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RESEARCH ARTICLE

EXISTENCE OF SOLUTIONS FOR IMPULSIVE NEUTRAL FUNCTIONAL INTEGRO DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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ABSTRACT

In this paper, by using fractional power of operators and Sadovskii's fixed point theorem, we study the existence of mild solution for a certain class of impulsive neutral functional integrodifferential equations with nonlocal conditions. The results we obtained are a generalization and continuation of the recent results on this issue.

Keywords: Sadovskii's fixed point theorem, integrodifferential equations.

1. INTRODUCTION

Impulsive differential equations, that are differential equations involving impulsive effect, appear as a natural description of several real world problems. Many evolution process that have a sudden change in their states such as mechanical systems with impact, biological systems such as heart beats, blood flows, population dynamics, theoretical physics, radiophysics, pharmacokinetics, mathematical economy, chemical technology, electric technology, metallurgy, ecology, industrial robotics, biotechnology process, chemistry, engineering, control theory, medicine and so on. Adequate mathematical models of such processes are systems of differential equations with impulses, see the monographs of Bainov and Simeonov (13), Bainov, Lakshmikantham and Simeonov (14), the papers (10,15) and the references therein.

The theory of integrodifferential equations can be used to describe a lot of natural phenomena arising from many fields such as electronics, fluid dynamics, biological models, and chemical kinetics. Most of these phenomena cannot be described through classical differential equations. That is Why in recent years they have attracted more and more attention of several mathematicians, physicists and engineers. Impulsive integrodifferential equations has undergone rapid development over the years and played very important role in modern applied mathematical models of real process. Recently, several authors (3,9,11) have investigated the impulsive integrodifferential equations in abstract spaces. We refer to the papers Wang and Wei (20) and Guo (21) and the references cited therein. Particularly, neutral (integro) differential equations arise in many areas of applied mathematics. For instance, the system of heat conduction with finite wave speeds, studied in (19) can be modeled in the form integrodifferential equation of neutral type. For more details on this theory and on its applications we refer to the monographs of Lakshmikantham *et al.* (14), and Samoilenko and Perestyuk (4) for the case of ordinary impulsive system and for partial differential and for partial functional differential equations with impulses.

The starting point of this paper is the work in papers (11, 12). Especially, authors in (12) investigated the existence of solutions for the system.

$$\frac{d}{dt} x\ t + F\ t, x\ t\ , x\ b_1 t\ , \dots , x\ b_m t + A\ t\ \ x t = G\ t, x\ t\ , x\ a_1\ t\ , \dots , x\ a_n\ t\ , 0 \le t \le a \quad (1)$$

$$x\ 0 + g\ x = x_0 \quad (2)$$

by using fractional powers of operators and Sadovskii's fixed point theorem. And in (11), authors studied the following impulsive functional integrodifferential equation with nonlocal conditions of the form

$$x'\ t = A\ t\ x\ t + F\ t, x\ \sigma_1\ t\ , \dots , x\ \sigma_n\ t\ , \int_0^t t, s, x\ \sigma_{n+1}\ s\ ds \quad (3)$$

$$t \in J = 0, b\ , t \ne t_k, k = 1, \dots , m$$

$$x\ \ 0\ \ + g\ \ x\ \ = x_0 \quad (4)$$

$$\Delta x\ \ t_k\ \ = I_k\ \ x\ \ t_k\ \ ,\ k = 1, \dots , m \quad (5)$$

by using Schaefer's fixed point theorem.

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Motivated by above mentioned works (11,12), the main purpose of this paper is to prove the existence of mild solutions for the following impulsive neutral functional integrodifferential equations in a Banach space X.

$$\frac{d}{dt}F\left(t, x(b_1(t)), \dots, x(b_m(t)), \int_0^t (t, s, x(b_{m+1}(s)))\,ds\right)$$

$$= A(t)x(t) + G\left(t, x(a_1(t)), \dots, x(a_n(t)), \int_0^t e(t, s, x(a_{n+1}(s)))\,ds\right) \quad (6)$$

$$t \in J = [0, b],\ t \neq t_k,\ k = 1, \dots, m$$

$$x(0) + g(x) = x_0 \quad (7)$$

$$\| (t)U(t,s) \| \leq \frac{C_\alpha}{(t-s)^\alpha} \quad 0 \leq t \leq b \quad (9)$$

For our convenience let us take $F(0, x(b_1(0)), \dots, x(b_m(0)), 0) = 0$.

