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RESEARCH ARTICLE

TWO-OUT DEGREE EQUITABLE DOMINATION IN THE MIDDLE, CENTRAL AND THE LINE GRAPHS OF P_N, C_N AND $K_{1,N}$

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ABSTRACT

Let G=(V,E) be a simple, finite, connected and undirected graph. A dominating set D of G is said to be two-out degree equitable dominating set if for any two vertices $u, v \in D$ such that $\left|od_D\ u \ - od_D(v)\right| \leq 2$, where $od_D\ u \ = \left|N\ v \ \cap (V-D)\right.$. The minimum cardinality of two -out degree equitable dominating set is called two- out degree equitable domination number and it is denoted by $\gamma_{2oe}(G)$. In this paper, we introduced the two-out degree equitable domination numbers in the middle, central and the line graphs of the path P_n, cycle C_n and star $K_{1,n}$ graphs.

Keywords: Two-out degree equitable domination number, Middle graph, Central graph, Line graph, Path P_n, Cycle C_n and Star graph $K_{1,n}$.

AMS Subject Classification: 05C9, 05C69, 05C70.

1. INTRODUCTION

The concept of domination was first studied by Ore and Berge (1962). A non-empty set $D \subseteq V$ is said to be a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality of the minimal dominating set D is = called the domination number and it is denoted by γ(G).

An equitable domination has interesting application in the context of social network. In a network, nodes with nearly equal capacity may interact with each other in a better way. In society, persons with nearly equal status, tend to be friendly. Ali Sahal and V.Mathad (Sahal,2013) introduced the concept of two out degree equitable domination in graphs. In this paper, we investigated the two out degree equitable domination number in the middle and the central graphs of P_n , C_n and $K_{1,n}$ graphs.

Definition 1.1 (8)

The middle graph of a connected graph G denoted by M(G) is the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if

(i) They are adjacent edges of G (or)

(ii) One is a vertex of G and the other is an edge incident with it.

Definition 1.2 (8)

For a given graph G=(V,E) of order n, the central graph C(G) is obtained, by subdividing each edge in E exactly once and joining all the non adjacent vertices of G. The central graph C(G) of a graph G is an example of a split graph, where a split graph is a graph whose vertex set V can be partitioned into two sets, V1 and V2, where each pair of vertices in V1 are adjacent, and no two vertices in V2 are adjacent.

Definition 1.3 (7)

A dominating set D in a graph G is called a two-out degree equitable dominating set if for any two vertices $u, v \in D$ such that $\left|od_D u - od_D(v)\right| \leq 2$, where $od_D u = \left|N\ V \cap V - D\right|$. The minimum cardinality of a two-out degree equitable dominating set is called the two-out degree equitable domination number of G and is denoted by γ_{2oe}(G).

In the consequent section, we obtained the two-out degree equitable domination number γ_{2oe}(G) of the middle graph of P_n, C_n and $K_{1,n}$ graphs.

Definition 1.4 (9)

Line graph L(G) of a graph G is defined with the vertex set E(G), in which two vertices are adjacent if and only if the corresponding edges are adjacent in G.

2. Two-out degree Equitable domination in the Middle graphs of P_n, C_n and $K_{1,n}$.

Example 2.1

Let G be the middle graph as in the figure. We obtained the two-out degree equitable domination number.

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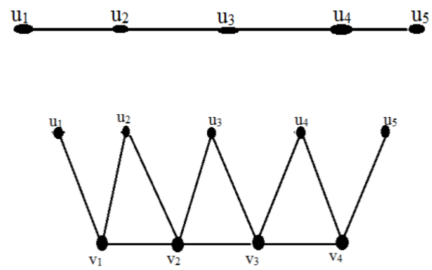


Middle graph of P_5 $M(P_5)$

Consider the set D= $\{v_1, v_3\}$.It is a dominating set and $V - D = \{v_2, v_4, u_1, u_2, u_3, u_4, u_5\}$

$od_D\, v_1 = |N\, v_1 \cap \{ v_2, v_4, u_1, u_2, u_3, u_4, u_5$

$= |\{u_1, u_2, v_2\} \cap \{ v_2, v_4, u_1, u_2, u_3, u_4, u_5\}|$

= 3

$od_D\, v_3 = |N\, v_3 \cap \{ V - D|$

$= |\{u_3, u_4, v_2, v_4\} \cap \{ v_2, v_4, u_1, u_2, u_3, u_4, u_5\}|$

= 4

From the above, any two vertices $u, v \in D$ such that $|od_D\, u - od_D\, (v)| \leq 2$.

