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RESEARCH ARTICLE

P_4-DECOMPOSITION OF LINE AND MIDDLE GRAPH OF SOME GRAPHS

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ABSTRACT

A decomposition of a graph G is a collection of edge-disjoint subgraphs G_1, G_2,… G_m of G such that every edge of G belongs to exactly one G_i, $1 \leq i \leq m$. $E(G) = E(G_1) \cup E(G_2) \cup ....\cup E(G_m)$. If every graph $G_i$is a path then the decomposition is called a path decomposition. In this paper, we have discussed the P_4-decomposition of line and middle graph of Wheel graph, Sunlet graph, Helm graph. The edge connected planar graph of cardinality divisible by 3 admits a P_4-decomposition.

Keywords: Decomposition, P_4-decomposition, Line graph, Middle graph.

Mathematics Subject Classification: 05C70

1. INTRODUCTION AND PRELIMINARIES

Let G = (V, E) be a simple graph without loops or multiple edges. A path is a walk where $v_i \neq v_j$, $\forall$ i ≠ j. In other words, a path is a walk that visits each vertex at most once. A decomposition of a graph G is a collection of edge-disjoint subgraphs G_1, G_2,… G_m of G such that every edge of G belongs to exactly one G_i, $1 \leq i \leq m$. $E(G) = E(G_1) \cup E(G_2) \cup ....\cup E(G_m)$. If every graph $G_i$is a path then the decomposition is called a path decomposition.

Heinrich, Liu and Yu (8) proved that a connected 4-regular graph admits a P_4decomosition if and only if $|E(G)| \equiv 0 \pmod 3$ by characterizing graphs of maximum degree 4 that admit a triangle-free Eulerian tour. Haggkvist and Johansson (5) proved that every maximal planar graph with atleast 4 vertices has a P_4-decomposition. C. Sunil Kumar (12) proved that a complete r-partite graph is P_4-decomposable if and only if its size is a multiple of 3. The name line graph comes from a paper by Harary & Norman (1960) although both Whitney (1932) and Krausz (1943) used the construction before this (9). The concept of middle graph was introduced by T. Hamada and I. Yoshimura (6) in 1974.

Definition 1.1. (10) A cycle graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed chain.

Definition 1.2. (10) A wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices can also be defined as the 1-skeleton of an (n-1)-gonal pyramid.

Definition 1.3. (2) The -sunlet graph is the graph on vertices obtained by attaching pendant edges to a cycle graph.

Definition 1.4. (1) TheHelm graphis obtained from a wheel by attaching a pendant edge at each vertex of the -cycle.

Definition 1.5. (7) Let G be a graph, its Line graph L(G), is defined with the vertex set E(G), in which two vertices are adjacent if and only if the corresponding edges are adjacent in G.

Definition 1.6. (1) The Middle graph of G, denoted by M(G), is defined with the vertex set V(G) E(G), in which two elements are adjacent if and only if either both are adjacent edges in G or one of the elements is a vertex and the other one is an edge incident to the vertex in G.

Theorem 1.1. (12) $C_n$is P_4-decomposable if and only if $n \equiv 0 \pmod 3$.

Theorem 1.2. (12) K_n is P_4-decomposableif and only if $n \equiv 0 \pmod 3$ or $n \equiv 1 \pmod 3$.

P_4-DECOMPOSITION OF LINE GRAPHS

P_4-Decomposition of Line graph of Wheel graph

Let G be the wheel graph W_n. In $L(W_n)$, there are 2n number of vertices and $\frac{n(n+5)}{2}$ number of edges. Its maximum degree is n+1 and minimum degree is 4.

Theorem 2.1. The graph $L(W_n)$ is P_4 decomposable if and only if $n \equiv 0 (\text{mod } 3)$ or $n \equiv 1(\text{mod}3)$.

