DOI:10.26524/krj231

Kong. Res. J. 1(2): 16-19, 2014
Kongunadu Arts and Science College, Coimbatore

NEW SEPARATION AXIOMS VIA *GENERALIZED PRE OPEN SETS

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ABSTRACT

In this Paper, we introduce the notion of *g-p open sets and *g-p continuity in topological spaces. By utilizing these notions we introduce some weak separation axioms. Also we show that some basic properties of (*g, p)-T_i, (*g, p)-D_i spaces, for i = 0, 1, 2,…

Keywords: *g-p open, *g-p continuity, (*g, p)-T_i, (*g, p)-D_i.

1. INTRODUCTION

In 2000, Jafari introduced the notion of pre-regular p-open sets and further investigated its fundamental properties in (Jafari, 2006). The concept of preopen sets and precontinuous functions in topological spaces are introduced by A. S. Mashhour *et al* in 1982.

M.K.R.S Veerakumar introduced the notion of *g-p open sets which are weaker than open sets. Since then *g-open sets have been widely used in order to introduce new spaces and functions. In this paper X and Y denote the topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by Int(A) and Cl(A) respectively.

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

2.1. Definition

A subset A of a space (X,τ) is called

i) a pre-open set (Mashhour *et al.,* 1982) if A⊆ int(cl(A))and a pre-closed set if cl(int(A)) ⊆A.

ii) a semi-open set (Levine,1963) if A ⊆cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.

iii) an α-open set (Njastad, 1965) if A ⊆ int(cl(int(A))) and an α-closed set if cl(int(cl(A))) ⊆ A.

iv) a semi-pre open set(Andrijevic,1986) if A ⊆ cl(int(cl(A))) and a semi-pre closed set if int(cl(int(A))) ⊆ A.

v) a regular open set (Stone,1937) if A = int(cl(A)) and a regular closed set if cl(int(A)) = A and

vi) δ–open set (Velicko, 1968) if for each x ∈ A there exists a regular open set G such that x ∈ G ⊆ A.

The pre-closure (resp. semi-closure, α-closure, semi-preclosure) of a subset A of a space (X,τ) is the intersection of all pre-closed (resp. semi-closed, α-closed, semi-preclosed) sets that contain A and is denoted by pcl(A) (resp. scl(A), αcl(A), spcl(A)).

2.2. Definition

A subset A of a space (X,τ) is called a g–closed set (Veera kumar, 2003) if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X,τ).

Let (X,τ) be a space and let A be a subset of X. A is called *g-closed set (Veera kumar, 2006) if cl(A) ⊆ U whenever A ⊆ U and U is g–open set of (X,τ). The complement of a *g-closed set is called *g-open. The intersection of all *g-closed (resp. δ-closed) sets containing A is called the *g-closure (resp. δ–closure) of A and is denoted by cl_{*g} (A) (resp. cl_δ (A)).

2.3. Definition

A subset A of a space (X,τ) is called a δ-preopen (Raychaudhuri and Mukherjee, 1993) if A int(cl_δ (A)). A family of δ-preopen sets in a topological space (X,τ) is denoted by $\delta PO(X,\tau)$.

3. *GENERALIZED PRE OPEN SETS

3.1. Definition

A subset A of a space (X,τ) is said to be *g-p-open if A int(cl_{*g} (A)). The complement of a *g-p-open sets is said to be *g-p-closed. The family of all *g-p-open (resp. *g-p-closed) sets in a topological space (X,τ) is denoted by $*gPO(X,\tau)$ (resp. $*gPC(X,\tau)$.

3.2. Definition

Let A be a subset of a topological space (X,τ). The intersection of all *g-p-closed (resp. δ-pre-closed) sets containing A is called the *g-p-closure (resp. δ–closure (Raychaudhuri and Mukherjee, 1993)) of A and it is denoted by pcl_{*g} (A) (resp. pcl_δ (A).

3.3. Definition

Let (X,τ) be a topological space. A subset U of X is called (*g,p) – neighbourbood of a point $x \in X$ if there exists a *g-p-open set V such that $x \in V \subseteq U$.