Let $M_0 = \| (-A)^{-\beta}(t) \|$ assume the following conditions

$(H1)$ $F: [0, b] \times X^{m+1} \to X$ is a continuous functions and there exists a $\beta \in (0,1)$ and $L > 0$ such that the function $(-A)^\beta F$ satisfies the condition:

$$\| (-A)^\beta F(s_1, x_1, x_2, \dots, x_m, y) - (-A)^\beta F(s_1, x_1, x_2, \dots, x_m, y) \|$$
$$\leq L \max_{i=1,\dots,m} \| x_i - x_i \| + \| y - y \|$$

for any $0 \leq s_1, s_2 \leq b$, $x_i, x_i, y, y \in X$, $i = 1,2, \dots, m$.

Moreover, there exists constant $N > 0$ such that

$$\left\| \int_0^t (t,s,x) - (t,s,y)\,ds \right\| \leq N \| x - y \| \text{ for } t,s \in [0,b],\ x,y \in X$$

(H2) The function $G: [0, b] \times X^{n+1} \to X$ satisfies the following conditions:

(i) For each $t \in [0, b]$, the function $G(t, .) : X^{n+1} \to X$ is continuous and for each $(x_1, x_2, \dots, x_n, y) \in X^{n+1}$ the function $G(., x_1, x_2, \dots, x_n, y) : [0, b] \to X$ is strongly measurable.

(ii) For each positive number $k \in \mathrm{N}$, there is a positive function $g_r \in L^1(J)$ such that

$$\sup_{\|x_1\|, \dots, \|x_n\| \leq r} \| G(s_1, x_1, x_2, \dots, x_n, y) \| \leq g_r(t) \quad \text{and}$$

$$\lim_{r \to \infty} \frac{1}{r} \int_0^b g_r(s)\,ds = \gamma < \infty$$

(H3) $a_i, b_j \in C([0, b], [0, b])$, $i = 1,2, \dots, n+1$, $j = 1,2, \dots, m+1$.

(H4) There exist positive constants L_1 and L_2 such that

$$\| g(x) \| \leq L_1 \| x \|_\Omega + L_2 \text{ for all } x \in \Omega$$

and $g : \Omega \to X$ is completely continuous.

(H5) $I_k : X \to X$ is completely continuous and there exist continuous non decreasing functions $L_k : R_+ \to R_+$ such that for each $x \in X$.

$$\| I_k(x) \| \leq L_k(\| x \|), \quad \liminf_{r \to \infty} \frac{L_k(r)}{r} = \lambda_k < \infty$$

Definition 2.1

A family of linear operator $\{U(t, s) : 0 \leq s \leq t \leq b\}$ on X is called an evolution system if the following conditions hold:

(i) $U(t,s) \in B(X)$ the space of bounded linear transformation on X whenever $0 \leq s \leq t \leq b$ and for each $x \in X$ the mapping $(t, s) \to U(t, s)$ is continuous;

(ii) $U(t,s)U(s,\tau) = U(t,\tau)$ whenever $0 \leq \tau \leq s \leq t \leq b$.

Theorem 2.1(Sadovskii)

Let P be a condensing operator on a Banach space, that is, P is continuous and takes bounded sets into bounded sets, and let $\alpha(P(B)) \leq \alpha(B)$ for every bounded set B of X with $\alpha(B) > 0$ of $P(H) \subset H$ for a convex, closed and bounded set H of X, then P has fixed point in H (where $\alpha(.)$ denotes Kuratowski's measure of noncompactness).

3. EXISTENCE RESULTS

In order to define the solution of the problem $(6) - (8)$, we consider the following space

$\Omega = \{x : J \to X : x(t)$ *is continuous at* $t = t_k$ *and left continuous at* $t = t_k$ *and the right limit* $x(t_k^+)$ *exists for* $k = 1, \dots, m\}$

which is Banach space with norm

$$\| x \|_\Omega = sup_{t \in J} \| x(t) \|.$$

Definition 3.1

A continuous function $x(.) : [0, b] \to X$ is said to be a mild solution of the nonlocal Cauchy problem $(6) - (8)$, if the function $A(s)U(t,s)F\left(s, x(b_1(s)), \dots x(b_m(s)), \int_0^s (s, \tau, x(b_{m+1}(\tau)))\,d\tau\right)$, $s \in [0, b)$

