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CONTRA αgμ-CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper we introduce and discuss some basic properties of contra αgμ-continuous functions.

Keywords : αgμ-open, contra αgμ-continuous

1. INTRODUCTION

The notion αgμ-closed sets in topological spaces was introduced by R. Devi, V. Vijayalakshmi and V. Kokilavani. The concept of Contra continuous mappings was introduced and investigated by J. Dontchev. In this paper we introduce the notion of contra αgμ-continuous functions and discuss their basic properties

2. PRELIMINARIES

2.1. Definition

A subset A of space (X,τ) is called

1. a generalized closed (briefly g-closed) set (Njastad,1965) if $cl(A) \subseteq U$ and U is open in (X, τ); the complement of a g-closed set is called a g-open set,

2. an α-generalized closed (briefly αg-closed) set (Maki *et al.,*1994) if $\alpha cl(A) \subseteq U$ whenever

$A \subseteq U$ and U is open in (X, τ),

3. a μ-closed set (Veera Kumar, 2005) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha^*$-open in (X, τ),

4. an αgμ-closed set (Devi *et al.,* 2007) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in (X, τ).

2.2. Definition

1. A space (X, τ) is said to be ${}_cT_{\alpha g\mu}$ (Devi *et al.,* 2007) if every αgμ-closed set is closed in X.

2. A space (X, τ) is said to be locally indiscrete (Atick, 1997) if every open subset of X is closed.

3. A function f : (X, τ) →(Y, σ) is said to be regular set connected (Atick, 1997) if $f^{-1}(V)$ is clopen in (X, τ) for every regular open set V of (Y, σ).

4. A function f : (X, τ) →(Y, σ) is said to be perfectly continuous (Atick, 1997) if $f^{-1}(V)$ is clopen in X for every open set V of Y.

2.3. Definition

A function f : (X, τ) → (Y, σ) is said to be contra continuous (Dontchev, 1996) if for every open set in (Y, σ) there exist a closed set in (X, τ).

2.4. Definition

A function f: (X, τ) → (Y, σ) is said to be αgμ-continuous (Devi *et al.,* 2007) if for every open set in (Y, σ) there exist an αgμ-open set in (X, τ).

3. CONTRA αGμ-CONTINUOUS FUNCTION

3.1. Definition

A function f: (X, τ) → (Y, σ) is said to be contra αgμ-continuous if for every open set in (Y, σ) there exist an αgμ-closed set in (X, τ).

3.2. Theorem

Every contra continuous function is contra αgμ- continuous.

Proof. Let V be open in (Y, σ). Since f : (X, τ) → (Y, σ) is contra continuous, $f^{-1}(V)$ is closed in (X, τ) and hence αgμ-closed By (Devi *et al.,* 2007). Thus, f is contra αgμ continuous.

Converse of the above theorem need not be true by the following example.

3.3. Example

Let X = Y = {a, b, c}, τ = {ϕ, X, {a}, {b, c}} and σ = {ϕ, Y, {a}, {a, b}}.

Define f : (X, τ) → (Y, σ) by f(a) = b, f(b) = c and f(c) = a. The αgμ-closed sets of X are ϕ, X, {a},{b},{c}, {a, b}, {b, c}, {a, c}. Here {a, b} is an open set of (Y, σ) but $f^{-1}(\{a, b\})$ = {b, c} is not a closed set of (X, τ). Hence f is contra αgμ- continuous but not contra continuous.

3.4. Lemma

The following properties hold for subsets A,B of a space X:

(a) $x \in \ker(A)$ if and only if $A \cap F \neq \phi$ for any $F \in C(X,x)$.

(b) $A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X.

(c) If $A \subseteq B$, then $\ker(A) \subseteq \ker(B)$.

3.5. Theorem

For a function $f : (X, \tau) \rightarrow (Y, \sigma)$ the following conditions are equivalent:

(1) f is contra $\alpha g\mu$- continuous;

(2) for every closed subset F of Y, $f^{-1}(F) \in \alpha g\mu O(X)$;

(3) for each $x \in X$ and each $F \in C(Y,f(x))$, there exists $U \in \alpha g\mu O(X,x)$ such that

$f(U) \subseteq F$;

(4) $f(\alpha g\mu cl(A)) \subseteq \ker(f(A))$ for every subset A of X;

(5) $\alpha g\mu cl(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$ for every subset B of Y.

Proof.

(1) $\Rightarrow$ (2) Since f is contra $\alpha g\mu$- continuous, inverse image of a closed subset F of Y is $\alpha g\mu O(X)$.