Therefore $\{v_1, v_3\}$ is the two-out degree equitable dominating set with the minimum cardinality is 2. That is $\gamma_{2oe} M(P_5) = 2$.

Theorem2.2

For any Path P$_n$, $\gamma_{2oe}\ M(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$ $where\ n \geq 5$.

Proof

Let P_n: $u_1, u_2, \ldots . u_{n+1}$ be the path of length n and $u_i u_{i+1} = v_i$, By the definition of middle graph, $M(P_n)$ has the vertex set $V\ P_n \cup E\ P_n = u_i / 1 \leq i \leq n+1 \cup \{v_i / 1 \leq i \leq n\}$ in which each u_i is adjacent to v_i and v_i is adjacent to u_{i+1}. The vertices $u_1,\ v_1,\ v_2 \ldots .\ v_{2k},\ v_{2k-1}$ of $M(P_n)$ induces a path of length 4k.

Let $D = \{(v_{i+1}, v_{i+3} / i = 1), (v_{i,} v_{i+2} / i = 2), (v_{i-1}, v_{i+1}, v_{i+3} / i = 3), \ldots . \}$ be a dominating set of $M(P_n)$ and

$V - D = \{(v_{i,} v_{i+2} / i = 1), (v_{i-1}, v_{i+1}, v_{i+3} / i = 2), . \ldots \cup (u_i / 1 \leq i \leq n)\}$.

Now $v_{i+1} \in D$ then $od_D\, v_{i+1} = |N\, v_{i+1} \cap V - D|$ $for\ i = 1, od_D\, v_2 = |N\, v_2 \cap V - D|$

$= |\{(v_i, v_{i+2}, u_{i+1}, u_{i+2} / i = 1), (v_{i-1}, v_{i+1}, u_{i-1}, u_{i+1} / i = 2), \ldots \} \cap \{(v_{i,} v_{i+2} / i = 1), (v_{i-1}, v_{i+1}, v_{i+3} / i = 2), . \ldots \cup \quad (u_i / 1 \leq i \leq n)\}|$

$|\{(v_i, v_{i+2}, u_{i+1}, u_{i+2}), (v_{i-1}, v_{i+1}, u_{i-1}, u_{i+1}), \ldots . . \}|$
$= 4$

Then $|od_D(v_i) - od_D(v_{i+1})| \leq 2$, for any $v_i\, v_{i+1} \in D$. Therefore D is the minimum two-out degree equitable dominating set, Hence, $\gamma_{2oe}\ M(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$ $where\ n \geq 5$. .

Theorem 2.3

For any Cycle C$_n$, $\gamma_{2oe} M(C_n) = \left\lceil \frac{n}{3} \right\rceil + 1$ $where\ n \geq 4$.

Proof

Let $V\ C_n = \{u_1, u_2 \ldots . u_n\}$ and $E\ C_n = \{v_1, v_2 \ldots . v_n\}$ Where $v_i = u_i u_{i+1}\ 1 \leq i \leq n-1$, $v_n = u_n u_1$. By the definition of middle graph, $M(C_n)$ has the vertex set $V\ C_n \cup E\ C_n$ in which each v_i is adjacent to $v_{i+1}\ (i = 1,2 \ldots .\ n-1)$ and v_n is adjacent to u_1. In $M(C_n)$, $\{u_1, v_1, u_2, v_2 \ldots . v_{n-1}, u_1\}$ induces a cycle of length 2n. That is $|V\ M\ C_n| = 2n$ and $|E\ M\ C_n| = 3n$.

Let $D = \{(v_i, v_{i+2} / i = 1), (v_{i-1}, v_{i+1}, v_{i+3} / i = 2), . . . \}$ be a dominating set of $M\ C_n$ and

$V - D = (v_{i+1}, v_{i+3} / i = 1), (v_i, v_{i+2} / i = 2), \ldots . \cup \{ui / 1 \leq i \leq n\}$.

Now $v_i \in D$ then $od_D\ v_i = |N\ v_i \cap V - D|$

$for\ i = 1,\ \ od_D\ v_1 = |N\ v_1 \cap V - D|$

$= |\{(v_{i+1}, v_{i+3,} u_i, u_{i+1} / i = 1), (v_i, v_{i+3,} u_{i-1}, u_i / i = 2), \ldots . \} \cap (v_{i+1}, v_{i+3} / i = 1), (v_i, v_{i+2} / i = 2), \ldots . \cup \{u_i / 1 \leq i \leq n\}|$

$= |\{(v_{i+1}, v_{i+3,} u_i, u_{i+1} / i = 1), , (v_i, u_{i-1}, u_i / i = 2), \ldots . \}|$

= 3 or 4.

