Kong. Res. J. 1(2): 20-22, 2014
Kongunadu Arts and Science College, Coimbatore

DOI:10.26524/krj32

ON b-mI-OPEN SETS AND b-mI- CONTINUOUS FUNCTIONS

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ABSTRACT

The purpose of this paper is to introduce b-mI-open sets in ideal minimal spaces and to investigate the relationships between minimal spaces and ideal minimal spaces. Furthermore, decomposition of continuous functions are established.

2000 Mathematics Subject Classification. 54A05, 54C10.

Keywords : b-mI-open sets ,b-I-open set, b-mI-continuous functions.

1. INTRODUCTION

An ideal (Kuratowski, 1996) I on a nonempty set X is a nonempty collection of subsets of X which satisfies (i) A ∈ I and B ⊂ I and (ii) A ∈ I and B ∈ I implies A ∪ B ∈ I. Given a topological space (X, τ) with an ideal I on X and if P(X) is the set of all subsets of X, a set operator (.)*: P(X) → P(X), called a local function (6) for A with respect to τ and I is defined as follows: for A ⊂ X , A*(I , τ) = {x ∈ X : U ∩ A ∉ I for every U ∈ τ (x)} where τ (x) = { U ∈ τ : x ∈ U}. A Kuratowski closure operator cl*(.) for a topology τ*(I ,τ), called the τ - topology, finer than τ is defined by cl*(A) = A ⊂ A* (I , τ) (Vaidyanathaswamy, 1945). A subset A of an ideal space is said to be *-dense in itself (Hayashi,1986). (resp. *-closed (Jankovic and Hamlett, 1986)) if A ⊂ A* (resp.A* ⊂ A). By a space (X, τ), we always mean a topological space (X, τ) with no separation properties assumed. If A ⊂ X, cl(A) will, respectively, denote the closure and interior of A in (X, τ) and int*(A) will denote the interior of A in (X, τ). The notion of I-open sets was introduced by Jankovic *et al*. in 1992, further it was investigated by Abd El-Momsef. In 1965, Njastad initiated the investigation of α- open sets, Hatir and Noiri introduced the notion of α-I-open sets in an ideal topological spaces (X, τ,I), where τ is a topology and I is an ideal.

Maki *et al*. (1996) introduced the notion of minimal structure and minimal spaces as a generalization of topological spaces on a given nonempty set. Also, generalized topologies which are other generalization of topology were defined by Csaszar in 2002. Further, it was studied by Popa and Noiri in 2000. A subfamily $\mathcal{M}$ of the power set P(X) of a non empty set X a minimal structure, if ϕ, X ∈ $\mathcal{M}$. (X, $\mathcal{M}$) is called a minimal space (m-space). A subset A of X is said to be m-open (Maki, *et al*., 1996) if A ∈ $\mathcal{M}$. The complement of a m-open set is called a m-closed set. Define m-int(A) = ∪{U : U ⊂ A,U ∈ $\mathcal{M}$} and m-c(A) = ∩{F : A⊂F, X-F∈ $\mathcal{M}$}. A minimal (X, $\mathcal{M}$) has the property[$\mathcal{U}$] (Popa and Noiri, 2000) if the arbitrary union of m-open sets is again a m-open set.

Ozbakir and Yildirim in 2009 have defined the minimal local function A_m^* in an ideal minimal space (X, $\mathcal{M}$, I). The notion of α-mI-open set, semi-mI-open set, β-mI-open set in (X, $\mathcal{M}$, I) were introduced and investigated by Parimala. In this paper, by using the local function A_m^* we introduce and investigate the notion of α-mI-open set in (X, $\mathcal{M}$, I). Furthermore, decompositions of continuous function are established.

2. PRELIMINARIES

2.1. Definition (Ozbakiri and Yildirim, 2009) Let (X, $\mathcal{M}$) be a minimal space with an ideal I and X (.)*m be a set operator from P(X) to P(X) (P(X) is the set of all subsets of X). For a subset A⊂X, A^*_m(I, $\mathcal{M}$) ={ x ∈ X: U_m∩ A ∉I; for every U_m ∈ U_m (x)} is called the minimal local function of A with respect to I and $\mathcal{M}$. We will simply write A^*_m for A^*_m(I, $\mathcal{M}$).

