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RESEARCH ARTICLE

ON THE CONTROLLABILITY OF IMPULSIVE FUNCTIONAL INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES

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ABSTRACT

In this paper, we established the some sufficient conditions for controllability of impulsive functional integrodifferential equations with nonlocal conditions by using the measure of noncompactness and Monch fixed point theorem.

Keywords: Controllability, impulsive functional integro-differential equations, Banach space.

1. INTRODUCTION

Impulsive differential equations are a class of important models which describes many evolution process that abruptly change their state at a certain moment,see the monographs of Bainov and Simonov [2], Lakshmikantham et al.[8] and have been studied extensively by many authors[3,4,10]. On the other hand, the concept of controllability is of great importance in mathematical control theory. Many authors have been studied the control of nonlinear systems with and without impulses; see for instance[5, 6, 7].

The starting point of this paper is the work in papers [5,9]. Especially, authors in [9] investigated the controllability results of mixed-type functional integro-differential evolution equations with nonlocal conditions

$$x'(t) = A(t)x(t) + f\left(t, x_t, \int_0^t h(t, s, x_s)\,ds, \int_0^b k(t, s, x_s)\,ds\right) + Bu(t), \quad (1.1)$$

$t \in J = [0, b],\ t \neq t_i,\ i = 1, \dots, s,$

$$\Delta x|_{t=t_i} = I_i(x_{t_i}),\ i = 1, \dots, s, \quad (1.2)$$

$$x_0 = \emptyset + g(x),\ t \in [-r, 0], \quad (1.3)$$

by using Monch fixed point theorem. And in[5], authors studied the following controllability of impulsive differential systems with nonlocal conditions of the form

$$x'(t) = A(t)x(t) + f(t, x(t)) + Bu(t) \text{ a.e on } [0, b] \quad (1.4)$$

$$\Delta x(t_i) = x(t_i^+) - x(t_i^-) = I_i(x(t_i)),\ i = 1, \dots, s. \quad (1.5)$$

$$x(0) + M(x) = x_0 \quad (1.6)$$

Motivated by above mentioned works[5,9],the main work of this paper is to prove the controllability results of impulsive integro-differential systems with nonlocal conditions.

$$x'(t) = A(t)x(t) + f\left(t, x(t), \int_0^t h(t, s, x(s))\,ds\right) + Bu(t) \quad (1.7)$$

$$\Delta x(t_i) = x(t_i^+) - x(t_i^-) = I_i(x(t_i)),\ i = 1, \dots, s. \quad (1.8)$$

$$x(0) + M(x) = x_0 \quad (1.9)$$

Where $A(t)$ is a family of linear operators which generates an evolution operator

$$U(t, s) : \Delta = \{(t, s) \in [0, b] \times [0, b] : 0 \leq s \leq t \leq b\} \to L(X),$$

here, X is a Banach space, $L(X)$ is the space of all bounded linear operators in $X; f: [0, b] \times X \to X; G: [0, b] \times X \to X; 0 < t_1 < \dots < t_s < t_{s+1} = b; I_i = X \to X, i = 1, \dots, s$ are impulsive functions; $M: PC([0, b]; X) \to X$; B is a bounded linear operators from a Banach space V to X and the control function $u(\cdot)$ is given in $L^2([0, b], V)$.

The paper is organized as follows: In section 2, we will recall some basic notations definition, hypothesis and necessary preliminaries. In section 3, we prove the controllability of impulsive integro-differential system with nonlocal system(1.7) –(1.9), using Monch fixed point theorem.

2. PRELIMINARIES

In this section, we recall some basic definitions and lemmas which will be used to prove our main results of this paper.

Let (X, ‖.‖) be a real banach space .We denote by C([0,b];X) the space of X- valued continuous function on [0,b]with the norm $\|x\| = \sup\{\|x(t)\|, t \in [0,b]\}$ and by L^1([0,b];X) the space of X- valued Bochner integrable functions on[0,b] with the norm $\|f\|_{L^1} = \int_0^b \|f(t)\|\,dt$.