Then $|od_D(v_i) - od_D(v_{i+1})| \leq 2$, for any $v_i\, v_{i+1} \in D$. Therefore D is the minimum two- out degree equitable dominating set. Hence, $\gamma_{2oe}\ M(C_n) = \left\lceil \frac{n}{3} \right\rceil + 1$ $where\ n \geq 4$..

Theorem 2.4

For any Star graph $K_{1,n}$, $\gamma_{2oe} M(K_{1,n}) = n$.

Proof

Let $V\ K_{1,n} = \{u, u_1, u_2 \ldots . u_n\}$ and $E(K_{1,n}) = \{v_1, v_2 \ldots . v_n\}$.By the definition of middle graph, we have $V\ M\ K_{1,n} = u \cup v_i / \leq i \leq n \cup u_i / \leq i \leq n$ in which the vertices $v_1, v_2 \ldots . v_n, u$

induces clique of order $n+1$. Let $D = \{v_i / 1 \leq i \leq n\}$ be the dominating set of $M(K_{1,n})$ and $V - D = (u, u_i)/1 \leq i \leq n$.

Now $v_i \in D$ then $od_D\, v_i = \left| N\, v_i \cap V - D \right|$ $for\ i = 1, od_D\, v_1 = \left| N\, v_1 \cap V - D \right|$

$= \left| v_{i+1}, \dots v_n \cup \frac{u_i}{\leq i} \leq n \cap \quad u, u_i \right.$

$= \left| u, u_i \right|$ $since\ i = 1$

=2

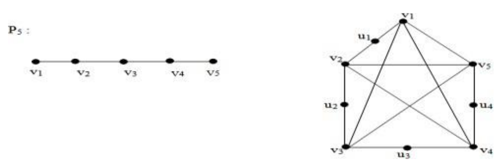
Then $\left| od_D(v_i) - od_D(v_j) \right| \leq 2$, for any $v_i\, v_j \in D$. Therefore D is the minimum two - out degree equitable dominating set, Hence, $\gamma_{2oe}\, M(K_{1,n}) = n$.

3. Two-out degree Equitable domination in the Central graphs of P_n, C_n and $K_{1,n}$.

In this section, we obtained the two-out degree equitable domination number $\gamma_{2oe}(G)$ of the central graphs of the path P_n, cycle C_n and the star graph $K_{1,n}$.

Example 3.1.

Let G be the central graph as in the figure. we obtained the two-out degree equitable domination number.



Central graph of P_5

Consider the set $D = \{v_1, u_2, u_3, u_4\}$ .It is a dominating set and $V - D = \{v_2, v_3, v_4, v_5, u_1\}$

$od_D\, v_1 = \left| N\, v_1 \cap V - D \right|$

$= \left| \{u_1, v_3, v_4, v_5\} \cap \{v_2, v_3, v_4, v_5, u_1\} \right|$

= 4

$od_D\, u_2 = \left| N\, u_2 \cap V - D \right|$

= 2

Similarly, $od_D\, u_3 = od_D\, u_4 = 2$

From the above, any two vertices $u, v \in D$ are such that $\left| od_D\, u - od_D(v) \right| \leq 2$.

Therefore $\{v_1, u_2, u_3, u_4\}$ is the minimum two-out degree equitable dominating set with the minimum cardinality 4. Hence, $\gamma_{2oe}\, C(P_5) = 4$.

Theorem 3.2.

For any Path P_n, $\gamma_{2oe}\, C(P_n) = n - 1\ for\ n \leq 5$.

Proof

Let P_n be the path of length $(n-1)$ with vertices $v_1, v_2 \dots .\ v_n$ .By the definition of central graph, the non-adjacent vertices $v_i\ and\ v_j$ of P_n are adjacent in $C\, P_n$.

Therefore,$V(C\, P_n = \{v_i / 1 \leq i \leq n\} \cup \{u_i / 1 \leq i \leq n - 1$ and

$E(C\, P_n = \{e_i / 1 \leq i \leq n - 1\} \cup \{e'_i / 1 \leq i \leq n - 1\} \cup e_{ij} : 1 \leq i \leq n - 2, i + 2 \leq j \leq n$.

Let $\{v_i\} \cup \{u_i / 2 \leq i \leq n - 1\}$ will be the dominating set.

