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RESEARCH ARTICLE

FUZZY ANTI-MAGIC LABELING ON SOME GRAPHS

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ABSTRACT

In this Paper, we introduced the concept of fuzzy anti-magic labeling in graphs. We defined Fuzzy Anti-Magic Labeling (FAML) for Cycle, Star, Path and Antiprism graphs. A fuzzy graph G: (σ, μ) is known as fuzzy anti-magic graph if there exists two bijective functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ with the property that the sum of the edge labels incident to each vertex, the sums will all be different. We investigated and verified that fuzzy Cycle graphs, fuzzy Star graphs, fuzzy Path graphs and fuzzy antiprism graphs admits fuzzy anti-magic labeling.Further some properties related to fuzzy bridge and fuzzy cut vertex have been discussed.

Keywords: Fuzzy Anti-Magic labeling, FAM Cycle, FAM Star, FAM Path, FAM Antiprism.

AMS Mathematical Subject Classification: 03E72, 05C72, 05C78, 05C38.

1. INTRODUCTION

We begin with a finite, connected and undirected graph G: (σ, μ)without loops and multiple edges. Throughout this paper $\sigma(G)$ and $\mu(G)$ denote the vertices and edges respectively. In recent years, graph theory has been actively implemented in the fields of Bio-chemistry, Electrical engineering, Computer science, Algebra, Topology and Operations Research. A Mathematical background to describe the phenomenon of uncertainty in real life situation has been suggested by Zadeh (1). The theory of fuzzy graphs was independently developed by Rosenfeld, Yeh and Bang. Fuzzy graph theory is finding extensive applications in modeling real time systems where the level of information congenital in the system varies with different levels of precision. Nagoorgani *et al.* (2) introduced the concept of fuzzy magic labeling and properties of fuzzy labeling. Akram *et al.* introduced interval valued fuzzy graphs, Strong intuitionistic fuzzy graphs, m-polar fuzzy graphs and novel properties of fuzzy labeling graphs (3).

Already we published two articles in fuzzy Bi-magic labeling and Interval valued fuzzy Bi-magic labeling (4,5). In this paper, we introduced the concept of Fuzzy anti-magic labeling on some standard graphs.

2. PRELIMINARIES AND OBSERVATIONS

Let U and V be two non-empty sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of UxV. A fuzzy graph G: (σ, μ) is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$where for all u,v $\in V$, we have $\mu(u, v) < \sigma(u) \wedge \sigma(v)$. A graph G: (σ, μ) admits fuzzy labeling and if the mapping σ: $V \to [0,1]$ and μ: $V \times V \to [0,1]$ are bijection such that the membership values of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all u,v $\in V$. Let $G:(\sigma, \mu)$ be a fuzzy graph. The strong degree of a vertex v is defined as the sum of membership values of all strong neighbours of v, then

$$d_s(v) = \sum_{u \in N_s(v)} \mu(u, v).$$

An edge uv is called a fuzzy bridge of G, if its removal reduces the strength of connectedness between some pair of vertices in G. Equivalently (u,v) is a fuzzy bridge iff there are nodes x,y such that (u,v) is a arc of every strongest x-y path.A vertex is a fuzzy cutvertex of $G:(\sigma, \mu)$ if removal of it reduces the strength of connectedness between some pair of vertices in G. A fuzzy graph admits anti-magic labeling,if the sum of the edge labels incident to each vertex, the sums all will be different and it is denoted by $Am_f(G)$. A fuzzy graph which admits an anti-magic labeling is called Fuzzy anti-magic labeling graphs.

3. RESULTS

Definition 3.1.

A Cycle or Circulant graph is a graph that consists of a single cycle. The number of vertices in a cycle graph C_n equals the number of edges and every vertex has degree 2.

A Cycle graph which admits fuzzy labeling is called a fuzzy labeling Cycle graph and anti-magic

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labeling exists then it is called a fuzzy anti-magic labeling cycle graph and it is denoted by $Am_0 C_n$.

Example 3.2.

