

RESEARCH ARTICLE

FIBONACCI MEAN ANTI-MAGIC LABELING OF SOME GRAPHS

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ABSTRACT

In this paper, we introduced Fibonacci mean anti-magic labeling in graphs. A graph G with p vertices and q edges is said to have Fibonacci mean anti-magic labeling if there is an injective function $f: E(G) \rightarrow F_j$, ie, it is possible to label the edges with the Fibonacci number F_j where $(j= 0,1,1,2\dots n)$ in such a way that the edge uv is labeled with

$$\begin{aligned} & \lfloor \frac{f_u + f_v}{2} \rfloor \text{ if } |f_u + f_v| \text{ is even,} \\ & \lfloor \frac{f_u + f_v + 1}{2} \rfloor \text{ if } |f_u + f_v| \text{ is odd and the resulting vertex labels admit mean} \end{aligned}$$

anti-magic labeling. In this paper, we discussed the Fibonacci mean anti-magic labeling for some special classes of graphs.

Keywords: Fibonacci mean labeling, circulant graph, Bistar, Petersen graph, Fibonacci mean anti-magic labeling.

AMS Subject Classification (2010): 05c78.

1. INTRODUCTION

The concept of Fibonacci labeling was introduced by David W. Bange and Anthony E. Barkauskas in the form Fibonacci graceful (1). The concept of skolem difference mean labeling was introduced by Murugan and Subramanian (2). Somasundaram and Ponraj have introduced the notion of mean labeling of graphs. Hartsfield and Ringel introduced the concept of anti-magic labeling which is an assignment of distinct values to different vertices in a graph that in such a way that when taking the sums of the labels, all the sums will be having different constants.

For various graph theoretic notations and terminology, we followed Gross and Yellen (3). Sridevi *et al.* (4) proved the path and cycle graphs are Fibonacci divisor cordial graphs. A dynamic survey of graph labeling is updated by Gallian (5). Rokad and Ghodasara (6) proved that Fibonacci cordial labeling exists for some special graphs. In this paper, we have discussed different families of graphs which satisfy the conditions of Fibonacci mean anti-magic labeling.

Definition 1.1.

Fibonacci number can be defined by the linear recurrence relation $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$ where $F_0 = 0, F_1 = 1$. This generates the infinite sequence of integers in the form $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots$

Definition 1.2.

A graph G with p vertices and q edges admits mean anti-magic labeling if there is an injective function f from the edges $E(G) \rightarrow \{0, 1, 1, \dots, q\}$ such that when each uv is labeled with

$$\begin{aligned} & \lfloor \frac{f_u + f_v}{2} \rfloor \text{ if } |f_u + f_v| \text{ is even and} \\ & \lfloor \frac{f_u + f_v + 1}{2} \rfloor \text{ if } |f_u + f_v| \text{ is odd} \end{aligned}$$

then the resulting vertices are distinctly labeled.

Note: A graph which admits mean anti-magic labeling is called mean anti-magic graph.

Definition 1.3.

A graph G is called anti-magic if the q edges of G can be distinctly labeled in such a way that when taking the sum of the edge labels incident to each vertex, they all will have different (distinct) constants.

2. RESULTS

Theorem 2.1.

The circulant graph C_n ($n \geq 6$) admits Fibonacci mean anti-magic labeling with the generating set $(1, 2)$.

Proof:

Let $G = C_n(1, 2)$ be the 4-regular graph with ($n \geq 6$).

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We define the labeling function $f: E(G) \rightarrow F_j$ where $(j=0,1,1\dots n)$

Then apply mean labeling for the edges so that the sum of the labels of the vertices are all distinct.

Thus, the above labeling pattern gives rise to a Fibonacci mean anti-magic labeling on the given graph $G = C_n(1,2)$.

Example 2.2.

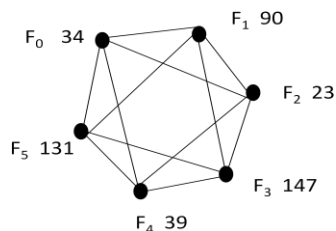


Fig. 1. Fibonacci mean anti-magic labeling of circulant graph C_6

Theorem 2.3.

Petersen graph admits Fibonacci mean anti-magic labeling.