Let $D = (v_1, u_{i+1}), (v_1, u_{i+1}), (v_1, u_{i-1}, u_i, u_{i+1})$, $where\ 1 \leq i \leq 3$ be the dominating set of $C\, P_n\ and$

$V - D = u_1, v_{i+1}, v_{i+2}, u_1, v_i, v_{i+1}, v_{i+2}, u_1, v_{i-1}, v_i, v_{i+1}, v_{i+2}$

$where\ 1 \leq i \leq 3$

Now $v_1 \in D$ then $od_D\, v_1 = \left| N(v_1) \cap V - D \right|$

$= \left| (u_1, v_{i+2}), (u_1, v_{i+1}, v_{i+2}), (u_1, v_i, v_{i+1}, v_{i+2}) \cap \{ u_1, v_{i+1}, v_{i+2}, (u_1, v_i, v_{i+1}, v_{i+2}), (u_1, v_{i-1}, v_i, v_{i+1}, v_{i+2})\} \right|$

$= \left| (u_1, v_{i+2}), (u_1, v_{i+1}, v_{i+2}), (u_1, v_i, v_{i+1}, v_{i+2}) \right|$

$= n - 1$

Now $u_i \in D$ then $od_D\, u_i = \left| N(u_i) \cap V - D \right|$

=2

Then $\left| od_D(v_1) - od_D(u_i) \right| \leq 2$, for any $v_1, u_i \in D$. Therefore D is the minimum two - out degree equitable dominating set. Hence, $\gamma_{2oe}\, C(P_n) = n - 1\ for\ n \leq 5$.

Theorem 3.3.

For any Cycle C_n, $\gamma_{2oe}\, C(C_n) = n - 1\ for\ n \leq 5$.

Proof

Let C_n be any cycle of length n and let $V\ C_n = \{v_1, v_2 \dots v_n\}$ and $E\ C_n = e_1, e_2 \dots e_n$ .By the definition of central graph $C(C_n)$ has the vertex set $V\ C_n \cup \{u_i : 1 \leq i \leq n\}$ where u_i is a vertex of subdivision of the edge $v_i v_{i+1}$ $(1 \leq i \leq n - 1)$ and u_n is a vertex of subdivision of the edge $v_n v_1$.

Let $D = (v_1, u_{i+1}), (v_1, u_i, u_{i+1}), (v_1, u_{i-1}, u_i, u_{i+1})$,$where\ 1 \leq i \leq 3$ be a dominating set of $C\, C_n$ and $V - D = (v_{i+1}, v_{i+2}, u_i, u_{i+2}), (v_i, v_{i+1}, v_{i+2}, u_{i-1}, u_{i+2})(v_{i-1}, v_i, v_{i+1}, v_{i+2}, u_{i-2}, u_{i+2})$, $where\ 1 \leq i \leq 3$.

Now $v_1 \in D$ then $od_D\, v_1 = \left| N\, v_1 \cap V - D \right|$

$= \left| (u_{i+2}), (u_{i-1}, u_{i+2}, v_{i+1}), (u_{i-2}, u_{i+2}, v_i, v_{i+1}) \cap (v_{i+1}, v_{i+2}, u_i, u_{i+2}), (v_i, v_{i+1}, v_{i+2}, u_{i-1}, u_{i+2})(v_{i-1}, v_i, v_{i+1}, v_{i+2}, u_{i-2}, u_{i+2}) \right|$

$= \left| (u_{i+2}), (u_{i-1}, u_{i+2}, v_{i+1}), (u_{i-2}, u_{i+2}, v_i, v_{i+1}) \right|$

$= n - 2 \text{ or } n - 1$

Similarly, $od_D\, u_i = od_D\, u_{i+1} = 2$

Then $\left| od_D(v_1) - od_D(u_{i+1}) \right| \leq 2$, for any $v_1 u_i \in D$. Therefore D is the minimum two - out degree equitable dominating set. Hence, $\gamma_{2oe}\, C(C_n) = n - 1 \text{ for } n \leq 5$.

Theorem 3.4.

For any Star graph $K_{1,n}$, $\gamma_{2oe}\, C(K_{1,n}) = 2$.

Proof

Let $V\ K_{1,n} = v, v_1, v_2, \ldots v_n$ where $\deg v = n$. By the definition of central graph of $K_{1,n}$ we denote the vertices of subdivision by $v'_1, v'_2, \ldots v'_n$. That is vv_i is subdivided by $u_i\ 1 \leq i \leq n$.