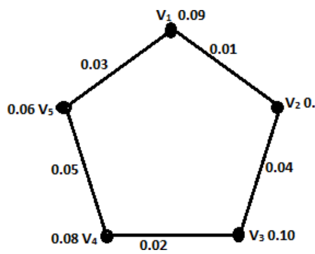

Fig. 1. $\boldsymbol{Am_0C_5}$.

Theorem 3.3.

If n is odd, then the cycle C_n admits a fuzzy anti-magic labeling.

Proof:

Let G be a cycle with odd number of vertices and $v_1, v_2, v_3, \dots v_n$ and $v_1v_2, v_2v_3, \dots v_nv_1$ be the vertices and edges of C_n respectively. Let $z \to [0,1]$ such that one can choose $z = 0.01$ if $n \ge 3$.

The fuzzy labeling is defined as follows:

$$\sigma(v_{2i}) = (n+1+i)z \quad \text{for} \quad 1 \le i \le \frac{n-1}{2}$$

$$\sigma(v_{2i-1}) = Max\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} + i(z) \quad \text{for} \quad 1 \le i \le \frac{n-1}{2}$$

$$\sigma(v_{2i-1}) = Min\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} - i(z) \quad \text{for} \quad i \le \frac{n+1}{2}$$

$$\mu(v_{n-i}, v_{n-i+1}) = \frac{1}{2} Max\{\sigma(v_i) / 1 \le i \le n\} \quad \text{for} \quad i = 1.$$

$$\mu(v_{n-i}, v_{n-i+1}) = \frac{1}{2} Max\{\sigma(v_i) / 1 \le i \le n\} - \{i(z) / 1 \le i \le n\}$$

for $i = 3,5,\dots.$

$$\mu(v, v_n) = \frac{1}{2} Min\{\sigma(v_i) / 1 \le i \le n\}$$

$$\mu(v_{n-i}, v_{n-i+1}) = \{\mu(v, v_n) - i(z) / 1 \le i \le n\} \quad \text{for}$$

i=2,4,.....

Here, we investigated the results for fuzzy anti-magic cycle $\widetilde{Am}_0(C_7)$ for n=7.

Case (i): i is even

Then i=2x for any positive integer x.

For each edge v_i, v_{i+1}, the fuzzy anti-magic labelings are as follows:

Subcase (i):

i=2x for any positive integer $\left(x \le \frac{n-5}{2}\right)$

$$\widetilde{Am}_0(C_7) \quad \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$$

$$\sigma(v_{2x}) + \mu(v_{2x}, v_{2x+1}) + \sigma(v_{2x+1})$$

$$= \sigma(v_2) + \mu(v_2, v_3) + \sigma(v_3)$$

$$\left\{(n+1+x)z / 1 \le i \le \frac{n-1}{2}\right\} + \frac{1}{2} Min\{\sigma(v_i) / 1 \le i \le n\} + (n-4-2x)z + Max\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} + (x+1)z$$

$$(2n-2)z + \frac{1}{2} Min\{\sigma(v_i) / 1 \le i \le n\} + Max\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\}$$

Subcase (ii):

i=2x for any positive integer $\left(x \le \frac{n-3}{2}\right)$

$$\widetilde{Am}_0(C_7) \quad \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$$

$$= \sigma(v_4) + \mu(v_4, v_5) + \sigma(v_5)$$

$$\left\{(n+1+x)z / 1 \le i \le \frac{n-1}{2}\right\} + \frac{1}{2} Min\{\sigma(v_i) / 1 \le i \le n\} + (n-1-2x)z + Max\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\} + (x+1)z$$

$$(2n+1)z + \frac{1}{2} Min\{\sigma(v_i) / 1 \le i \le n\} + Max\left\{\sigma(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right\}$$

Subcase (iii):

i=2x for any positive integer $\left(x \le \frac{n-1}{2}\right)$

$$\tilde{A}m_0(C_7) = \sigma(v_6) + \mu(v_6, v_7) + \sigma(v_7)$$

$$= \left\{(n+1+x)z/1\le i\le \frac{n-1}{2}\right\} + \frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}+$$

$$(n-5-2x)z+Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+(x+1)z$$

$$(2n-3)z + \frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}+$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}$$

Case (i): i is odd

Then i=2x-1 for any positive integer x.