2.2. (*Theorem* Ozbakiri and Yildirim, 2009) Let (X, $\mathcal{M}$) be a minimal space with I,I′ ideals on X and A, B be subsets of X. Then

(i) A⊂B ⇒ $A^*_m \subset B^*_m$,

(ii) I⊂I′⇒ $A^*_m(I') \subset A^*_m(I)$,

(iii) A^*_m=m-cl(A^*_m) ⊂m-cl(A),

(iv) $A^*_m \cup B^*_m \subseteq (A\cup B)^*_m$

(v) $(A^*_m)^*_m \subseteq A^*_m$

2.3. Remark (Ozbakiri and Yildirim, 2009) If (X, $\mathcal{M}$) has property (Lashien and Nasef, 1992) then $A^*_m \cup B^*_m = (A\cup B)^*_m$

Definition 2.4. (Ozbakiri and Yildirim, 2009) Let (X, $\mathcal{M}$) be a minimal space with an ideal I on X. The set operator m-cl* is called a minimal *-closure and is defined as m-cl*(A) = A∪ A_m^* for A⊂X. We will denote by $\mathcal{M}$*(I, $\mathcal{M}$) the minimal structure generated by m-cl*, that is, $\mathcal{M}$*(I, $\mathcal{M}$)={ U⊂X: m-cl*(X-U)=X-U}. $\mathcal{M}$*(I, $\mathcal{M}$) is called *-minimal structure which is finer than $\mathcal{M}$. The elements of $\mathcal{M}$*(I, $\mathcal{M}$) are called minimal *-open (briefly, m*-open) and the complement of an m*-open set is called minimal *-closed (briefly, m*-closed).

Throughout the paper we simply write $\mathcal{M}$* for $\mathcal{M}$*(I, $\mathcal{M}$). If I is an ideal on X, then (X, $\mathcal{M}$,I) is called an ideal minimal space.

2.5. Proposition (Ozbakiri and Yildirim, 2009) The set operator m-cl* satisfies the following conditions:

(i) A⊂m-cl*(A),

(ii) m-cl*(ϕ)= ϕ and m-cl*(X)=X,

(iii) If A⊂B, then m-cl*(A) ⊂ m-cl*(B),

(iv) m-cl*(A)∪ m-cl(B) ⊂ m-cl*(A∪B).

2.6. Remark

If (X, $\mathcal{M}$) has property (Lashien and Nasef, 1992) then m-cl*(m-cl*(A))=m-cl*(A) and m-cl*(A)∪ m-cl*(B)= m-cl*(A∪B).

2.7. Lemma (Renukadevi, *et al.,* 2005) Let (X, τ,I) be an ideal space and A⊂ X. If A⊂A*, then A* = cl(A*) = cl(A) = cl*(A).

2.8. Definition

A subset A of an ideal minimal space (X, $\mathcal{M}$, I) is said to be

(i) α-mI-open set (Parimala, 2010) if A⊂ m-int(m-cl*(m-int(A))).

(ii) semi-mI-open set (Parimala, 2010) if A⊂ m-cl*(m-int(A))).

(iii) β-mI-open set (Parimala, 2010) if A⊂ m-cl(m-int(m-cl*(A))).

(iv) mI-open set (Ozbakiri and Yildirim, 2009) if A⊂ m-int(A_m^*)

(v) pre-mI-open set (Parimala, 2010) if A⊂m-int(m-cl*(A)).

3. B-MI-OPEN SET AND B-MI-CLOSED SET

3.1.Definition

A subset A of an ideal minimal space (X, $\mathcal{M}$, I) is said to be a b-mI-open set if A⊂m-cl(m-int(A)) ∪ m-int(m-cl(A)). The complement of a b-mI-open set is a b-mi-closed set.

3.2. Theorem.

For a subset of an ideal minimal space, the following condition hold.

(i) Every b-mI-open set is b-m-open.

(ii) SmIO(X, $\mathcal{M}$) ∪ PmIO(X, $\mathcal{M}$) ⊂ BmIO(X, $\mathcal{M}$).

(iii) Every m-open set is b-mI-open.

Proof. (i) Let A be b-mI-open set. Then we have

A⊂m-int(m-cl*(A)) ∪ cl*(m-int(A))

⊂m-int(A_m^*∪ A) ∪((m-int(A))* ∪(m-int(A)))

⊂m-int(m-cl(A) ∪ A) ∪(m-cl(m-int(A)) ∪(m-int(A)))

⊂m-int(m-cl(A) ∪m-cl(m-int(A))

Therefore this shows that A is b-m-open.

The proof is obvious for (ii),(iii).

3.3.Theorem

For a subset of an ideal minimal space, the following conditions hold.

(i) Every pre-mI-open set is b-mI-open.

(ii) Every semi-mI-open set is b-mI-open.

(iii) Every b-mI-open set is β–mI-open.

Proof. The proof is obvious for (i), (ii).