Let $D = \{v, v_1\}$ be the dominating set of $C(K_{1,n})$ and $V - D = (v'_i, v_{i+1}, \ldots v_n\}, 1 \leq i \leq n$

Now $v \in D$ then $od_D\, v = \left| N\, v \cap V - D \right|$

$= \left| \{v'_1, v'_2, \ldots v'_n\} \cap (v'_i, v_{i+1}, \ldots v_n\} \right|$

$= \left| \{v'_1, v'_2, \ldots v'_n\} \right| = n$

Similarly, $od_D\, v_1 = n$

Then $\left| od_D(v) - od_D(v_1) \right| \leq 2$, for any $v, v_1 \in D$. Therefore D is the minimum two - out degree equitable dominating set. Hence, $\gamma_{2oe}\, C(K_{1,n}) = 2$.

4. Two-out degree Equitable domination in the Line graphs of P_n, C_n, and $K_{1,n}$.

In this section, we obtained the two-out degree equitable domination number $\gamma_{2oe}(G)$ of the line graphs of the path P_n, cycle C_n and the star graph $K_{1,n}$.

Theorem 4.1.

For any Path P_n, $\gamma_{2oe}\, L(P_n) = n - 3$.

Proof

Let P_n has n vertices n-1 edges. Let $V = \{v_1, v_2, \ldots . v_{n-1}\}$ be the vertices of $L(P_n)$ and let $E = \{u_1, u_2, \ldots . u_{n-2}\}$ be the edges of $L(P_n)$.

Since the degree of any vertex in $L(P_n)$ is 2 except the initial and terminal vertices.

Let us consider $D = \{v_1, v_2, \ldots . v_{n-3}\}$ be a dominating set of $L(P_n)$ and $V - D = \{v_{n-2}, v_{n-1}\}$`

Now $v_i \in D$ then $od_D\, v_i = \left| N\, v_i \cap V - D \right|$

$od_D\, v_{i+1} = \left| N\, v_{i+1} \cap V - D \right| = 0$

Similarly, $od_D\, v_{n-3} = 1$. Hence for every $v_i, v_j \in D$ then $\left| od_D\, u - od_D(v) \right| \leq 2$. So D is the minimum two-out degree equitable dominating set. Then $\gamma_{2oe}\, L(P_n) = n - 3$.

Theorem 4.2.

For any Cycle C_n, $\gamma_{2oe}\, L(C_n) = n - 2$.

Proof

Let $L(C_n)$ have n vertices and n edges in which each vertex is of degree 2. That is each vertex dominates two vertices. Let $V = \{u_1, u_2, \ldots . u_n\}$ be the vertices of $L(C_n)$.

Let us consider $D = \{u_1, u_2 \ldots . u_{i-1}, u_{i+1}, \ldots u_n\}$ be the dominating set of $L(C_n)$ and $V - D = \{u_i, u_{i+1}\}$.

Now $od_D(u_j) = 0, j = 1,2 \ldots i-2, i+3, \ldots n$

$od_D(u_i) = 1$ and $od_D(u_{i+1}) = 1$

Then $|od_D(u_i) - od_D(u_j)| \leq 2$

Then D is the minimum two out degree equitable dominating set

So $\gamma_{2oe}\, L(C_n) = n - 2$.

Note: The line graph of C_n, $L\, C_n$ is C_n itself.

Theorem 4.3.

For any Star graph $K_{1,n}$, $\gamma_{2oe}\, L(K_{1,n}) = 1 \text{ for } n \leq 3$.

Proof

Let $V = \{v, u_1, u_2, \ldots u_n\}$ be the vertices of $K_{1,n}$. Let $V = \{v, u_1, u_2, \ldots u_{n-1}\}$ be the vertices of $L\, K_{1,n}$. Let us consider $D = \{v\}$ be a dominating set of $L\, K_{1,n}$ and $V - D = u_1, u_2$.

Now $v \in D$ then $od_D\, v = \left| N\, v \cap V - D \right|$

$= \left| u_1 u_2 \cap (u_1 u_2) \right| = 2$

Then $\left| od_D(u) - od_D(v) \right| \leq 2$, for any $u, v \in D$. Therefore D is the minimum two - out degree equitable dominating set. Hence, $\gamma_{2oe}\, L(K_{1,n}) = 1 \text{ for } n \leq 3$.

5. CONCLUSION

In this paper, we introduced two-out degree equitable domination number in the middle, central and line graph of P_n, C_n, and $K_{1,n}$ graphs. We extend this study on some more special classes of graphs.

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