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RESEARCH ARTICLE

DOMINATION AND TOTAL DOMINATION IN INTUITIONISTIC TRIPLE LAYERED SIMPLE FUZZY GRAPH

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ABSTRACT

In this paper, we discussed domination and total domination in Intuitionistic fuzzy graph. We determined the domination number γ and total domination number γ_t in Intuitionistic Triple Layered Simple fuzzy graph (ITLFG) and also verified the existence of 2-domination in intuitionistic Triple Layered simple fuzzy graph.

Keywords: Intuitionistic Triple Layered Simple fuzzy graphs, Domination, Total domination, 2- Domination.

1. INTRODUCTION

Atanassov introduced the concept of intuitionistic fuzzy graphs (1). Parvathi and Karunambigai are also studied the concept of intuitionistic fuzzy graphs and its properties (2). Some of the properties of intuitionistic fuzzy graphs are introduced by Nagoorgani and Shajitha Begum (3).

Pathinathan and Jesintha Roseline defined the Triple Layered fuzzy graph and its properties (4,5). In this paper, Jethurth Emelda Mary and Ameenalbibi introduced the concept of domination γ and total domination γ_t in Intuitionistic Triple Layered Simple fuzzy graph (ITLFG) under certain conditions and illustrated with some examples.

2. PRELIMINARIES

In this section, we presented some of the basic definitions.

2.1. Fuzzy graph

A fuzzy graph G is a pair of functions G: (σ,μ) where σ is a fuzzy subset of a non-empty vertex set S and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ,μ) is denoted by G*: (σ*,μ*)

2.2. Complement of Fuzzy graph

The Complement of the fuzzy graph G: (σ,μ) is a fuzzy graph $G = (\sigma, \mu)$ where $\sigma = \sigma$ and $\mu\ u, v\ = 0\ if$ μ u, , v > 0 and $\mu\, u, v = \sigma(u) \wedge \sigma(v)$ otherewise.

2.3. Intuitionistic Triple Layered Fuzzy Graph (ITLFG)

Let G: v_i , μ_1, ν_1 , e_{ij} , μ_2, ν_2 be an intuitionistic fuzzy graph with the underlying crisp graph G^*: σ^*, μ^* . The pair $TL\ G$: v_i , μ_{TL_1} , ν_{TL_1} , e_{ij} , μ_{TL_2} , ν_{TL_2} is called the Intuitionistic Triple Layered Fuzzy graph and is defined as follows. The Vertex set of ITL (G) be μ_{TL_1}, ν_{TL_1} . and The fuzzy subset μ_{TL_1}, ν_{TL_1} is defined as

$$\mu_{TL_1,} \gamma_{TL_1} = \begin{matrix} \mu_1 u\ , \nu_1 u\ if\ u \in \sigma^* \\ \mu_2 uv\ , \nu_2 uv\ if\ uv \in \mu^* \end{matrix}$$

Where, $0 \le \mu_{TL_1} + \nu_{TL_1} \le 1$.

The fuzzy relation $\mu_{TL_2,} \nu_{TL_2}\ on\ \sigma^* \cup \mu^* \cup \mu^*$ is defined as

$$\langle \mu_{TL_2,} \nu_{TL_2} \rangle = \begin{cases} \langle \mu_2(uv), \gamma_2(uv) \rangle & if\ u,v \in \sigma^* \\ \langle \mu_2(e_i) \wedge \mu_2(e_j), \nu_2(e_i) \vee \nu_2(e_j) \rangle & if\ the\ edges e_i and\ e_j\ have\ a\ vertex\ in\ common\ between\ them \\ \langle \mu_1(u_i) \wedge \mu_2(e_j), \nu_1(u_i) \vee \nu_2(e_i) \rangle & if\ u_i \in \sigma^*\ and\ e_i \in \mu^*\ and\ each\ e_i\ is\ incident\ with\ sigle\ u_i \\ & either\ clockwise\ or\ anticlockwise \\ 0 & otherwise \end{cases}$$

By definition $0 \le \mu_2\ uv\ + \nu_2(uv) \le 1$ for all (u,v) in $\sigma^* \cup \mu^* \cup \mu^*$. Here $\mu_{TL_2,} \nu_{TL_2}$ is a fuzzy relation on the fuzzy subset $\mu_{TL_1,} \nu_{TL_1}$

2.4. Domination in Intuitionistic Fuzzy Graph

Let $G = \sigma, \mu$ be an intuitionistic fuzzy graph on the set V. Let $x, y \in V$, we say that x dominates y in G if $\mu\ x, y = \sigma(x) \wedge \sigma(y)$ and $\mu\ x, y = \sigma\ x\ v \sigma\ y$. A Subset D of V is called a dominating set of G if for every vertex $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma\ G\ or\ \gamma$.