(2)$\Rightarrow$(3) It is given F is closed subset of Y and $f^{-1}(F)$ is $\alpha g\mu O(X)$. Hence for $x \in X$, there exists $U \in \alpha g\mu O(X)$ such that $f(U) \subseteq F$.

(3) $\Rightarrow$ (2) Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \alpha g\mu O(X,x)$ such that $f(U_x) \subseteq F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x / x \in f^{-1}(F)\}$ and $f^{-1}(F)$ is $\alpha g\mu$-open.

(2) $\Rightarrow$ (4) Let A be any subset of X. Suppose that $y \notin \ker(f(A))$. Then by the lemma 3.4, there exists $F \in C(Y,f(x))$ such that $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and since $f^{-1}(F)$ is $\alpha g\mu$-open, we have $\alpha g\mu cl(A) \cap f^{-1}(F) = \phi$.

Therefore, we obtain $f(\alpha g\mu cl(A)) \cap F = \phi$ and $y \notin f(\alpha g\mu cl(A))$. This implies that $f(\alpha g\mu cl(A)) \subseteq \ker(f(A))$.

(4) $\Rightarrow$ (5) Let B be any subset of Y. By lemma 3.4, we have

$f(\alpha g\mu cl(f^{-1}(B))) \subseteq \ker(f(f^{-1}(B))) \subseteq \ker(B)$ thus $\alpha g\mu cl(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$.

(5) $\Rightarrow$ (1) Let V be any open set of Y. Then by lemma 3.4, we have

$\alpha g\mu cl(f^{-1}(V)) \subseteq f^{-1}(\ker(V)) = f^{-1}(V)$ and $\alpha g\mu cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\alpha g\mu$-closed in X.

3.6. Theorem

If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\alpha g\mu$-continuous and Y is regular, then f is $\alpha g\mu$-continuous.

Proof. Let x be an arbitrary point of X and let V be an open set of Y containing f(x). Since Y is regular, there exists an open set W in Y containing f(x) such that $cl(W) \subseteq V$. Since f is $\alpha g\mu$-continuous, so by theorem3.5, there exists $U \in \alpha g\mu O(X,x)$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. Hence, f is $\alpha g\mu$-continuous.

3.7. Corollary

If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\alpha g\mu$-continuous and Y is regular, and then f is continuous.

We introduce the following definitions

3.8. Definition

1) A space (X, τ) is said to be locally $\alpha g\mu$-indiscrete if every $\alpha g\mu$-open set is

closed.

2) A function $f: X \rightarrow Y$ is called almost $\alpha g\mu$-continuous if for each $x \in X$ and each

open set V of Y containing f(x), there exists $U \in \alpha g\mu O(X, x)$ such that $f(U) \subseteq \alpha g\mu int(cl(V))$.

3.9. Theorem

If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\alpha g\mu$-continuous and X is a ${}_cT_{\alpha g\mu}$, then f is $\alpha g\mu$-continuous.

Proof. Let V be a closed set in Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(V)$ is $\alpha g\mu$ open in X. Since X is ${}_cT_{\alpha g\mu}$, $f^{-1}(V)$ is open in X. Hence f is contra-continuous.

3.10. Theorem.

Let X be locally $\alpha g\mu$-indiscrete. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra $\alpha g\mu$-continuous, then f is continuous.

Proof. Let V be a closed set in Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(V)$ is $\alpha g\mu$-open in X. Since X is locally $\alpha g\mu$-indiscrete, $f^{-1}(V)$ is closed in X. Hence f is continuous.

3.11. Theorem

A function $f : X \rightarrow Y$ is almost $\alpha g\mu$-continuous if and only if for each $x \in X$ and each regular open set V of Y containing f(x), there exists $U \in \alpha g\mu O(X, x)$ such that $f(U) \subseteq V$.

Proof. Let V be regular open set of Y containing f(x) for each $x \in X$. Since every regular open set is open

(Njastad, 1965), V be an open set of Y containing f(x) for each x $\in$X.

Since f is almost $\alpha g\mu$-continuous, there exists U $\in\alpha g\mu O(X, x)$ such that f(U) $\subseteq\alpha g\mu int(cl(V))\subseteq$ V. Therefore f(U) $\subseteq$ V.

Conversely, if for each x $\in$X and each regular open set V of Y containing f(x), there exists U $\in\alpha g\mu O(X, x)$ such that f(U) $\subseteq$ V. This implies V is an open set of Y containing f(x), such that f(U) $\subseteq$ V $=\alpha g\mu int(cl(V))$. Therefore f is almost $\alpha g\mu$-continuous.