Proof.

Petersen graph is a three regular graph with 10 vertices and 15 edges.

Let $u_0, u_1, u_2 \dots u_{14}$ be the edges and

Let $v_0, v_1, v_2 \dots v_9$ be the vertices of Petersen graph.

We define the labeling function $f: E(G) \rightarrow F_j$ where $(j=0,1,1\dots n)$ such that each uv is labeled with $\frac{|f u + f v|}{2}$ if $|f u + f v|$ is even and $\frac{|f u + f v| + 1}{2}$ if $|f u + f v|$ is odd then the resulting vertices are distinctly labeled.

By applying the above mean labeling to the edges, we obtained the sum of the vertex labels are all distinct (different constants).

Hence Petersen graph admits Fibonacci mean anti-magic labeling.

Example 2.4.

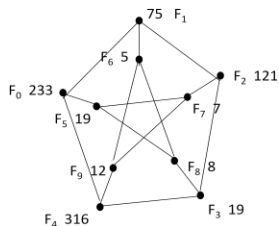


Fig. 2. Fibonacci mean anti-magic labeling of Petersen graph.

Theorem 2.5.

The Wheel graph W_n admits Fibonacci mean anti-magic labeling.

Proof:

Let $u_0, u_1, u_2 \dots u_{2n}$ be the edges of W_n and

Let $v_0, v_1, v_2 \dots v_n$ be the vertices of the Wheel graph W_n .

We defined the labeling function $f: E(G) \rightarrow F_j$ where $(j=0,1,1\dots n)$ such that each uv is labeled with $\frac{|f u + f v|}{2}$ if $|f u + f v|$ is even and $\frac{|f u + f v| + 1}{2}$ if $|f u + f v|$ is odd then the resulting vertices are distinctly labeled.

By applying mean labeling to the edges of W_n we obtained the sum of the vertex labels are all distinct (different constants).

Hence the Wheel graph W_n admits Fibonacci mean anti-magic labeling.

Example 2.6:

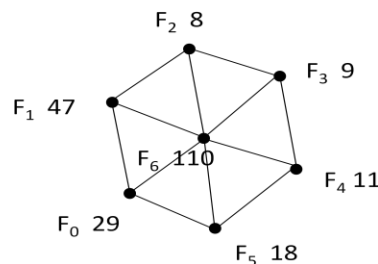


Fig. 3. Fibonacci mean anti-magic labeling of Wheel graph W_6 .

Theorem 2.7.

Bistar B_m admits Fibonacci mean anti-magic labeling.

Proof:

Let $v_{1,0}$ and $v_{2,0}$ be the apex (central) vertices of $B_{n,n}$.

Let $v_{1,1} \dots v_{1,n}$ be the pendent vertices adjacent to the vertex $v_{1,0}$.

Let $v_{2,1} \dots v_{2,n}$ be the pendent vertices adjacent to the vertex $v_{2,0}$.

Let u_0 be the edge of the two apex vertices.

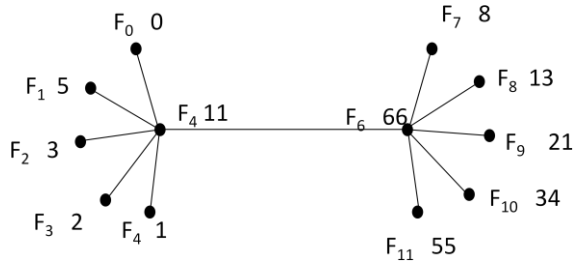
Let $u_1, u_2 \dots$ be the edges of all the pendent vertices.

We defined the labeling function $f: E(G) \rightarrow F_j$ where $(j=0,1,1\dots n)$ such that each uv is labeled with $\frac{|f u + f v|}{2}$ if $|f u + f v|$ is even and

$\frac{|f u + f v| + 1}{2}$ if $|f u + f v|$ is odd then the resulting vertices are distinctly labeled.

Hence Bistar $B_{n,n}$ admits Fibonacci mean anti-magic labeling.

Example 2.8.



3. CONCLUSION

In this paper, we have obtained Fibonacci mean anti-magic labeling for the circulant graph, the Wheel graph, Petersen graph and the Bistar. Further study on some more special graphs is under progress.

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