Proof: By definition of $L(W_n)$, let e_i, $1 \leq i \leq n$ands_i, $1 \leq i \leq n$be the vertices of W_n joining the vertices

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corresponding to the edges v_iv_{i+1}&v_nv_1 $(1 \le i \le n-1)$ and $vv_i$$(1 \le i \le n)$ respectively.

E(L(W_n))= $\{e_ie_{i+1} / 1 \le i \le n-1\} \cup \{e_ne_1\} \cup \{e_is_i / 1 \le i \le n\} \cup$

$\{e_is_{i+1} / 1 \le i \le n-1\} \cup \{e_ns_1\} \cup \{s_is_j / 1 \le i \le n-1 , 2 \le j \le n , i \ne j\}$

Case I : For n ≡ 0(mod 3) , n > 3

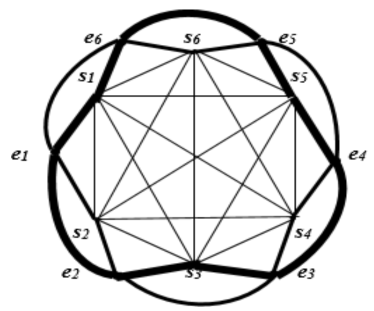

Fig.2.1. P_4-decomposition of L(W_6).

$< s_i > \cong K_n$, $n \equiv 0 (\text{mod } 3)$

$< s_ie_ie_{i+1}s_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< s_{n-1}e_{n-1}e_ns_1 > \cong P_4$

$< s_ne_ne_1s_2 > \cong P_4$

Hence $E(L(W_n))=E(K_n) \cup E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $L(W_n)$ is P_4-decomposable.

Case II: For n ≡ 1(mod 3)

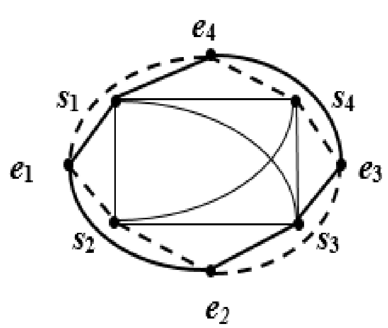

Fig.2.2. P_4-decomposition of L(W_4).

$< s_i > \cong K_n$, $n \equiv 1 (\text{mod } 3)$

$< s_ie_ie_{i+1}s_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< s_{n-1}e_{n-1}e_ns_1 > \cong P_4$

$< s_ne_ne_1s_2 > \cong P_4$

Hence $E(L(W_n))=E(K_n) \cup E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $L(W_n)$ is P_4-decomposable.

Conversely, suppose that $L(W_n)$ is P_4-decomposable.

Then $|E(L(W_n))| \equiv 0 (\text{mod } 3)$ which implies that $\frac{n\ n+5}{2} \equiv 0 (\text{mod } 3)$ and thus $n \equiv 0 (\text{mod } 3)$ or $n \equiv 1 (\text{mod } 3)$.

P_4-Decomposition of Line graph of Sunlet graph

Let G be the sunlet graph S_n. In $L(S_n)$, there are 2n number of vertices and 3n number of edges. Its maximum degree is 4 and minimum degree is 2.

Theorem 2.2. The graph $L(S_n)$ is P_4-decomposable for all values of n.

Proof: By definition of $L(S_n)$, let f_i , $1 \le i \le n$ and e_i, $1 \le i \le n$ be the vertices of S_n joining the vertices corresponding to the edges v_iu_i $(1 \le i \le n)$ and v_iv_{i+1}&$v_nv_1$$(1 \le i \le n-1)$ respectively.

E(L(S_n))= $\{e_ie_{i+1} / 1 \le i \le n-1\} \cup \{e_ne_1\} \cup \{e_if_i / 1 \le i \le n\} \cup$

$\{e_if_{i+1} / 1 \le i \le n-1\} \cup \{e_nf_1\}$

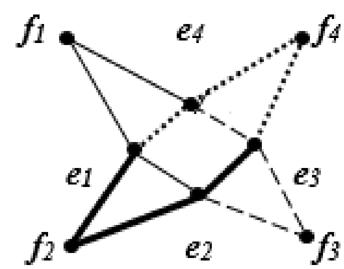

Fig.2.3. P_4-decomposition of L(S_4).