is integrable on $[0, b)$ and the integral equation

$$x(t) = U(t,0)[x_0 - g(x)] - F(t, x(b_1(t)), \dots, x(b_m(t)), \int_0^t (t, s, x(b_{m+1}(s)))ds) - \int_0^t A(s)U(t,s)F(s, x(b_1(s)), \dots, x(b_m(s)), \int_0^s (s,\tau, x(b_{m+1}(\tau)))d\tau)ds + \int_0^t U(t,s)G(s, x(a_1(s)), \dots, x(a_n(s)), \int_0^t e(s,\tau, x(a_{n+1}(\tau)))d\tau)ds$$

$$+ \sum_{0<t_k<t} U(t, t_k) I_k(x(t_k)) \quad (10)$$

Theorem 3.1

If assumptions $H_1 - (H_5)$ are satisfied and $x_0 \in X$, then the nonlocal Cauchy problem $(6) - (8)$ has a mild solution provided that

$$L_0 = L(N+1)\left[M_0 + \frac{1}{\beta} C_{1-\beta} b^{\beta}\right] < 1 \quad (11)$$

and

$$M\left[L_2 + \gamma + \sum_{k=1}^{m} \lambda_k\right] + L(N+1)\left[M_0 + \frac{1}{\beta} C_{1-\beta} b^{\beta}\right] < 1 \quad (12)$$

Proof:

Let us write,

$$(t, x(b_1(t)), \dots, x(b_m(t)), \int_0^t (t, s, x(b_{m+1}(s)))ds) = (t, v(t))$$

and

$$(t, x(a_1(t)), \dots, x(a_n(t)), \int_0^t e(t, s, x(a_{n+1}(s)))ds) = (t, u(t))$$

Define the operator P on Ω by the formula

$$Px(t) = U(t,0)[x_0 - g(x)] - F(t, v(t)) - \int_0^t A(s)U(t,s)F(s, v(s)) + \int_0^t U(t,s)G(s, u(s))ds + \sum_{0<t_k<t} U(t, t_k) I_k(x(t_k)) \quad 0 \le t \le b$$

For each positive integer r, let

$$B_r = \{x \in \Omega : \|x(t)\| \le r, 0 \le t \le b\}.$$

Then for each r, B_r is clearly a bounded closed convex set in Ω. Since by (9) and (H1) the following relation holds:

$$\|A(t)U(t,s)F(s, v(s))\| = \|(-A)^{1-\beta}(t)U(t,s)(-A)^{\beta}(t)F(s, v(s))\|$$

$$\le \frac{C_{1-\beta}}{(t-s)^{\beta}} [L(N+1)r + LC_1 + C_2]$$

Where,
$C_1 = b \| (t, s, 0) \|$, $C_2 = \|(-A)^{\beta}(t)\| \|F(t, 0,0, \dots ,0,0)\|$

then from Bocher's theorem (22) it follows that $A(t)U(t,s)F(s, v(s))$ is integrable on $[0, b]$, so P is well defined on B_r. We claim that there exist a positive integer r such that $PB_r \subseteq B_r$. If it is not true, then for each positive integer r, there is a fuction $x(.) \in B_r$, but $Px_r \notin B_r$, that is $\|Px_r(t)\| > r$ for some $t(r) \in [0, b]$, where $t(r)$ denotes t is dependent of r. However, on the otherhand, we have

$$r < \|Px_r(t)\| = \|U(t,0)[x_0 - g(x_r)] - F(t, v_r(t)) - \int_0^t A(s)U(t,s)F(s, v_r(s))ds + \int_0^t U(t,s)G(s, u_r(s))ds + \sum_{0<t_k<t} U(t, t_k) I_k(x_r(t_k))\|$$

$$\le \|U(t,0)[x_0 - g(x_r)]\| + \|(-A)^{-\beta}(t)(-A)^{\beta}(t)F(t, v_r(t))\|$$

$$+ \int_0^t \|(-A)^{1-\beta}(s)U(t,s)(-A)^{\beta}(s)F(s, v_r(s))\|ds + \int_0^t \|U(t,s)G(s, u_r(s))\|ds + \sum_{0<t_k<t} \|U(t, t_k) I_k(x_r(t_k))\|$$

$$\le M[\|x_0\| + L_1 r + L_2] + M_0[L(N+1)r + LC_1 + C_2] + \int_0^t \frac{C_{1-\beta}}{(t-s)^{\beta}}[L(N+1)r + LC_1 + C_2]ds + M \int_0^t g_r(s)ds + M \sum_{k=1}^{m} L_k(r)$$

$$\le M[\|x_0\| + L_1 r + L_2] + M_0[L(N+1)r + LC_1 + C_2] + \frac{C_{1-\beta}}{\beta}[L(N+1)r + LC_1 + C_2]b^{\beta} + M\int_0^t g_r(s)ds + M \sum_{k=1}^{m} L_k(r)$$