For each edge v_i, v_{i+1}, the fuzzy anti-magic labelings are as follows:

Subcase (i):

i=2x-1 for any positive integer $\left(x \le \frac{n-5}{2}\right)$

$$\tilde{A}m_0(C_7) = \sigma(v_i)+\mu(v_i,v_{i+1})+\sigma(v_{i+1})$$

$$= \sigma(v_{2x-1})+\mu(v_{2x-1},v_{2x})+\sigma(v_{2x})$$

$$= \sigma(v_1)+\mu(v_1,v_2)+\sigma(v_2)$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+(x+1)z+$$

$$\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}-$$

$$(2x+1)z/1\le i\le n-1+\left\{(n+x)z\right\}$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}+nz$$

Subcase (ii):

i=2x-1 for any positive integer $\left(x \le \frac{n-3}{2}\right)$

$$\tilde{A}m_0(C_7) = \sigma(v_i)+\mu(v_i,v_{i+1})+\sigma(v_{i+1})$$

$$= \sigma(v_3)+\mu(v_3,v_4)+\sigma(v_4)$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+(x+1)z+$$

$$\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}-$$

$$(2x-2)z/1\le i\le n-1+\left\{(n+x)z\right\}$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}+(n+3)z$$

Subcase (iii):

i=2x-1 for any positive integer $\left(x \le \frac{n-1}{2}\right)$

$$\tilde{A}m_0(C_7) = \sigma(v_i)+\mu(v_i,v_{i+1})+\sigma(v_{i+1})$$

$$= \sigma(v_{2x-1})+\mu(v_{2x-1},v_{2x})+\sigma(v_{2x})$$

$$= \sigma(v_5)+\mu(v_5,v_6)+\sigma(v_6)$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+(x+1)z+$$

$$\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}-$$

$$(2x-5)z/1\le i\le n-1+\left\{(n+x)z\right\}$$

$$= Max\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}+(n+6)z$$

Subcase (iv):

i=2x-1 for any positive integer $\left(x \le \frac{n+1}{2}\right)$

$$\tilde{A}m_0(C_7) = \sigma(v_7)+\mu(v_7,v_1)+\sigma(v_1)$$

$$= Min\left\{\sigma(v_{2i})/1\le i\le \frac{n-1}{2}\right\}+(x+1)z+$$

$$\frac{1}{2}Min\left\{\sigma(v_i)/1\le i\le n\right\}-$$

$$(2x-3)z/1\le i\le n-1+\left\{(n+x)z\right\}$$

$$Min\left\{\sigma(v_{2i})/1\le i\le\frac{n-1}{2}\right\}+$$

$$=\frac{1}{2}Min\{\sigma(v_i)/1\le i\le n\}+(n+4)z$$

Therefore, from the above cases, we verified that C_n is a fuzzy anti-magic graph if C_n has odd number of vertices.

Definition 3.4.

A fuzzy Star graph consists of two vertex sets V and U with $|V| = 1$ and $|U| > 1$ such that $\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0$ for all $1 \le i \le n$.

In a fuzzy Star graph, if an anti-magic labeling exists then it is called a fuzzy anti-magic labeling Star graph and it is denoted by $\widetilde{Am}_0(S_{1,n})$.

Example 3.5.

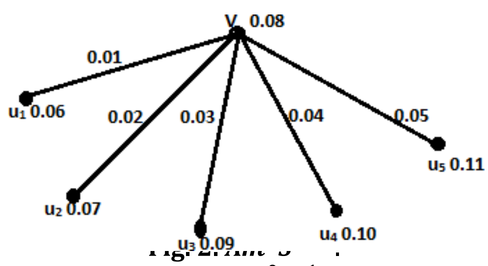

Fig. 2: $\widetilde{Am}_0(S_{1,n})$

Theorem 3.6.

For n ≥ 3, the Star graph $S_{1,n}$ admits a fuzzy anti-magic labeling.