$< e_if_{i+1}e_{i+1}e_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< e_{n-1}f_ne_ne_1 > \cong P_4$

$< e_nf_1e_1e_2 > \cong P_4$

Hence $E(L(S_n)) = E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $L(S_n)$ is P_4-decomposable.

P_4-Decomposition of Line graph of Helm graph

Let G be the helm graph H_n. In $L(H_n)$, there are 3n number of vertices and $\frac{n(n+11)}{2}$ number of edges. Its maximum degree is n+2 and minimum degree is 3.

Theorem 2.3. The graph $L(H_n)$ is P_4-decomposable if and only if $n \equiv 0 (\text{mod } 3)$ or $n \equiv 1 (\text{mod } 3)$.

Proof: By definition of $L(H_n)$, let f_i , $1 \le i \le n$; e_i,$1 \le i \le n$ and s_i , $1 \le i \le n$ be the vertices of H_n joining the vertices corresponding to the edges v_iu_i $(1 \le i \le n)$;v_iv_{i+1}&$v_nv_1$$(1 \le i \le n-1)$ and $vv_i$$(1 \le i \le n)$ respectively.

E(L(H_n))= $\{f_is_i / 1 \le i \le n\} \cup \{f_ie_i / 1 \le i \le n\} \cup \{e_if_{i+1} / 1 \le i \le n-1\} \cup$

$\{e_nf_1\} \cup \{s_ie_i \,/\, 1 \le i \le n\} \cup \{e_is_{i+1} \,/\, 1 \le i \le n-1\} \cup \{e_ns_1\} \cup$

$\{e_ie_{i+1} \,/\, 1 \le i \le n-1\} \cup \{e_ne_1\} \cup \{s_is_j \,/\, 1 \le i \le n-1 , 2 \le j \le n , i \ne j\}$

Case I : For n ≡ 1(mod 3)

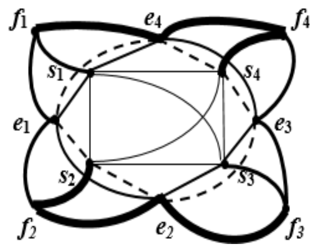

Fig.2.4. P_4-decomposition of $L(H_4)$.

$< s_i > \cong K_n$, n ≡ 1(mod 3)

$< s_if_ie_if_{i+1} > \cong (n-1)P_4$, $1 \le i \le n-1$

$< s_nf_ne_nf_1 > \cong P_4$

$< s_ie_ie_{i+1}s_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< s_{n-1}e_{n-1}e_ns_1 > \cong P_4$

$< s_ne_ne_1s_2 > \cong P_4$

Hence $E(L(H_n)) = E(K_n) \cup E((n-1)P_4) \cup E(P_4) \cup E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $L(H_n)$ is P_4-decomposable.

Case II : For n ≡ 0(mod 3) , n > 3

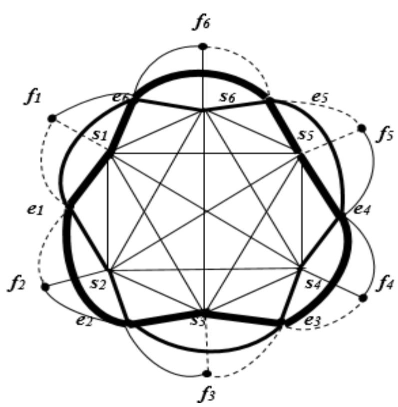

Fig.2.5. P_4-decomposition of $L(H_6)$.

$< s_i > \cong K_n$, n ≡ 0(mod 3)

$< s_if_ie_if_{i+1} > \cong (n-1)P_4$, $1 \le i \le n-1$

$< s_nf_ne_nf_1 > \cong P_4$

$< s_ie_ie_{i+1}s_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< s_{n-1}e_{n-1}e_ns_1 > \cong P_4$

$< s_ne_ne_1s_2 > \cong P_4$

Hence $E(L(H_n)) = E(K_n) \cup E((n-1)P_4) \cup E(P_4) \cup E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $L(H_n)$ is P_4-decomposable.