3.12. Theorem

If a function f: $(X, \tau) \to (Y, \sigma)$ is pre $\alpha g\mu$-open and contra $\alpha g\mu$-continuous, then f is almost $\alpha g\mu$-continuous.

Proof. Let x be any arbitrary point of X and V be an open set containing f(x).Since f is contra $\alpha g\mu$-continuous, then there exists U $\in\alpha g\mu O(X, x)$ such that f(U) $\subseteq$ cl(V). Since f is pre $\alpha g\mu$-open, f(U) is pre $\alpha g\mu$-open in Y. Therefore, f(U) = $\alpha g\mu int(f(U)) \subseteq \alpha g\mu int\ (cl(f(U))) \subseteq \alpha g\mu int\ (cl(V))$. This shows that f is almost $\alpha g\mu$-continuous.

3.13. Definition

The graph of a function f: X→Y is said to be contra $\alpha g\mu$-closed if for each

$(x, y) \in (X \times Y) - Gr(f)$, there exists U $\in\alpha g\mu O(X, x)$ and V $\in$ C(Y, y) such that $(U \times V) \cap Gr\ (f) = \phi$.

3.14. Theorem

If f: X →Y is contra $\alpha g\mu$-continuous and Y is Urysohn, then f is C$\alpha g\mu$-closed in the product space X × Y.

Proof. Let $(x, y) \in (X \times Y) - Gr(f)$. Then y ≠ f(x) and there exists open sets A and B such that f(x) $\in$A, y $\in$ B and $cl(A) \cap cl(B) = \phi$. Then there exists V $\in\alpha g\mu O(X, x)$ such that f(V) $\subseteq$ cl(A). Therefore, we obtain $f(V) \cap cl(B) = \phi$. This shows that f is C$\alpha g\mu$-closed.

3.15. Theorem

If f : X→Y is contra $\alpha g\mu$-continuous with X as locally $\alpha g\mu$-indiscrete then f is continuous.

Proof. Let V be an open set in Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(V)$ is $\alpha g\mu$-closed set in X. Since X is locally $\alpha g\mu$-indiscrete every $\alpha g\mu$-closed set is open. Hence $f^{-1}(V)$ is open in X. Therefore f is continuous.

3.16. Theorem

If f: X→Y is contra $\alpha g\mu$-continuous and X is ${}_cT_{\alpha g\mu}$-space, then f is contra- continuous.

Proof. Let V be open set in Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(V)$ is $\alpha g\mu$-closed in X. Since X is a ${}_cT_{\alpha g\mu}$ space, every $\alpha g\mu$-closed set is closed. Hence $f^{-1}(V)$ closed in X. Therefore f is contra- continuous.

3.17. Theorem

If f: X→Y is a surjective pre-closed contra $\alpha g\mu$-continuous with X as ${}_cT_{\alpha g\mu}$ space, then Y is locally indiscrete.

Proof. Let V be an open subset in Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(V)$ is $\alpha g\mu$-closed in X. Since X is ${}_cT_{\alpha g\mu}$ space $f^{-1}(V)$ is closed in X. Since f is pre-closed,V is pre-closed in Y. Now we have cl(V) = cl(int(V)) $\subseteq$ V which implies cl(V) = V. This means V is closed in X and hence Y is locally indiscrete.

3.18. Definition

A space X is said to be $\alpha g\mu$-connected if X cannot be written as a disjoint union of two non-empty $\alpha g\mu$-open sets.

3.19. Theorem

A contra $\alpha g\mu$-continuous image of a $\alpha g\mu$-connected space is connected.

Proof. Let f: X→Y be a contra $\alpha g\mu$-continuous map of a $\alpha g\mu$-connected space X on to a topological space Y. If possible, let Y be disconnected. Let A and B form a disconnection of Y. Then A and B are clopen and Y = $A \cup B$, where $A \cap B = \phi$. Since f is contra $\alpha g\mu$-continuous map, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and f^{-1} B are non-empty $\alpha g\mu$-open sets in X. Also $f^{-1}(A) \cap f^{-1} B = \phi$. Hence X is not $\alpha g\mu$-connected. This is a contradiction. Therefore Y is connected.

3.20. Theorem

If f is contra $\alpha g\mu$-continuous map from a $\alpha g\mu$-connected space X on to any space Y, then Y is not a discrete space.