Proof:

Let $S_{1,n}$ be the Star graph with $v, u_1, u_2, \ldots u_n$ as vertices and $vu_1, vu_2, \ldots vu_n$ as edges. Let $z \to [0,1]$ such that one can choose z=0.01 if n ≥ 3. The fuzzy labeling is defined as follows:

$\sigma(u_i) = (n+i)z$ for i=1,2

$\sigma(u_i) = [(n+i)+1]z$ for $3 \le i \le n$

$\sigma(v) = \sum \frac{\sigma(u_i)}{n}$ for $1 \le i \le n$

$\mu(v,u_1) = Max\{\sigma(v),\sigma(u_1)\} - Min\{\sigma(v),\sigma(u_1)\} - z$ for i=1

$\mu(v,u_2) = Max\{\sigma(v),\sigma(u_2)\} - Min\{\sigma(v),\sigma(u_2)\} + z$ for i=2

$\mu(v,u_i) = Max\{\sigma(v),\sigma(u_i)\} - Min\{\sigma(v),\sigma(u_i)\} + 2z$ for $3 \le i \le n$

Here, we investigated the results for fuzzy anti-magic labelings of $\widetilde{Am}_0(S_{1,n})$.

Case (i):

$$\widetilde{Am}_0(S_{1,n}) = \sigma(v) + \mu(v,u_1) + \sigma(u_1)$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_1)/i=1\} - Min\{\sigma(v),\sigma(u_1)/i=1\} - z + (n+1)z.$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_1)/i=1\} - Min\{\sigma(v),\sigma(u_1)/i=1\} + nz$$

Case (ii):

$$\widetilde{Am}_0(S_{1,n}) = \sigma(v) + \mu(v,u_2) + \sigma(u_2)$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_1)/i=2\} - Min\{\sigma(v),\sigma(u_1)/i=2\} + z + (n+2)z.$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_1)/i=2\} - Min\{\sigma(v),\sigma(u_1)/i=2\} + (n+3)z$$

Case (iii):

$$\widetilde{Am}_0(S_{1,n}) = \sigma(v) + \mu(v,u_i) + \sigma(u_i)$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_i)/3\le i\le n\} - Min\{\sigma(v),\sigma(u_i)/3\le i\le n\} + 2z + \{(n+i)+1\}z.$$

$$= \left\{\sum\frac{\sigma(u_i)}{n}/1\le i\le n\right\} + Max\{\sigma(v),\sigma(u_i)/3\le i\le n\} - Min\{\sigma(v),\sigma(u_i)/3\le i\le n\} + \{(n+3+i)z/3\le i\le n\}.$$

Definition 3.7.

A Path with atleast two vertices is connected and has two terminal vertices (vertices that have degree 1) while all others (if any) have degree 2.

In a graph which admits fuzzy labeling is called a fuzzy path graph and anti-magic labeling exists then it is called as a fuzzy anti-magic Path graph.

Example: 3.8

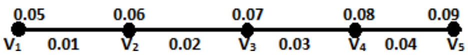

Fig. 3. $\tilde{Am}_0\, P_5$.

Theorem: 3.9

For n ≥ 3, the Path graph P_n has (n-1) fuzzy anti-magic labeling.

Proof:

Let P be a Path with length n ≥ 1 and $v_1, v_2, v_3, \ldots v_n$ and $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$ are the vertices and edges of P.

Let $z \to [0,1]$ such that one can choose z=0.01 if n ≥ 3. If the length of the path P is Odd, then the fuzzy labeling is defined as follows:

$$\sigma(v_{2i-1}) = (n+2i-2)z \quad \text{for } 1 \le i \le \frac{n+1}{2}$$

$$\sigma(v_{2i}) = Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + (2i-1)z$$

for $1 \le i \le \frac{n+1}{2}$

$$\mu(v_i, v_{i+1}) = Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} - (n-i-1)z \quad \text{for } 1 \le i \le n$$

Here, we investigated the results for fuzzy anti-magic labeling of Path graph for n=5.