Conversely, suppose that $L(H_n)$ is P_4-decomposable.

Then $|E(L(H_n))| \equiv 0 (\text{mod } 3)$ which implies that $\frac{n\ n+11}{2} \equiv 0(\text{mod } 3)$ and thus $n \equiv 0(\text{mod } 3)$ or $n \equiv 1(\text{mod } 3)$.

MIDDLE GRAPH OF CYCLE RELATED GRAPHS

P_4-Decomposition of Middle graph of Wheel graph

Let G be the wheel graph W_n. In $M(W_n)$, there are 3n+1 number of vertices and $\frac{n(n+13)}{2}$ number of edges. Its maximum degree is n+3 and minimum degree is 3.

Theorem 3.1. The graph of$M(W_n)$ is P_4-decomposable if and only if n ≡ 0(mod 3) or n ≡ 2(mod 3).

Proof: Let $V(G) = \{v, v_1, v_2, \ldots, v_n\}$ be the vertices of W_n.

By definition of $M(W_n)$,let e_i, $1 \le i \le n$ and s_i, $1 \le i \le n$ be the newly introduced vertices of W_n joining the vertices v_iv_{i+1}&v_nv_1($1 \le i \le n-1$)and vv_i ($1 \le i \le n$) respectively.

$\mathbf{E(M(W_n))} = \{v_ie_i \,/\, 1 \le i \le n\} \cup \{e_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{e_nv_1\} \cup \{v_is_i \,/\, 1 \le i \le n\} \cup$

$\{e_ie_{i+1} \,/\, 1 \le i \le n\} \cup \{e_ne_1\} \cup \{vs_i \,/\, 1 \le i \le n\} \cup \{s_ie_i \,/\, 1 \le i \le n\} \cup$

$\{e_is_{i+1} \,/\, 1 \le i \le n-1\} \cup \{e_ns_1\} \cup \{s_is_j \,/\, 1 \le i \le n-1, 2 \le j \le n, i \ne j\}$

Case I : For n ≡ 2(mod 3)

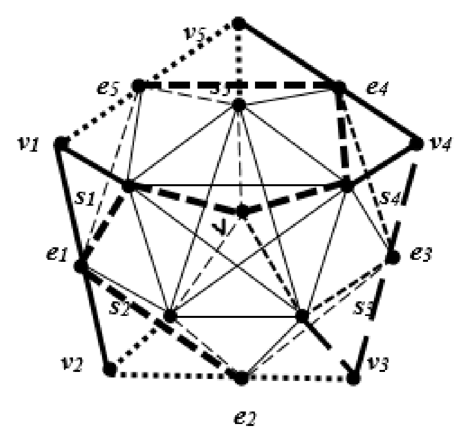

Fig.3.1. P_4-decomposition of $M(W_5)$.

$< s_iv_ie_iv_{i+1} > \cong (n-1)\ P_4$, $1 \le i \le n-1$

$< s_nv_ne_nv_1 > \cong P_4$

$< vs_ie_ie_{i+1} > \cong (n-1)\ P_4$, $1 \le i \le n-1$

$< vs_ne_ne_1 > \cong P_4$

$< e_is_{i+1}s_is_{i+2} > \cong (n-2)\ P_4$, $1 \le i \le n-2$

$< e_{n-1}s_ns_{n-1}s_1 > \cong P_4$

$< e_ns_1s_ns_2 > \cong P_4$

Hence $E(M(W_n)) = E((n-1)P_4) \cup E(P_4) \cup E((n-1)P_4) \cup E(P_4) \cup$

$E((n-2)P_4) \cup E(P_4) \cup E(P_4)$.

Thus $M(W_n)$ is P_4-decomposable.