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For the sake of simplicity, we put J=[0,b] ; J_0 = [0,t_1]; $J_i = (t_i , t_{i+1}]$,i=1,……,s. In order to define the mild solution of problem (1.7)-(1.9), we introduce the set PC([0,b];X) = {u : [0,b]→X : u is]; continuous on J_i ,i = 0,1,….,s and the right limit u(t_i^+) exists, i = 1…..,s}. It is easy to verify that PC([0,b];X) is a banach space with the norm u_{PC} = sup{ $u(t)$,t ε [0,b]} .

Definition 2.1: Let E^+ be the positive cone of an order Banach space (E,≤) . A function Φ defined on the set of all bounded subsets of the Banach space X with values in E^+ is called a measure of noncompactness (MNC) on X if Φ($co\Omega$) = Φ(Ω) for all bounded subsets $\Omega \subset X$,where $co\Omega$ stands for the closed convex hull of Ω. The MNC Φ is said:

(1) Monotone if for all bounded subsetsΩ_1, Ω_2 of X we have:

($\Omega_1 \subseteq \Omega_2$) ⇒ (Φ($\Omega_1$)≤ Φ($\Omega_2$));

(2) Nonsingular if Φ({a}∪ Ω) = Φ(Ω) for every aϵX ,$\Omega \subset X$;

(3) Regular if Φ(Ω) = 0 if only if Ω is relatively compact in x.

One of the most important examples of MNC is the noncompactness measure Of Hausdorff β defined on each bounded subset Ω of X by β(Ω)= inf { ε> 0;Ω can be covered by a finite number of balls of radii smaller than ε}.for all bounded subset Ω, Ω_1, Ω_2 of X,

(1) $\beta(\Omega_1 + \Omega_2) \leq \beta(\Omega_1) + \beta(\Omega_2)$, where $\Omega_1 + \Omega_2$ = {x+y : x $\epsilon\Omega_1$, y$\epsilon\Omega_2$}

(2) $\beta(\Omega_1 \cup \Omega_2) \leq \max\{\beta(\Omega_1) , \beta(\Omega_2) \}$;

(3) $\beta(\lambda\Omega) \leq \lambda\ \beta\ \Omega$ for any $\lambda\epsilon$ R;

(4) If the map Q : D(Q) ⊆ X →Z is Lipschitz continuous with constants k,then β_Z(QZ) ≤ kβ(Ω) for any bounded subset $\Omega \subset D(Q)$, where Z is a Banach space.

Definition 2.2: A two parameter family of bounded linear operators U(t, s), 0 ≤ s ≤ t ≤ b on X is called an evolution system if the following two conditions are satisfied:

(i) U(s, s) = I, U(t, r)U(r, s) = U(t, s) for 0 ≤ s ≤ r ≤ t ≤ b;

(ii) U(t, s) is strongly continuous for 0 ≤ s ≤ t ≤ b

and there exists M_1> 0 such that U(t, s) ≤ M_1 for any (t ,s) ϵ T.

Definition 2.3: A function x (·) ϵ PC([0,b];X) is a mild solution of(1.7)-(1.9) if

$$X\ t = U\ t, 0\ x_0 \quad - M\ x + \int_0^t U(t,s)\ f\ s, x\ s + \int_0^s (s,\tau,x(\tau))\,d\tau + Bu(s)\ ds$$

$$+ \sum_{0<t_i<t} U\ t, t_i\ I_i\,(x(t_i)), \text{for all t } \epsilon\ [0,b],$$

where $x\ 0 + M\ x = x_0$.

Definition 2.4: The system (1.7) –(1.9) is said to be controllable on the interval J if for every initial function $\varphi\epsilon$D and $x_1\epsilon$ X, there exists a control u ϵL_2 (J, V) such that themild solution x(·) of (1.7) − (1.9) satisfies.$x\ b = x_1 + M\ x$.

Definition 2.5: A countableset $\{f_n\}_{n=1}^{+\infty} \subset L^1$([0,b];X) is said to be semicompact if:

(1) The sequence $\{f_n\}_{n=1}^{+\infty}$ is relatively compact in X for a.e. t ϵ [0,b]

(2) There is a function $\mu\epsilon L^1$([0,b]; R^+) satisfying $sup_{n\geq 1}\ f_n(t) \leq \mu(t)$ for a.e.

tϵ [0,b].