Dividing on both sides by r and taking the lower limit as $r \to \infty$, we get

$$ML_2 + M_0 L(N+1) + \frac{C_{1-\beta}}{\beta} L(N+1) b^{\beta} + M\gamma + M \sum_{k=1}^{m} \lambda_k \ge 1$$

$$ML_2 + \gamma + \sum_{k=1}^{m} \lambda_k + L(N+1)\left(M_0 + \frac{C_{1-\beta}}{\beta} b^{\beta}\right) \geq 1$$

This contradicts (12). Hence for some positive integer r, $PB_r \subseteq B_r$.

Next we will show that the operator P has a fixed point on B_r, which implies eq $6-(8)$ has a mild solution. To this end, we decompose P as $P = P_1 + P_2$, where the operator P_1, P_2 are defined on B_r respectively, by

$$P_1 x\, t = -F\, t, v\, t - \int_0^t A\, s\, U\, t, s\, F\, s, v\, s$$

and

$$P_2 x\,(t) = U\, t, 0\, x_0 - g\, x + \int_0^t U(t,s)\, G\, s, u(s)\, ds + \sum_{0<t_k<t} U\, t, t_k\, I_k\, x\, t_k$$

for $0 \leq t \leq b$, and we will verify that P_1 is a contraction while P_2 is a compact operator.

To prove that P_1 is a contraction, we take $x_1, x_2 \in B_r$. Then for each $t \in [0, b]$ and by condition (H1) and (11), we have

$$\|(P_1 x_1)\, t - (P_2 x_2)\, t\| \leq \|F\, t, v_1\, t - F\, t, v_2\, t\|$$

$$+ \left\| \int_0^t A\, t\, U(t,s)\, F\, s, v\, s_1 - F\, s, v\, s_2\, ds \right\|$$

$$\leq \| -A^{-\beta}\, t - A^{\beta}\, t\, F\, t, v_1\, t - -A\beta(t)Ft,v2t \|$$

$$+ \left\| \int_0^t -A^{1-\beta}\, s\, U\, t, s - A^{\beta}\, s\, F\, s, v\, s_1 - -A\beta(s)Fs,v2sds \right\|$$

$$\leq M_0 L\, max_{i=1,2,\ldots,m} \|x_1\, s - x_2\, s\| + N\|x_1\, s - x2s\|$$

$$+ \int_0^t \frac{C_{1-\beta}}{(t-s)^{\beta}} \left(L\, max_{i=1,2,\ldots,m} \|x\, s_1 - x\, s_2\| + N\|x1s-x2s\| \right) ds$$

$$\leq M0Lsup_{0\leq s\leq b}\|x1s-x2s\| + Nsup_{0\leq s\leq b}\|x1s-x2s\|$$

$$+ \frac{1}{\beta} C_{1-\beta}\, b^{\beta}\, L\, sup_{0\leq s\leq b} \|x_1\, s - x_2\, s\| + Nsup_{0\leq s\leq b}\|x1s-x2s\|$$

$$\leq L(N+1)\left(M_0 + \frac{1}{\beta} C_{1-\beta}\, b^{\beta}\right) sup_{0\leq s\leq b} \|x\, s_1 - x2s\|$$

$$= L_0\, sup_{0\leq s\leq b} \|x_1\, s - x_2\, s\|$$

Thus,

$$\|P_1 x_1 - P_1 x_2\|_{\Omega} \leq L_0 \|x_1 - x_2\|_{\Omega}$$

So by assumption $0 < L_0 < 1$, we see that P_1 is contraction.

To prove that P_2 is compact, firstly we prove that P_2 is continuous on B_r. Let $\{x_n\}_{n=0}^{\infty} \subseteq B_r$ with $x_n \to x$ in B_r, then by H2(i) and H5

$(i)\ I_k, k = 1,2,\ldots,m$ is continuous

$(ii)\ G\, s, u_n\, s \to G\, s, u\, s, \quad n \to \infty$

Since

$$\|G\, s, u_n\, s - G\, s, u\, s\| \leq 2g_r(s)$$

By the dominated convergence theorem, we have

$$\|P_2 x_n - P_2 x\| = \sup_{0\leq t\leq b} \| U(t,0)\, g\, x_n - g(x)$$

$$+ \int_0^t U(t,s)\, G\, s, u\, s_n - G(s,u(s))ds$$

$$+ \sum_{0<t_k<t} U(t,t_k)\, I_k\, x_n\, t_k - Ikxtk\|$$

$$\leq M\| g\, x_n - g(x)\|$$

$$+ M\int_0^t \|G\, s, u\, s_n - G(s,u(s))\|\, ds$$

$$+ M \sum_{0<t_k<t} \|I_k\, x_n\, t_k - I_k\, x\, t_k\|$$

$\to 0$ as $n \to \infty$

ie, P_2 is continuous.