2.5. Total Domination in Intuitionistic Fuzzy Graph

Let $G = \sigma, \mu$ be an intuitionistic fuzzy graph without isolated vertices. A Subset S of V is said to be total dominating set if every vertex in V is dominated by a vertex in D.

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2.6. 2-Domination in Intuitionistic Fuzzy graph

The 2-Domination number of a fuzzy graph G denoted by $\gamma_2 G$, is the minimum cardinality of a 2-dominating set of G.

3. DOMINATION IN INTUITIONISTIC TRIPLE LAYERED FUZZY GRAPH

In this section, we introduced the Domination in Intuitionistic Triple Layered Fuzzy graph and illustrate with some examples.

3.1. Definition

Let G: v_i, μ_1, ν_1 , e_{ij}, μ_2, ν_2 be an intuitionistic fuzzy graph with the underlying crisp graph G^*: σ^*, μ^* . Let $ITL\ G : v_i, \mu_{TL_1}, \nu_{TL_1}$, e_{ij} , μ_{TL_2}, ν_{TL_2} be an Intuitionistic Triple Layered Fuzzy graph on the vertex set $(\sigma^* \cup \mu^* \cup \mu^*)$. Let $x, y \in (\sigma^* \cup \mu^* \cup \mu^*)$, we say that x dominates y in G if $\mu_{TL_1} x, y = \sigma_{TL}\, x \wedge \sigma_{TL}\, y$ and $\nu_{TL_1} x, y = \sigma_{TL}(x) \vee \sigma_{TL}(y)$. A subset D of $(\sigma^* \cup \mu^* \cup \mu^*)$ is called adominating set in G if every vertex $v \notin D$ there exists $u \in D$ such that u dominates v.

The minimum cardinality of the minimal dominating set in G is called the domination number of G and is denoted by $\gamma\ G\ or\ \gamma$.

Example 3.1.

Consider the Intuitionistic fuzzy graph G: (σ, μ) with n=3 vertices whose crisp graph is a cycle.

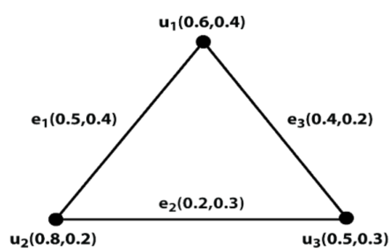

Fig. 1. Intuitionstic Fuzzy Graph$G$: (σ, μ)

Then the Intuitionistic Triple Layered Fuzzy graph is given by

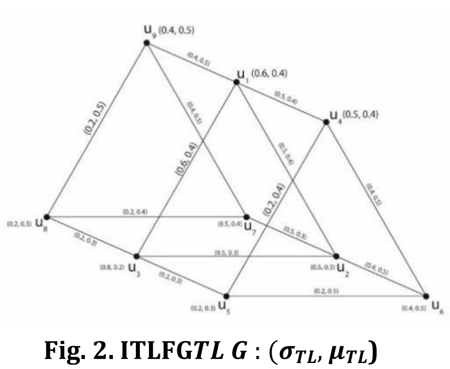

Fig. 2. ITLFG$TL\ G$: (σ_{TL}, μ_{TL})

The minimal cardinality of the minimal dominating set of the Intuitionistic Triple Layered fuzzy graph is $D = \{u_3, u_6, u_9\}$ is 3. ie) The domination number is $\gamma\ G = 3$

Theorem 3.1.

For any intuitionistic triple layered fuzzy graph ITL (G) $\gamma\ G + \gamma\ G \leq 2p$, where p – number of vertices and $p \geq 3$.