Case II : For n ≡ 0(mod 3)

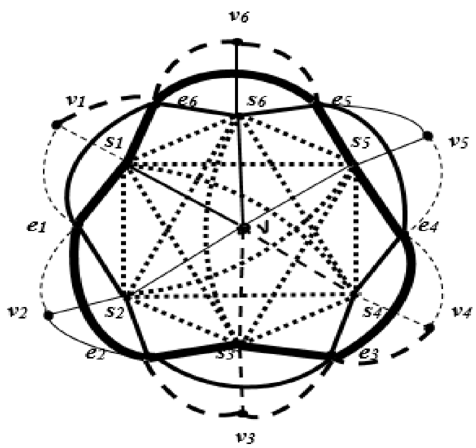

Fig.3.2. P_4-decomposition of $M(W_6)$.

$< s_i > \cong K_n$, $n \equiv 0(\text{mod } 3)$

$< s_ie_ie_{i+1}s_{i+2} > \cong (n-2)P_4$, $1 \le i \le n-2$

$< s_{n-1}e_{n-1}e_ns_1 > \cong P_4$

$< s_ne_ne_1s_2 > \cong P_4$

$< s_iv_ie_iv_{i+1} > \cong P_4$, $1 \le i \le n-1$ & $i = i+3$

$< v_is_ivs_{i+1} > \cong P_4$, $3 \le i \le n-3$ & $i = i+3$

$< v_ns_nvs_1 > \cong P_4$

$< e_iv_is_iv > \cong P_4$, $2 \le i \le n-1$ & $i = i+3$

$< e_iv_{i+1}e_{i+1}v_{i+2} > \cong P_4$, $2 \le i \le n-2$ & $i = i+3$

$< e_{n-1}v_ne_nv_1 > \cong P_4$

Hence $E(M(W_n)) = E(K_n) \cup E((n-2)P_4) \cup E(P_4) \cup E(P_4) \cup E(P_4) \cup$

$E(P_4) \cup E(P_4) \cup E(P_4) \cup E(P_4) \cup E(P_4)$.

Thus $M(W_n)$ is P_4-decomposable.

Conversely, suppose that $M(W_n)$ is P_4-decomposable.

Then $|E(M(W_n))| \equiv 0(\text{mod } 3)$ which implies that $\frac{n\ n+13}{2} \equiv 0(\text{mod } 3)$ and thus $n \equiv 0(\text{mod } 3)$ or $n \equiv 2(\text{mod } 3)$.

P_4-Decomposition of Middle graph of Sunlet graph

Let G be the sunlet graph S_n. In $M(S_n)$, there are 4n number of vertices and 7n number of edges. Its maximum degree is 6 and minimum degree is 1.

Theorem 3.2. ThegraphM(S_n) is P_4-decomposable if and only if $n \equiv 0(\text{mod } 3)$.

Proof: Let $V(G) = \{v_1,v_2,\dots,v_n\}$ and $\{u_1,u_2,\dots,u_n\}$ be the vertices of S_n. By definition of $M(S_n)$, let f_i, $1 \le i \le n$ and e_i, $1 \le i \le n$ be the newly introduced vertices of S_n joining the vertices v_iu_i $(1 \le i \le n)$ and v_iv_{i+1} & v_nv_1 $(1 \le i \le n-1)$ respectively.

$E(M(S_n))$ $= \{v_ie_i / 1 \le i \le n\} \cup \{e_iv_{i+1} / 1 \le i \le n-1\} \cup \{e_nv_1\} \cup \{u_if_i / 1 \le i \le n\} \cup$

$\{f_iv_i / 1 \le i \le n\} \cup \{f_ie_i / 1 \le i \le n\} \cup \{e_if_{i+1} / 1 \le i \le n-1\} \cup$