Lemma 2.1: Let $\{f_n\}_{n=1}^{+\infty}$ be a sequence of function in L^1([0,b];R^+).Assume that there exist $\mu,\eta\epsilon L^1$([0,b];R^+) satisfying $\sup_{n\geq 1} f_n(t) \leq \mu(t)$ and β ($f_n\ t\ _{n=1}$) $\leq \eta(t)$ a.e. t ϵ [0,b]. Then for all t ϵ [0,b], we have$\beta(\{\int_0^t U\ t, s\ f_n\ s : n \geq 1\}) \leq 2M_1 \int_0^t \eta(s)\,ds$.

Lemma 2.2: Let (Gf)(t) = $\int_0^t U\ t,\ s\ \ f(s)ds$. If $\{f_n\}_{n=1}^{+\infty} \subset L^1$([0,b];X) is semicompact then the set $\{Gf_n\}_{n=1}^{+\infty}$is relatively compact in C([0,b];X) and moreover , if $f_n \to f_0$, then for all t ϵ [0,b] ,

$(Gf_n)\ t \to (Gf_0(t)$as n→ +∞.

Lemma 2.3: Let D be a closed convex subset of a Banach space X and 0ϵD. Assume that F: D→X is a continuous map which satisfies Monch's condition, that is, M⊆ $Discountable$, $M \subseteq co\ \{0\} \cup F(M) \Rightarrow M$ is compact. Then, there exists xϵD with x = F(x).

3. CONTROLLABILITY RESULTS

We first give the following hypothesis:

(H1) A(t) is a family of linear operators , A(t): D(A) → x, D(A) not depending on t and dense subset of X, generating an equicontinuous evolution system {U(t,s) : (t,s) ∈ Δ}, i.e.,

(t,s) → $\{U\ t, s\ x: x \in B\}$ is equicontinuous for t > 0 and for all bounded subsets B.

(H2) The function f:[0,b]× $X \to X$ satisfies:

(i) For a.e. $t\epsilon[o, b]$,the function f(t,·):X →X is continuous and for all xϵX,the function $f(\cdot, x)$:[0,b] →X is measurable;

(ii) There exists a function mϵL^1([0,b];R^+) and a nondecreasing continuous function

Ω: $R^+ \to R^+$such that $f(t, x) \leq m\ \ t\ \ \Omega(\ x\ )$, x ϵX, $t\epsilon[0, b]$and

$\lim_{n\to+\infty}$ 0

$inf \frac{\Omega(n)}{n} = 0.$

(iii) There exists $h\epsilon L^1([0,b];R^+)$ such that , for any bounded subset $D\subset X$,

$\beta\ f\ t,x\ t\ \leq\ t\ \beta(x(t))$ for a.e. $t\epsilon[0,b]$,where β is the Hausdorff MNC

(H3) The function h:[0,b]× $X\to X$ satisfies:

(i) For each t, $s\epsilon[0,b]$,the function h(t,s,·):X →X is continuous and for all xϵX,the

function $h(\cdot,\cdot,x)$:[0,b] →X is measurable;

(ii) There exists a function $m\epsilon L^1([0,b];R^+)$ such that

$(t,s,x(s))\ \leq m\ t,s\ \ x(s)$,

$x\epsilon X$, t, $s\epsilon[0,b]$and $\lim_{n\to+\infty}\ inf\frac{x(n)}{n}=0$.

(iii) There exists $\zeta\epsilon L^1([0,b];R^+)$ such that , for any bounded subset $D\subset X$,

$\beta\ \ t,s,x\ s\leq\zeta\ t,s\ \beta\ x\ s$ for a.e $t\epsilon J$,

For convenience let us take $L_0=max\ ^t_0\ m(t,s)ds$

and $\zeta^*=max\ ^t_0\ \zeta(t,s)ds$

(H4)M : PC(J,X) → X is a continuous compact operator such that

$$\lim_{y\ _{PC}\to+\infty}\frac{M(y)}{y\ _{PC}}=0;$$

(H5) The linear operator W:$L^2 J,V\to X$ is defined byWu = $^b_0 U\ b,s\ Bu(s)ds$ such that:

(i) W has an invertible operators W^{-1} which take values in $L^2 J,V\ kerW$ and there

exist positive constants M_2 , M_3 such that $B\leq M_2$ and $W^{-1}\leq M_3$;

(ii) there is $K_W\in L^1 J,R^+$ such that , for any bounded set $Q\subset X$

$\beta\ W^{-1}Q\ t\leq K_W\ t\ \beta(Q)$

(H6) Let $I_i : X\to X$, $isi=1,\ldots,s$ be a continuous

operator such that:

(i) There are nondecreasing functions I_i: $R^+\to R^+$, $i=1,\ldots,s$ such that

$I_i(x)\ \ \leq I_i\ \ X$ and $\lim_{n\to+\infty} inf\frac{I_i(n)}{n}=0$, i=1,..... ,s.

(ii) There exist constants $K_i\geq 0$,such

that$\beta\ I_i\ x\ \ (t\leq K_i\beta(x(t))$. i=1,...,s.

(H7) The following estimation holds true:
L = $(M_1+\ 2M^2M_2\ \ K_W\ _{1}\ \ _1)\ \ \ ^s_{i=1}K_i+4M_1+$
$8M12M2KWL1\ \ \ \ L1+\zeta*b<1$

Where $M_1=sup\{\ U\ t,s\ ,(t,s)\epsilon\Delta\}$

Theorem: Assume that (H1) – (H7) are satisfied,then the impulsive integrodifferential system

(1.7)-(1.9) is nonlocally controllable on J, provided that

$\frac{1}{n}[C_1+C_2\ M(x_n\ )+C_3\Omega\ n+C_4\ x_n(\tau)+$
$C_5\ ^s\ _{1i}I_i(n)]<1$.

Proof : Using hypothesis (H5)(i),for every $x\epsilon PC(J,X)$,define the control

$$u_x\ t=W^{-1}\ x_1-M\ x_n-U\ b,0\ x_0-M\ x_n$$
$$-\ ^b_0 U\ b,s\ f\ s,x_n\ s$$
$$+\ ^s_0\ \ s,\tau,x_n\ \tau\ d\tau\ ds$$
$$-\sum_{0<t_i<t} U\ t,t_i\ I_i\ x_n\ t_i$$

We shall show that, when using this control, the operator, defined by

$$Gx\ t=U(t,0)(x_0-M(x))$$
$$+\ ^t_0 U(t,s)\ f\ s,x\ s$$
$$+\ ^s_0\ \ (s,\tau,x(\tau))\ d\tau+Bu_x(s)\ \ ds$$
$$+\sum_{0<t_i<t} U\ \ t,t_i\ \ I_i\ x\ t_i \qquad (3.1)$$

has afixed point. This fixed point is then a solution of the system(1.7)-(1.9).Clearly

$x\ b=x_1-M\ x=G\ x\ (b)$which implies that the system (1.7)-(1.9) is controllable.

We define G=G_1+G_2 where

$G_1x\ t=U(t,0)(x_0-M\ x\ )+\ _{0<t_i<t}U\ t,t_i\ I_i(x(t_i))$

$G_2x\ \ t\ \ =\ \ ^t_0\ U(t,s)\ f\ \ s,x\ \ s\ \ +\ \ ^s_0\ \ (s,\tau,x(\tau))\ d\tau\ +$
$Bux(s)ds$

for all tϵ $0,\ b$ .subsequently,we will prove that Ghas a fixed point by using lemma2.3. (Monch fixed point theorem).

Step1: There exist a positive integer $n_0\geq 1$ such that $G(B_{n_0})\subseteq B_{n_0}$,where $B_{n_0}=\{x\epsilon PC\ J,X:x\leq n_0\}$.

Suppose the contrary. Then we can find $x_n\ \epsilon PC\ J,X$,$y_n=Gx_n\ \epsilon PC\ J,X$,such that $x_n\ _{PC}\leq n$

and $y_n\ _{PC}>n$ for every $n\geq 1$.

