy topology τ = {0, 1, C}. It is clear that A and B are*gα-fuzzy closed set. But A ∧ B is not a *gα-fuzzy closed set.

REFERENCES

Balasubramanian, G. and P. Sundram, (1997). On some generalizations of fuzzy continuous functions, *Fuzzy sets and systems* **86**:93–100.

Chang C.L., (1968). Fuzzy topological spaces. *J. Math. Anal. Appl.* **24**:182 – 190.

Devi, R and K. Bhuvaneswari, (2006). On Fuzzy generalized α-continuous functions and its homeomorphisms, *Bull. Kerala Math. Assoc.* **3**(1):1-22.

Levine, N., (1970). Generalized closed sets in topology, *Red. Circ. Math. Palermo* **19**:89-96.

Maki, H., R. Devi and K. Balachandran, (1993). Generalized α-closed sets in topology, *Bull. Fukuoka Univ. Ed.* Part III, **42**:13-21.