$\{e_nf_1\} \cup \{e_ie_{i+1} / 1 \le i \le n-1\} \cup \{e_ne_1\}$

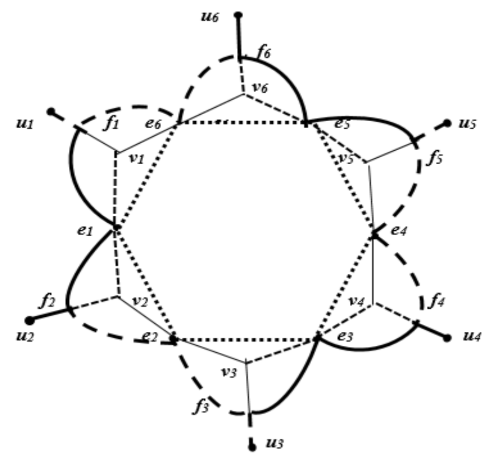

Fig.3.3. P_4-decomposition of $M(S_3)$.

$< u_if_ie_if_{i+1} > \cong (n-1)P_4$, $1 \le i \le n$

$< u_nf_ne_1f_1 > \cong P_4$

$< f_iv_ie_iv_{i+1} > \cong (n-1)P_4$, $1 \le i \le n$

$< f_nv_ne_nv_1 > \cong P_4$

$< e_i > \cong C_n$, $n \equiv 0(\text{mod } 3)$

Hence $E(M(S_n)) = E((n-1)P_4) \cup E(P_4) \cup E((n-1)P_4) \cup E(P_4) \cup E(C_n)$.

ThusM(S_n) is P_4-decomposable.

Conversely, suppose that $M(S_n)$ is P_4-decomposable.

Then $|E(M(S_n))| \equiv 0(\text{mod } 3)$ which implies that $7n \equiv 0(\text{mod } 3)$ and thusn $\equiv 0(\text{mod } 3)$.

P_4-Decomposition of Middle graph of Helm graph

Let G be the helm graph H_n. In $M(H_n)$, there are 5n+1 number of vertices and $\frac{n(n+23)}{2}$ number of edges. Its maximum degree is n+4 and minimum degree is 1.

Theorem 3.3. The graph $M(H_n)$ is P_4-decomposable if and only if $n \equiv 0 (\text{mod } 3)$ or$n \equiv 1(\text{mod}3)$.

Proof: Let $V(G) = \{v, v_1, v_2, \ldots, v_n\}$ and $\{u_1, u_2, \ldots., u_n\}$ be the vertices of H_n.

By definition of $M(H_n)$, let f_i, $1 \le i \le n$; e_i, $1 \le i \le n$and s_i, $1 \le i \le n$ be the newly introduced vertices of H_n joining the vertices $v_i u_i$ $(1 \le i \le n)$;$v_i v_{i+1}$ &$v_n v_1 (1 \le i \le n-1)andv v_i (1 \le i \le n)$ respectively.

$E(M(H_n))$ $= \{v_i e_i / 1 \le i \le n\} \cup \{e_i v_{i+1} / 1 \le i \le n-1\} \cup \{e_n v_1\} \cup \{u_i f_i / 1 \le i \le n\} \cup$

$\{f_i v_i / 1 \le i \le n\} \cup \{v_i s_i / 1 \le i \le n\} \cup \{v s_i / 1 \le i \le n\} \cup \{f_i s_i / 1 \le i \le n\} \cup$

$\{f_i e_i / 1 \le i \le n\} \cup \{e_i f_{i+1} / 1 \le i \le n-1\} \cup \{e_n f_1\} \cup \{e_i e_{i+1} / 1 \le i \le n-1\} \cup$

$\{e_n e_1\} \cup \{s_i e_i / 1 \le i \le n\} \cup \{e_i s_{i+1} / 1 \le i \le n-1\} \cup \{e_n s_1\} \cup$

$\{s_i s_j / 1 \le i \le n-1, 2 \le j \le n, i \ne j\}$

Case I : For $n \equiv 0$(mod 3)