Now we have

$y_n\, t = U(t,0)(x_0 - M(x_n)$

$+ \int_0^t U(t,s)\, f\, s, x_n\, s$

$+ \int_0^s (s,\tau,x_n(\tau))\, d\tau + Bu_{x_n}(s)$

$+ \sum_{0<t_i<t} U\, t, t_i\, I_i\,(x_n\,(t_i))$

$y_n\,_{PC} \le M_1\, x_0 + M\, x_n + M_1\Omega\, x_n\,_{PC}\, m_{L^1} + M_1\, bL_0\, x_n\, \tau\,_{PC}$

$u_{x_n}\,_{L^2} \le M_3[\, x_1 + M_1\, x_0 + (1+M_1)\, M\, x_n$

$+ M_1\Omega(\, x_n\,_{PC})\, m_{L^1}$

$+M_1 bL_0\, x_n(\tau)\,_{PC}$

$+ M_1 \sum_{i=1}^{s} I_i\, x_n\,_{PC}$ (3.3)

Substituting (3.3) in (3.2) we get

$1 \le \frac{1}{n} C_1 + C_2\, M\, x + C_n\, \Omega\, n + C_3\, x\, \tau_4\, n$

$+ C_5 \sum_{i=1}^{s} I_i\, n$ (3.4)

where $C_1 = M_1 + M_1^2 M_2 b^{\frac{1}{2}} M_3\, x_0 + M_1 M_2 b^{\frac{1}{2}} M_3\, x_1$

$C_2 = M_1 + M_1 M_2 b^{\frac{1}{2}} M_3 + M_1^2 M_2 b^{\frac{1}{2}} M_3$, $C_3 = M_1\, m_{L^1} + M_1 M_2 b^{\frac{1}{2}} M_3\, m_{L^1}$

$C_4 = M_1 bL_0 + M_1^2 M_2 b^{\frac{3}{2}} M_3 L_0$, $C_5 = M_1 + M_1^2 M_2 b^{\frac{1}{2}} M_3$

by passing to the limit as $n \to +\infty$ in (3.4),we get $1 \le 0$, which is a contradiction. Thus we deduce that there is $n_0 \ge 1$ such that $G(B_{n_0}) \subseteq B_{n_0}$.

Step2: The operators G is continuous on $PC\, 0, b\, ; X$ For thispurpose, we assume that

$x_n \to x$ in $PC\, 0, b\, ; X$.Then by hypothesis (H4) and (H6), we have

$G_1 x_n \to G_1 x\,_{PC} \le M_1\, M\, x_n - M\, x$

$+ M_1 \sum_{i=1}^{s} I_i\, x_n\, t_i - I_i\, x\, t_i$ (3.5)

$G_2 x_n \to G_2 x\,_C$

$\le M_1 \int_0^b f\, s, x_n\, s - f\, s, x\, s\, ds$

$+ M_1 \int_0^b \int_0^s [\, s,\tau,x_n\, \tau - (s,\tau,x(\tau))] d\tau\, ds$

$+ M_1 M_2 b^{\frac{1}{2}} u_{x_n} - u_x\,_{L^2}$ (3.6)

$u_{x_n} - u_x\,_{L^2} \le M_3[\, M\, x_n - M\, x$

$+ M_1\, M\, x_n - M\, x$

$+M_1 \int_0^b f\, s, x_n\, s - f\, s, x\, s\, ds$

$+M_1 \int_0^b \int_0^s [\, s,\tau,x_n\, \tau - (s,\tau,x(\tau))] d\tau\, ds$

$+ M_1 \sum_{i=1}^{s} I_i\, x_n\, t_i - I_i\, x\, t_i$ (3.7)

By domination convergence theorem, we have

$Gx_n \to Gx\,_{PC} \le G_1 x_n \to G_1 x\,_{PC} + G_2 x_n \to G_2 x\,_C \to 0$, as $n \to +\infty$, ie., G is continuous.