3.4. Theorem

For the *g-p-closure of subsets A, B in a topological space (X,τ), the following properties hold:

(i) A is *g-p-closed in (X,τ) if and only if $A = pcl_{*g}(A)$,

(ii) If $A \subset B$, then $pcl_{*g}(A) \subset pcl_{*g}(B)$

(iii) $pcl_{*g}(A)$ is *g-p-closed, that is $pcl_{*g}(A) = pcl_{*g}(pcl_{*g}(A))$ and

(iv) $x \in pcl_{*g}(A)$if and only if $A \cap V \neq \phi$ for every $V \in$ *$gPO(X,\tau)$ containing x.

Proof: It is obvious.

3.5. Theorem

For a family of subsets of a topological space (X,τ), the following properties hold:

(i) $pcl_{*g}(\cap \{A_\beta : \beta \in \Delta\}) \subset \cap\{pcl_{*g}(A_\beta): \beta \in \Delta\}$

(ii) $pcl_{*g}(\cup \{A_\beta : \beta \in \Delta\}) \supset \cup\{pcl_{*g}(A_\beta): \beta \in \Delta\}$

Proof:

(i) Since $\cap_{\beta \in \Delta} A_\beta \subset A_\beta$ for each $\beta \in \Delta$, by theorem 3.4, we have $pcl_{*g}(\cap_{\beta \in \Delta} A_\beta) \subset pcl_{*g}(A_\beta)$ for each $\beta \in \Delta$ and hence $pcl_{*g}(\cap_{\beta \in \Delta} A_\beta) \subset \cap_{\beta \in \Delta} pcl * gA_\beta$.

(ii) Since $A_\beta \subset \cup_{\beta \in \Delta} A_\beta$ for each $\beta \in \Delta$, by theorem 3.4, we have $pcl_{*g}(A_\beta) \subset pcl_{*g}(\cup_{\beta \in \Delta} A_\beta)$ for each $\beta \in \Delta$ and hence $\cup_{\beta \in \Delta} pcl * gA_\beta \subset pcl_{*g}(\cup_{\beta \in \Delta} A_\beta)$.

3.6. Theorem

Every *g-p-open sets is pre-open.

Proof: It follows from the definitions. The converse of the above theorem need not be true by the following example.

3.7. Example

Let X = {a, b, c} and τ = { X , ϕ , {a,b}}. Here {a,c} is not *g-p-open but however it is pre-open, since the 8g-p-open sets are X, ϕ, {a}, {b}, {a,b} and the pre-open sets are X, ϕ, {a}, {b}, {a,b}, {a,c}, {b,c}.

3.8. Theorem

(i) Every pre-open set is δ–pre-open (Caldas, 2010).

(ii) Every *g-p-open set is δ–pre-open.

Proof (ii): It follows from (i) and theorem 3.6.

3.9. Definition

A subset A of a topological space (X,τ) is called a $D_{(*g,p)}$ – set (resp. D_p – set , $D_{(\delta,p)}$ – set (Caldas, 2010)) if there are two U, V $\in$ *$gPO(X,\tau)$ (resp. $PO(X,\tau)$, $\delta PO(X,\tau)$) such that $U \neq X$ and $A = U - V$.

It is true that every *g-p-open (resp. pre-open) set U different from X is a $D_{(*g,p)}$ – set (resp. D_p – set) if A = U and V = ϕ.

3.10. Definition

A topological space (X,τ) is said to be

(1) (*g, p)-D_0 (resp. pre-D_0 (Caldas,2001; Jafari, 2001), (δ,p)- D_0 (Caldas, 2010)) if for any distinct pair of points x and y of X there exist a $D_{(*g,p)}$ – set (resp. D_p – set, $D_{(\delta,p)}$ – set) of X containing x but not y or a $D_{(*g,p)}$ – set (resp. D_p – set, $D_{(\delta,p)}$ – set) of X containing y but not x.

(2) (*g, p)-D_1 (resp. pre-D_1 (Caldas,2001; Jafari,2001), (δ,p)- D_1 (Caldas, 2010)) if for any distinct pair of points x and y of X there exist a $D_{(*g,p)}$ – set (resp. D_p – set, $D_{(\delta,p)}$ – set) of X containing x but not y or a $D_{(*g,p)}$ – set (resp. D_p – set, $D_{(\delta,p)}$ – set) of X containing y but not x.