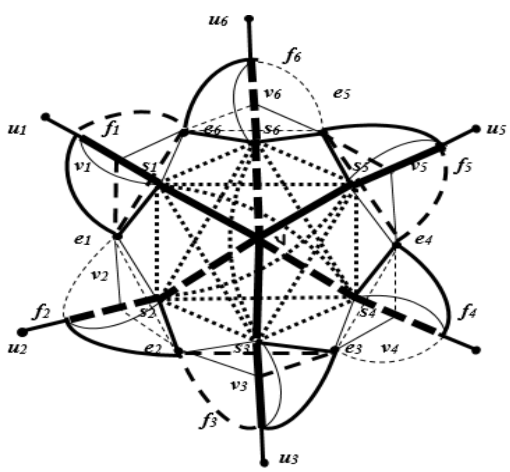

Fig.3.4. P_4-decomposition of $M(H_6)$.

$\langle s_i \rangle \cong K_n$, $n \equiv 0(\text{mod } 3)$

$\langle u_i f_i e_i s_i \rangle \cong nP_4$, $1 \le i \le n$

$\langle v s_i v_i f_i \rangle \cong nP_4$, $1 \le i \le n$

$\langle f_1 s_1 e_n v_1 \rangle \cong P_4$

$\langle f_i s_i e_{i-1} v_i \rangle \cong (n-1)P_4$, $2 \le i \le n$

$\langle v_1 e_1 e_n f_1 \rangle \cong P_4$

$\langle v_i e_i e_{i-1} f_i \rangle \cong (n-1)P_4$, $2 \le i \le n$

Hence $E(M(H_n)) = E(K_n) \cup E(nP_4) \cup E(nP_4) \cup E(P_4) \cup E((n-1)P_4) \cup$

$E(P_4) \cup E((n-1)P_4)$.

Thus $M(H_n)$ is P_4-decomposable.

Case II : For $n \equiv 1$(mod 3)

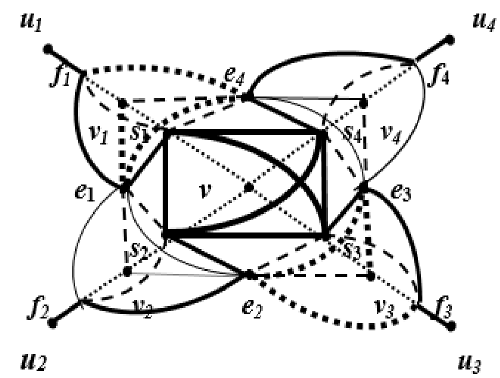

Fig.3.5. P_4-decomposition of $M(H_4)$.

$\langle s_i \rangle \cong K_n$, $n \equiv 1(\text{mod } 3)$

$\langle u_i f_i e_i s_i \rangle \cong nP_4$, $1 \le i \le n$

$\langle v s_i v_i f_i \rangle \cong nP_4$, $1 \le i \le n$

$\langle f_1 s_1 e_n v_1 \rangle \cong P_4$

$\langle f_i s_i e_{i-1} v_i \rangle \cong (n-1)P_4$, $2 \le i \le n$

$\langle v_1 e_1 e_n f_1 \rangle \cong P_4$

$\langle v_i e_i e_{i-1} f_i \rangle \cong (n-1)P_4$, $2 \le i \le n$

Hence $E(M(H_n)) = E(K_n) \cup E(nP_4) \cup E(nP_4) \cup E(P_4) \cup E((n-1)P_4) \cup$

$E(P_4) \cup E((n-1)P_4)$.

Thus $M(H_n)$ isP_4-decomposable.

Conversely, suppose that $M(H_n)$ is P_4-decomposable.

Then $|E(M(H_n))| \equiv 0(\text{mod } 3)$ which implies that $\frac{n\ n+23}{2} \equiv 0(\text{mod } 3)$ and thus$n \equiv 0(\text{mod } 3)$ or $n \equiv 1(\text{mod } 3)$.

4. CONCLUSION

In this paper, we have obtained the pattern for P_4-decomposition of line and middle graph of Wheel graph, Sunlet graph and Helm graph.

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