Step 3: G(D) is equicontinuous on every J_i, i=1,...s. ie., $D \subseteq co\, 0 \cup G\, D$ is also equicontinuous on every J_i.To this end, let $y \epsilon G(D)$ and $t_1, t_2 \epsilon J_i$, $t_1 \le t_2$. There is $x\, \epsilon$ D such that

$y\, t_2 - y\, t_1 \le \| U\, t_2, 0 - U\, t_1, 0\, x_0 - M\, x \|$

$+ \int_0^{t_1} \| U\, t_2, s - U\, t_1, s\, f\, s, x\, s$

$+ \int_0^s (s,\tau,x(\tau))\, d\tau + Bu_x(s) \| ds$

$+ \int_{t_1}^{t_2} U\, t_2, 0\, f\, s, x\, s$

$+ \int_0^s s,\tau, x\, \tau\, d\tau$

$+ Bu_x\, s\, ds$ (3.8)

By the equicontinuity property of $U\, \cdot, s$ and the absolute continuity of the lebesgue integral, right hand side of the inequality equation(3.8) tends to zero independent of y as $t_2 \to t_1$.

Therefore G(D) is equicontinuous on every J_i

Step 4: Assume that D= $\{x_n\}_{n=1}^{+\infty}$ since G maps D into an equicontinuous family,G(D) is equicontinuous on J_i.Hence $D \subseteq co\, 0 \cup G\, D$ is also equicontinuous on every J_i.

Now we shall show that (GD)(t) is relatively compact in X for each t∈ J.

From the compactness of M(·) ,we have

$\beta\, (G_1 x_n\, (t)\}_{n=1}^{\infty}$

$\le M_1 \sum_{i=1}^{s} K_i \beta\, x\, t_i$ (3.9) for $t\epsilon\ 0, b$ .by lemma(2.1),we have

$$\beta_V(\ u_{x_n}\ \ s\ \ {}_{n=1}^{\infty})$$

$$\leq K_W(s)\ 2M_1 \int_0^b s\ \beta\ x\ s\ ds + 2M_1\zeta^* b\ \beta\ x\ s$$

$$+ M_1 \sum_{i=1}^{s} K_i\,\beta\ \ x\ \ t_i \qquad (3.10)$$

Then this implies that

$$\beta\ (G_2x_n\ (t)\}^{\infty}{}_{n=1}$$

$$\leq 2M_1 \int_0^b s\ \beta\ x\ s\ ds$$

$$+ 4M_1^2M_2 \int_0^b K_W\ \ s\ ds\ (\int_0^b s\ \beta\ x\ s\ \ ds)$$

$$+ 2M_1\zeta^* b\ \beta\ x\ s + 4M^2M_2\,{}_1\int_0^b K_W\ s\ ds\ \zeta^* b\beta\ x\ s$$

$$+ 2M_1^2M_2 \int_0^b K_W\ \ \eta\ \ \ d\eta \sum_{i=1}^{s} K_i\,\beta\ \ x\ \ t_i \qquad (3.11)$$

There fore

$$\beta((\mathrm{GD})(\mathrm{t}))$$

$$\leq M_1 \sum_{i=1}^{s} K_i\beta\ x(t_i)$$

$$+ 2M_1$$

$$+ 4M_1^2M_2 \int_0^b K_W\ s\ ds \int_0^b s\ \beta\ x\ s\ ds$$

$$+ 2M_1$$

$$+ 4M_1^2M_2 \int_0^b K_W\ s\ ds\ \zeta^* b\beta\ x\ s$$

$$+ 2M_1^2M_2 \int_0^b K_W\,\eta\ d\eta \sum_{i=1}^{s} K_i\ \beta\ x\ t_i \qquad (3.12)$$

we have

$$\beta\ \ GD\ \ = \qquad M_1 + 2M^2_1M_2\ K_{W\ 1}\ {}^{s}{}_{1}\ K_{L}\ {}_{i=} + 4M_1 +$$

$$8M_1^2M_2\ K_{W\ 1_L} \qquad {}_1 +_L \zeta^* b\ \beta\ x\ s$$

$$= L\beta\ x\ s$$

Where L is defined in (H7). Thus,from the Monch's condition, we get

$$\beta(D) \leq \beta(\ \varpi\ \ \ 0 \cup G\ D = \beta(G(D)) \leq L\beta(D)$$

Which implies that β(D) = 0, since hypothesis (H7) holds. So we have that D is relatively compact.Finally,due to lemma ,G has atleast a fixed point and thus the system (1.7)-(1.9) is nonlocally controllable on [0,b].

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