Proof:

Let G: v_i , μ_1, ν_1 , e_{ij} , μ_2, ν_2 be a cycle graph with p vertices and the vertex set of G be σ*.Let $ITL\ G$: v_i , μ_{TL_1} , ν_{TL_1} , e_{ij} , μ_{TL_2} , ν_{TL_2} be an intuitionistic triple layered fuzzy graph of G with 3p vertices and a vertex set of ITL(G) be $(\sigma^* \cup \mu^* \cup \mu^*)$.

Let D be the Dominating set of the Intuitionistic Triple Layered fuzzy graph. The minimuml cardinality of the minimal dominating set D is p.

Ie) $\gamma(G) = p$ ....(1)

Let ITL $(G$) be the complement of the intuitionistic triple layered fuzzy graph $ITL\ G$ and a vertex set of ITL $(G$) be $(\sigma^* \cup \mu^* \cup \mu^*)$.

Let D be the dominating set of the complement of the intuitionistic triple layered fuzzy graph. The minimum cardinality of the minimal dominating set D is$\leq p$.

$\gamma(G) \leq p$................................(2)

The sum of the dominating set $ITL\ G\ and$ ITL(G) is,

$$\gamma + \gamma \leq p + p$$

$$\leq 2p$$

$\gamma\ G + \gamma\ G \leq 2p$. (by (1) and (2))

Example: 3.2.

Consider the Intuitionistic fuzzy graph G: (σ, μ) with p = 3 vertices whose crisp graph is a cycle.

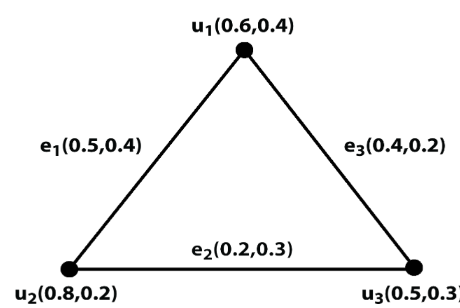

Fig. 3.1. Intuitionstic Fuzzy Graph$G$: (σ, μ)

The Intuitionistic Triple Layered Fuzzy graph of G with vertices $u_1 = (0.6,0.4)$, $u_2 = (0.8,0.2)$, $u_3=(0.5,0.3)$; and edges $u_1u_2 = (0.5,0.4)$, $u_2u_3 = (0.2,0.3)$ and $u_3u_1 = (0.4,0.3)$

The Intuitionistic Triple Layered Fuzzy graph is given by

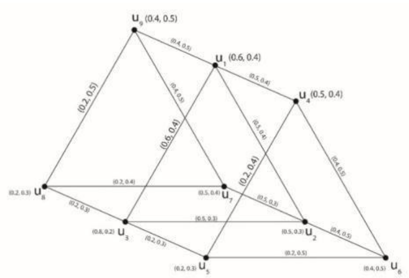

Fig. 3.2. ITL (G)

The minimal dominating set of an Intuitionistic Triple Layered fuzzy graph is

$$D = \{u_3, u_6, u_9\}$$

$$\gamma(G) = 3$$

The Complement of the Intuitionistic Triple Layered Fuzzy graph is given by

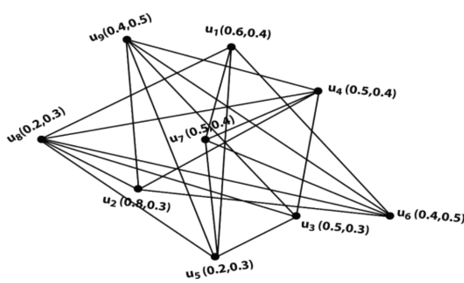

Fig. 3.3. ITL ($\boldsymbol{G}$)

The minimal dominating set of the complement of an Intuitionistic Triple Layered fuzzy graph is

$$D = \{u_1, u_7\}$$

$$\gamma(G\ ) = 2$$

The sum of the dominating set $ITL\ G\ and$ ITL(G) is $\gamma(G) + \gamma(G\ ) \leq 2p$.

$$\gamma(G) + \gamma(G\ ) = 2 + 3$$

$$= 5$$

$$2p\ = 2(3)$$

$$= 6$$

$$5 \leq 6.$$

$$\gamma(G) + \gamma(G\ ) \leq 2p.$$

Remark: 3.1.