(3) (*g, p)-D_2 (resp. pre-D_2 (Caldas,2001; Jafari, 2001), (δ,p)- D_2 (Caldas, 2010)) if for any distinct pair of points x and y of X there exists disjoint $D_{(*g,p)}$ – set (resp. D_p – set, $D_{(\delta,p)}$ – set) G and E of X containing x and y, respectively.

3.11. Definition

A topological space (X,τ) is said to be

(1) (*g, p)-T_0 (resp. pre-T_0 (Kar and Bhattacharyya, 1990; Nour, 1989) (δ,p)- T_0 (Caldas, 2005)) if for any distinct pair of points x and y of X there exist a *g-p-open (resp. pre-open, δ–pre-open) set U in X containing x but not y or a *g-p-open (resp. pre-open, δ–open) set V in X containing y but not x.

(2) (*g, p)-T_1 (resp. pre-T_1 (Kar and Bhattacharyya, 1990; Nour, 1989), (δ,p)- T_1 (Caldas, 2005)) if for any distinct pair of points x and y of X there exist a *g-p-open (resp. pre-open, δ–pre-open) set U in X containing x but not y and a *g-p-open (resp. pre-open, δ–pre-open) set V in X containing y but not x.

(3) (*g, p)-T_2 (resp. pre-T_2 (Kar and Bhattacharyya, 1990; Nour, 1989), (δ,p)- T_2 (Caldas, 2005)) if for any distinct pair of points x and y of X there exist a *g-p-open (resp. pre-open, δ–pre-open) sets U and V in X containing x and y, respectively, such that $U \cap V = \phi$.

3.12. Remark

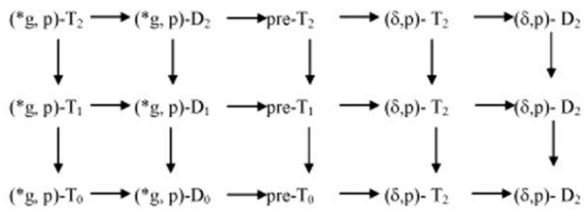
(i) If (X,τ) is (*g, p)-T_i, then it is (*g, p) – T_{i-1}, i = 1, 2.

(ii) If (X,τ) is (*g, p)-T_i, then it is (*g, p) – D_i, i = 0,1, 2.

(iii) If (X,τ) is (*g, p)-D_i, then it is (*g, p) – D_{i-1}, i = 1, 2.

(iv) If (X,τ) is (*g, p)-D_i, then it is pre–T_i, i = 0,1, 2.

By the above Remark 3.12 and [4], we have the following diagram.



3.13.Theorem

For a topological space (X,τ), the following properties hold: (X,τ) is (*g, p)-D_1 if and only if it is (*g, p)-D_2.

Proof:

Sufficiency Part: This follows from Remark 3.12.

Necessity Part: Suppose X is a (*g, p)-D_1. Then for each distinct pair x, y ∈ X, we have $D_{(*g,p)}$ – sets G_1 and G_2 such that $x \in G_1$, $y \notin G_1$; $y \in G_2$, $x \notin G_2$. Let $G_1 = U_1/ U_2$, $G_2 = U_3/ U_4$, where $U_1, U_2, U_3, U_4 \in$ *gPO(X,τ). From $x \notin G_2$ we have either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$. We discuss the two cases separately.

(1) $x \notin U_3$. From $y \notin G_1$ we have two subcases:

(a) $y \notin U_1$. From $x \in U_1/ U_2$ we have $x \in U_1/ (U_2 \cup U_3)$ and from $y \in U_3/ U_4$ we have $y \in U_3/ (U_1 \cup U_4)$. It is easy to see that $(U_1/ (U_2 \cup U_3)) \cap (U_3/ (U_1 \cup U_4)) = \phi$.

(b) $y \in U_1$ and $y \in U_2$. We have $x \in U_1/ U_2$, $y \in U_2$ and $(U_1/ U_2) \cap U_2 = \phi$.

(2) $x \in U_3$ and $x \in U_4$. We have $y \in U_3/ U_4$, $x \in U_4$ and $(U_3/ U_4) \cap U_4 = \phi$.

From the discussion above we know that the space X is (*g, p)-D_2.

3.14. Definition.

A point x ∈ X which has only X as the (*g, p) – neighbourhood is called a (*g, p)- neat point.