Next, we prove that $\{P_2 x : x \in B_r\}$ is a family of equicontinuous functions. To see this we fix $t_1 > 0$ and let $t_2 > t_1$, and $\epsilon > 0$ be enough small. Then

$$\|(P_2 x)(t_2) - (P_2 x)(t_1)\| \leq \|U\, t_2, 0 - U\, t_1, 0\|\, \|x_0 - g(x)\|$$

$$+ \int_0^{t_1-\epsilon} \|U\, t_2, s - Ut1,s\|\, \|G(s,u(s))\|\, ds$$

$$+ \int_{t_1-\epsilon}^{t_1} \|U\, t_2, s - Ut1,s\|\, \|G(s,u(s))\|\, ds$$

$$+ \int_{t_1}^{t_2} \|U\, t_2, s\|\, \|G(s,u(s))\|\, ds$$

$$+ \sum_{0<t_k<t_1} \|U\, t_2, t_k - Ut1,tk\|\, \|Ikxtk\|$$

$$+ \sum_{t_1<t_k<t_2} \|U\, t_2, t_k\|\, \|I_k\, x\, t_k\|$$

Noting that $\| G(s, u(s)) \| \leq g_r(s)$ and $g_r(s) \in L^1$, we see that $\| (P_2 x)(t_2) - (P_2 x)(t_1) \|$ tends to zero independently of $x \in B_r$ as $t_2 - t_1 \to 0$, since the compactness of $U(t, s)$ for $t - s > 0$, implies the continuity in the uniform operator topology. We can prove that the functions $P_2 x, x \in B_r$ are equicontinuous at $t = 0$. Hence P_2 maps B_r into a family of equicontinuous functions.

It remains to prove that $V\ t = (P_2 x)\ t : x \in B_r$ is relatively compact in X. $V(0)$ is relatively compact in X. Let $0 < t \leq b$ be fixed and $0 < \epsilon < t$. For $x \in B_r$, we define

$$(P_{2,\epsilon} x)(t) = U\ t, 0\ x_0 - g\ x + \int_0^{t-\epsilon} U(t,s)\ G\ s, u(s)\ ds$$

$$+ \sum_{0<t_k<t-\epsilon} U\ t, t_k\ I_k\ x\ t_k$$

$$= U\ t, 0\ x_0 - g\ x + U(t, t-\epsilon) \int_0^{t-\epsilon} U(t - \epsilon, s) G s, u(s) ds$$

$$+ U(t, t-\epsilon) \sum_{0<t_k<t-\epsilon} U\ t-\epsilon, t_k\ I_k\ x\ t_k$$

Then from the compactness of $U(t, s)$ for $t - s > 0$, we obtain $V_\epsilon t = (P_{2,\epsilon}\ x)\ t : x \in B_r$ is relatively compact in X for every ϵ, $0 < \epsilon < t$. Moreover, for every $x \in B_r$, we have

$$\| (P_2 x)\ t - (P_{2,\epsilon} x)\ t \| \leq$$

$$\int_{t-\epsilon}^{t} \| U(t,s)\ G\ s, u(s) \| ds$$

$$+ \sum_{t-\epsilon<t_k<t} \| U\ t, t_k \| \| I_k\ x\ t_k \|$$

$$\leq M \int_{t-\epsilon}^{t} g_r\ s\ ds + M \sum_{t-\epsilon<t_k<t} L_k(r)$$

Therefore, there ara relatively compact sets arbitrarily close to the set $V\ t$. Hence the set $V\ t$ is also relatively compact in X.

Thus, by Arzela-Ascoli theorem, P_2 is a compact operator. Those arguments enable us to conclude that $P = P_1 + P_2$ is a condensing map on B_r, and by the fixed point theorem of Sadovskii there exists a fixed point $x(.)$ for P on B_r. Therefore, the nonlocal Cauchy problem 6 $-$ (8) has a mild solution and the proof is completed.

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