Proof. Suppose that Y is discrete. Let A be a proper non-empty open and closed subset of Y. Since f is contra $\alpha g\mu$-continuous, $f^{-1}(A)$ is a proper non-empty $\alpha g\mu$-open and $\alpha g\mu$-closed subset of X, which is a contradiction to the fact that X is $\alpha g\mu$-connected space. Therefore Y is not a discrete space.

3.21. Theorem

If f: X→Y is $\alpha g\mu$-irresolute map with Y as locally $\alpha g\mu$-indiscrete space and g : Y→Z is contra

$\alpha g\mu$-continuous map, then $g_o f$: X→Z is $\alpha g\mu$-continuous.

Proof. Let A be any closed set in Z. Since g : Y→Z is contra $\alpha g\mu$-continuous, $g^{-1}(A)$ is $\alpha g\mu$-open set in Y. Since Y is locally $\alpha g\mu$-indiscrete, $g^{-1}(A)$ is closed in Y. Hence $g^{-1}(A)$ is $\alpha g\mu$-closed set in Y. Since f is $\alpha g\mu$-irresolute $(g_o f)^{-1}(A)) = f^{-1}(g^{-1}(A)$ is $\alpha g\mu$-closed in X. Therefore $g_o f$ is $\alpha g\mu$-continuous.

3.22. Theorem

If g : Y→Z is continuous and f : X→Y is contra $\alpha g\mu$-continuous then $g_o f$: X→Z is contra $\alpha g\mu$-continuous.

Proof. Let A be a closed set in Z. Since g: Y→ *Z* is continuous, $g^{-1}(A)$ is closed in Y. Since f is contra $\alpha g\mu$-continuous, $(g_o f)^{-1}(A)) = f^{-1}(g^{-1}(A)$is $\alpha g\mu$-open in X. Thus $g_o f$ is contra $\alpha g\mu$-continuous.

3.23. Theorem

If f: X→Y and g: Y→Z are $\alpha g\mu$-continuous and Y is locally $\alpha g\mu$-indiscrete, then $g_o f$: X→Z is contra $\alpha g\mu$-continuous.

Proof. Let A be a closed set in Z. Since g is $\alpha g\mu$-continuous, $g^{-1}(A)$ is $\alpha g\mu$-closed in Y and hence open. Since f is $\alpha g\mu$-continuous $(g_o f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $\alpha g\mu$-open in X. Hence $g_o f$ is contra $\alpha g\mu$-continuous.

3.24. Theorem

If f: X→Y is surjective $\alpha g\mu$-irresolute and pre$\alpha g\mu$-open and g: Y→Z is any function, then $g_o f$: X→Z is contra $\alpha g\mu$-continuous if and only if g is contra $\alpha g\mu$-continuous.

Proof. To prove if part, let g be contra $\alpha g\mu$-continuous and A be a closed set of Z.

Since g is contra $\alpha g\mu$-continuous, g^{-1} A is $\alpha g\mu$-open in Y. Since f is $\alpha g\mu$-irresolute, $(g_o f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $\alpha g\mu$-open in X. Hence $g_o f$: X→Z is contra $\alpha g\mu$-continuous.

To prove only if part, let $g_o f$: X→Z is contra $\alpha g\mu$-continuous and let A be a closed set in Z. Then $(g_o f)^{-1}(A)$ is $\alpha g\mu$-open of X. That is $f^{-1}(g^{-1}(A))$ is an $\alpha g\mu$-open subset of X. Since f is pre $\alpha g\mu$-open, $f(f^{-1}(g^{-1}(A)))$ is $\alpha g\mu$-open subset of Y. So, $g^{-1}(A)$ is an $\alpha g\mu$-open subset of Y. Hence g is contra $\alpha g\mu$-continuous.

3.25. Theorem

If f : X→Y is contra $\alpha g\mu$-continuous, closed injection and Y is ultra normal, then X is $\alpha g\mu$-normal.

Proof. Let A and B be disjoint closed subsets of X. Since f is closed injectve, f(A) and f(B) are disjoint closed subsets of Y. Since Y is ultra normal, f(A) and f(B) are separated by disjoint clopen sets V and W respectively. Hence $A \subseteq f^{-1}(V)$ and $B \subseteq f^{-1}(W)$, $f^{-1}(V) \subseteq \alpha g\mu O(X)$ and $f^{-1}(W) \in \alpha g\mu O(X)$. Also $f^{-1}(V) \cap f^{-1}(W) = \phi$. Thus X is $\alpha g\mu$-normal.

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