Since there does not exist any set which satisfying the condition of 2-domination and so 2-Domination is not applicable for the Intuitionistic Triple Layered simple fuzzy graph.

4. TOTAL DOMINATION IN INTUITIONISTIC TRIPLE LAYERED FUZZY GRAPH

In this section, we introduced the Total Domination in Intuitionistic Triple Layered Fuzzy graph and illustrated with some example.

Definition: 4.1.

Let G: v_i , μ_1, v_1 , e_{ij} , μ_2, v_2 be an intuitionistic fuzzy graph without isolated vertices the underlying crisp graph G^*: σ^*, μ^* . Let $ITL\ G$: v_i , μ_{TL_1} , v_{TL_1} , e_{ij} , μ_{TL_2} , v_{TL_2} be an Intuitionistic Triple Layered Fuzzy graph on the vertex set $(\sigma^* \cup \mu^* \cup \mu^*)$. A subset $D_t$of $(\sigma^* \cup \mu^* \cup \mu^*)$ is said to be a total dominating set if every vertex in $\sigma^* \cup \mu^* \cup \mu^*$ is dominated by a vertex in D_t.

The minimum intuitionistic cardinality of the minimal total dominating set in G is called total domination number of G and is denoted by γ_t.

Theorem. 4.1.

For any Intuitionistic Triple Layered Simple Fuzzy graph ITL (G), the total domination number $\gamma_t(G) = p$ if and only if every vertex of ITL(G) has a unique neighbor.

Proof:

Let G: v_i , μ_1, v_1 , e_{ij} , μ_2, v_2 be a cycle graph with p vertices. A vertex set of G be σ*.

Let $ITL\ G$: $v_i, \mu_{TL_1}, v_{TL_1}$, $e_{ij}, \mu_{TL_2}, v_{TL_2}$ be an intuitionistic triple layered fuzzy graph of G with 3p vertices. A vertex set of ITL(G) be$(\sigma^*\ \cup \mu^* \cup \mu^*)$.

Let D_t be the total Dominating set of the Intuitionistic Triple Layered fuzzy graph which is the minimum cardinality of the minimal dominating set $D_t\, of\ ITL(G)$ is p.

$$\gamma_t\,(G) = p$$

Conversely,

Suppose $\gamma_t\,(G) = p$. If there exists a vertex v with two neighbors x and y then V – {x} is a total dominating set of G so that $\gamma_t(G) < p$ which is a contradiction.

Example:

Consider the Intuitionistic fuzzy graph G: (σ, μ) with p = 5 vertices whose crisp graph is a cycle.

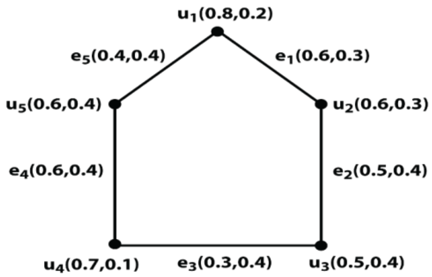

Fig. 4.1. Intuitionstic Fuzzy Graph$G$: (σ, μ)

The Intuitionistic Triple Layered Fuzzy graph is given by

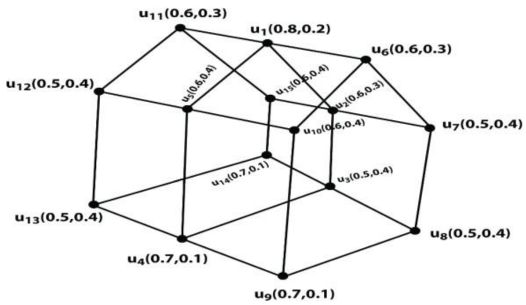

Fig. 4.2. ITL(G)

The Total dominating set is $D_t = \{u_1, u_2, u_3, u_4, u_5\}$

The minimum cardinality of the minimal total dominating set $\gamma_t(G) = 5$

5. CONCLUSION

In this paper, the Domination and the total domination in Intuitionistic Triple layered fuzzy graph is found under certain conditions and illustrated with some example. This work further can be extended to any simple Intuitionistic Triple Layered Fuzzy graph.

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