Case (i): i is even

Then i=2x for any positive integer x

For each edge v_i, v_{i+1}

Subcase (i):

i=2x for any positive integer $\left(x \le \frac{n-3}{2}\right)$

$$\tilde{Am}_0(P_5) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$$

$$= \sigma(v_2) + \mu(v_2, v_3) + \sigma(v_3)$$

$$= Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + (2n-x)z + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} - (n+x+2)z + (n+2x-2)z$$

$$= Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} + (2n-4)z$$

Subcase (ii):

i=2x for any positive integer $\left(x \le \frac{n-1}{2}\right)$

$$\tilde{Am}_0(P_5) = \sigma(v_4) + \mu(v_4, v_5) + \sigma(v_5)$$

$$= Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} + (2n+2)z$$

Case (ii): i is odd

Then i=2x for any positive integer x

For each edge v_i, v_{i+1}

Subcase (i):

i=2x-1 for any positive integer $\left(x \le \frac{n-3}{2}\right)$

$$Am_0(P_5) = \sigma(v_i) + \mu(v_i, v_{i+1}) + \sigma(v_{i+1})$$

$$= \sigma(v_{2x-1}) + \mu(v_{2x-1}, v_{2x}) + \sigma(v_{2x})$$

$$= \sigma(v_1) + \mu(v_1, v_2) + \sigma(v_2)$$

$$= (n-2x)z + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} - (n+x+2)z + Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + (n-x)z$$

$$= (n-2)z + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} + Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\}$$

Subcase (ii):

i=2x-1 for any positive integer $\left(x \le \frac{n-1}{2}\right)$

$$\tilde{Am}_0(P_5) = \sigma(v_3) + \mu(v_3, v_4) + \sigma(v_4)$$

$$= (n-2x)z + Max\{\sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} - (n+x+2)z + Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\} + (n-x)z$$

$(n+4)z + Max\{\sigma(v_i)/1 \le i \le n-1\} -$

$= Min\{\sigma(v_i)/1 \le i \le n-1\} +$ If the length

$Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n+1}{2}\right\}$

of the path P is even then it has the following membership functions:

$\sigma(v_{2i-1}) = (n+2i-2)z$ for $1 \le i \le \frac{n}{2}$

$\sigma(v_{2i}) = Min\left\{\sigma(v_{2i-1})/1 \le i \le \frac{n}{2}\right\} + (2i-1)z$ for $1 \le i \le \frac{n}{2}$

$\mu(v_i, v_{i+1}) = \{Max\ \sigma(v_i)/1 \le i \le n-1\} - Min\{\sigma(v_i)/1 \le i \le n-1\} - (n-i-1)z$ for $1 \le i \le n$

By using the above membership functions, we can prove that the Path with even length also admits fuzzy anti-magic labeling.

Definition 3.10.

An Antiprism graph is a graph corresponding to the skeleton of an Antiprism. Antiprism graphs are therefore polyhedral and planar graphs.

The n-antiprism graph has 2n-vertices and 4n-edges and is isomorphic to the circulant graph $C_{i2n}(1,2)$.

An Antiprism graph is a special case of Circulant graph. In a graph which admits fuzzy labeling is called a fuzzy antiprism graph and anti-magic labeling exists then it is called as a fuzzy anti-magic antiprism graph.

Example 3.11.

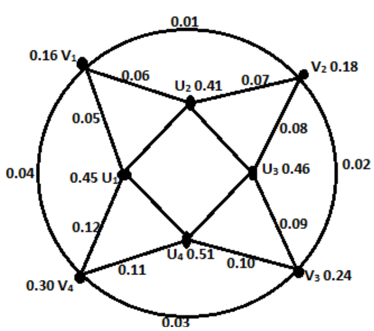

Fig. 4. $Am\ AP_0 ._4$

Theorem 3.12.

For n ≥ 2, the antiprism graph AP_n has 4n fuzzy antimagic labelings.

Proof:

Let AP_n be any n-sided Antiprism graph with 2n vertices and 4n edges.

It consists of two vertex sets U and V with $|U| > 1$ and $|V| > 1$ such that $\mu(u_i, v_i) > 0$ and $\mu(u_{i+1}, v_i) > 0$, $\mu(v_i, v_{i+1}) > 0$ and $\mu(u_i, u_{i+1}) > 0$.