3.15.Theorem

If a topological spaces (X,τ) is (*g, p)-D_1, so each point x of X is contained in a $D_{(*g,p)}$ – set O = U / V and thus in U. By definition U ≠ X. This implies that x is not a (*g, p)-neat point.

3.16. Definition

A topological space (X,τ) is (*g, p)-symmetric if x and y in X, $x \in pcl_{*g}(\{y\})$ implies $y \in pcl_{*g}(\{x\})$.

3.17. Theorem

For a topological space (X,τ), the following properties hold.

(1) If {x} is *g-p-closed for each x ∈ X, then (X,τ) is (*g, p)-T_1.

(2) Every (*g, p)-T_1 space is (*g, p)-symmetric.

Proof:

(1) Suppose {p} is *g-p-closed for every p ∈ X. Let x, y ∈ X with x ≠ y. Now x ≠ y implies y ∈ X / {x}. Hence X / {x} is a *g-p-open set contained in y but not containing x. Similarly X/{y} is a *g-p-open set contained in x but not containing y. Accordingly X is a (*g, p)-T_1 space.

(2) Suppose that $y \notin pcl_{*g}(\{x\})$. Then, since x ≠ y, there exists a *g-p-open set U containing x such that $y \notin U$ and hence $x \notin pcl_{*g}(\{y\})$. This shows that $x \in pcl_{*g}(\{y\})$ implies $y \in pcl_{*g}(\{x\})$. Therefore (X,τ) is (*g, p)-symmetric.

3.18. Definition

A function f: $(X,\tau) \to (Y, \sigma)$ is said to be *g-precontinuous if for each x ∈ X and each *g-p-open set V containing f(x), there is a *g-p-open set U in X containing x such that f(U) ⊆ V.

3.19. Theorem.

If f: $(X,\tau) \to (Y, \sigma)$ is a *g-precontinuous surjective function and E is a $D_{(*g, p)}$-set in Y, then the inverse image $f^{-1}(E)$ is a $D_{(*g, p)}$-set in X.

Proof:

Let E be a $D_{(*g, p)}$-set in Y. Then there are *g-p-open sets U_1 and U_2 in Y such that $E = U_1/U_2$ and $U_1 \neq Y$. By the *g-precontinuity of f, $f^{-1}(U1)$ and $f^{-1}(U_2)$ are *g-p-open in X. Since $U_1 \neq Y$, we have $f^{-1}(U1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U1) / f^{-1}(U_2)$ is a $D_{(*g, p)}$-set.

3.20. Theorem

If (Y, σ) is (*g, p)-D_1 and f: $(X,\tau) \to (Y, \sigma)$ is a *g-precontinuous bijection, then (X,τ) is (*g, p)-D_1.

Proof:

Suppose that Y is a (*g, p)-D_1 space. Let x and y be any pair of distinct points in X. Since F is injective and Y is (*g, p)-D_1, there exist $D_{(*g, p)}$-sets G_x and G_y of Y containing f(x) and f(y), respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By theorem 3.19, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are $D_{(*g,\,p)}$-sets in X containing x and y, respectively, such that $y \notin f^{-1}(G_x)$ and $x \notin f^{-1}(G_y)$. This implies that X is a (*g, p)-D_1 space.

3.21. Theorem

A topological space (X,τ) is (*g, p)-D_1 if and only if for each pair of distinct points x, y ∈ X, there exists a *g-pre-continuous surjective function f: $(X,\tau) \rightarrow (Y, \sigma)$ such that f(x) and f(y) are distinct, where (Y, σ) is a (*g, p)-D_1 space.

Proof:

Necessity: For every pair of distinct points of X, it suffices to take the identity function on X.

Sufficiency: Let x and y be any pair of distinct points in X. By hypothesis there exists a *g-pre-continuous, surjective function f of a space X onto a (*g, p)-D_1 space Y such that $f(x) \neq f(y)$. By theorem 3.13, there exist disjoint $D_{(*g,\,p)}$-sets G_x and G_y in Y such that $f(x) \in G_x$ and $f(y) \in G_y$. Since f is *g-pre-continuous and surjective, by theorem 3.20, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint $D_{(*g,\,p)}$-sets in X containing x and y, respectively, hence by theorem 3.13, X is a (*g, p)-D_1 space.

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