Let $z \to [0,1]$ such that one can choose z=0.01 if n ≥ 3. The fuzzy labeling is defined as follows:

$\mu(u_i, v_i)$= (n+2i-1)z for i=1,2,…

$\mu(u_{i+1}, v_i)$ = (n+2i)z for i=1,2,…

$\mu(v_i, v_{i+1})$ = iz for i=1,2,…

$\mu(u_i, u_{i+1})$=(3n+j)z for i=1,3,…, j=1,2,… for n ≥ 2

$\mu(u_i, u_{i+1})$=(3n+j+2)z for i=2,4,…, j=1,2,… for 2 ≤ n ≤ 4

$\mu(u_i, u_{i+1})$=(3n+j+3)z for i=2,4,…, j=1,2,… for 5 ≤ n ≤ 6

$\mu(u_i, u_{i+1})$=(3n+j+4)z for i=2,4,…, j=1,2,… for 7 ≤ n ≤ 8

$\mu(u_i, u_{i+1})$=(3n+j+5)z for i=2,4,…, j=1,2,… for 9 ≤ n ≤ 10

and so on.

For vertex labeling,

$\sigma(u_i) = \mu(u_i, u_{i+1}) + \mu(u_i, u_n) + \mu(u_i, v_i) + \mu(u_i, v_n)$ for i=1

$\sigma(u_i) = \mu(u_i, u_{i-1}) + \mu(u_i, u_{i+1}) + \mu(u_i, v_i) + \mu(u_i, v_{i-1})$ for 2 ≤ i ≤ n-1

$\sigma(u_i) = \mu(u_i, u_{i-1}) + \mu(u_n, u_{n-i+1}) + \mu(u_n, v_n) + \mu(u_n, v_{i-1})$ for i = n

and

$\sigma(v_i) = \mu(v_i, v_{i+1}) + \mu(v_i, v_n) + \mu(v_i, u_i) + \mu(v_i, u_{i+2})$ for i=1 $\sigma(v_i) = \mu(v_i, v_{i-1}) + \mu(v_i, v_{i+1}) + \mu(v_i, u_i) + \mu(v_i, u_{i+1})$ for 2 ≤ i ≤ n-1 $\sigma(v_i) = \mu(v_i, v_{i-1}) + \mu(v_n, v_{n-i+1}) + \mu(v_n, u_n) + \mu(v_n, u_{i-1})$ for i=n

In view of the above labeling pattern, the edges are distinctly labeled in such a way that when taking the sum of the edge labels incident to each vertex, the sums will be different.

Hence, the Antiprism graph admits fuzzy anti-magic labelings.

4. PROPERTIES OF FUZZY ANTI-MAGIC GRAPHS

Proposition 4.1.

For every fuzzy anti-magic graph G, there exists atleast one fuzzy bridge.

Proposition 4.2.

Removal of a fuzzy cut vertex from a fuzzy anti-magic Star graph G, the resulting graph G^* also admits a fuzzy anti-magic labeling if $n \geq 4$.

Proof:

Since G is a Star graph, there exists atleast one fuzzy cut vertex. Now if we remove that fuzzy cut vertex from G, then it becomes a smaller Star G^*. However, G^* remains to be admit fuzzy anti-magic labeling if $n \geq 4$.

Hence, we conclude that removal of a fuzzy cut vertex from a fuzzy anit-magic Star graph results in a fuzzy anti-magic Star graph if $n \geq 4$.

Proposition 4.3.

Removal of a fuzzy cut vertex from a fuzzy anti-magic graph G such that G* is a path is also a fuzzy anti-magic graph.

Observation 4.4.

1. Every fuzzy anti-magic graph is a fuzzy labelled graph, but the converse is not true.
2. If G is a fuzzy anti-magic graph then $d(u) \neq d(v)$ for any pair of vertices $u, v \in V(G)$.
3. For all fuzzy anti-magic cycle graph G, there exists a subgraph G^* which is a cycle with odd number of vertices and there exists atleast one pair of vertices u and v such that $d_s(u) = d_s(v)$

5. CONCLUSION

In this paper, the concept of fuzzy anti-magic labeling has been introduced. Fuzzy anti-magic labeling for cycle, Star, Path and Antiprism graphs have been discussed.Properties of fuzzy anti-magic graphs are investigated. We further extend this study on some more special classes of graphs.

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