(iii) Let A be an b-mI-open set. Then we have

A⊂m-int(m-cl*(A)) ∪cl*(m-int(A))

⊂m-cl(m-int(m-cl*(A))) ∪[(m-int(A))* ∪m-int(A)]

⊂m-cl(m-int(m-cl*(A))) ∪(m-cl(m-int(A)) ∪m-int(A))

⊂m-cl(m-int(m-cl*(A))) ∪(m-cl(m-int(A)))

⊂m-cl(m-int(m-cl*(A)))

Therefore this shows that A is an β–mI-open.

3.4. Example

(i) Let X={a,b,c,d}, $\mathcal{M}$={X ϕ,{a,b},{b,c},{c,d}} and I= ϕ. Let A={a,b,c} is b-mI-open but not semi-mI-open set.

(ii) Let X={a,b,c,d}, $\mathcal{M}$={X ϕ,{a},{b},{a,b,c},{b,c},{a,c}} and I={{a}, ϕ}. Let A={a,c,d} is β–mI-open but not pre-mI-open set.

(iii) Let X={a,b,c,d}, $\mathcal{M}$={X ϕ,{a},{b},{b,c,d}} and I={ ϕ , {b},{c},{b,c}}. Let A={a,b,c} is β–mI-open but not b-mI-open set.

3.5. Theorem

Let A be a b-mI-open set such that int(A)= ϕ, then A is pre-mI-open.

Proof. Since A⊂m-int(m-cl*(A)) ∪cl*(m-int(A))=m-int(m-cl*(A)) ∪m-cl*(ϕ)=m-int(m-(cl*(A))). Then A is pre-mI-open.

3.6.Theorem

Let (X, τ, $\mathcal{M}$) be an ideal minimal space and A,B subset of X.

(i) If $U_\alpha \in$ BmIO(X, τ) for each $\alpha \in \Delta$, then $\cup$ { U_α: $\alpha \in \Delta$} $\in$ BmIO(X, τ).

(ii) If $A_\alpha \in$ BmIO(X, τ) and B$\in \mathcal{M}$, then A$\cap B \in$ BmIO(X, τ)

Proof. (i)Since $U_\alpha \in$ BmIO(X, τ), we have $U_\alpha \subset$m-int(m-cl*(U_α)) $\cup$ cl*(m-int(U_α)) for every $\alpha \in \Delta$.
$\subset \cup_{\alpha \in \Delta}$[{(m-int($U_\alpha$))* $\cup$m-int(U_α)} $\cup$(m-int(($U^*_\alpha \cup$ U_α))]

$\subset$[{(m-int($\cup_{\alpha \in \Delta}$ U_α))* $\cup$m-int($\cup_{\alpha \in \Delta}$ U_α)} $\cup$(m-int(($\cup_{\alpha \in \Delta} U^*_\alpha \cup$ ($\cup_{\alpha \in \Delta} U\alpha$))]
[$cl*(m-int(\cup_{\alpha \in \Delta} U\alpha))$ $\cup$m-int(m-cl*($\cup_{\alpha \in \Delta} U\alpha$))]
Therefore, $\cup_{\alpha \in \Delta} U\alpha$ is b-mI-open.

(ii) Let A$\in$BmIO(X, $\mathcal{M}$) and B$\in \mathcal{M}$.Then A$\subset$m-int(m-cl*(A)) $\cup$cl*(m-int(A))

A$\cap B \subset$[m-int(m-cl*(A)) $\cup$ $cl*m-int\ A$] $\cap B$

$\subset$[{m-int(A))* $\cup$m-int(A)} $\cup$(m-int(A*$\cup$A))] $\cap B$

$\subset$[{m-int(A$\cap B$))* $\cup$m-int(A$\cap$B)} $\cup$(m-int((A$\cap B$)* $\cup$(A$\cap B$)))]

$\subset$m-int(m-cl*(A$\cap B$)) $\cup$cl*(m-int(A$\cap B$)). Then A$\cap B$ is b-mI-open.

(i.e.) A$\cap B$ BmIO(X, τ).

3.7. Definition

A subset A of a space (X, $\mathcal{M}$, I) is said to be a b-mI-closed set if its complement is b-mI-open.

3.8. Theorem

If a subset A of a space (X, τ, $\mathcal{M}$) is b-mI-closed then m-int(m-cl*(A)) $\cap$m-cl*m-int(A)) $\subset$A.

Proof. Since A is b-mI-closed, X-A BmIO(X, $\mathcal{M}$) and since $\mathcal{M}^*$ is finer than $\mathcal{M}$ X-A$\subset$ $cl*(m-int(X-A)$ $\cup$ m-int(m-cl*(X-A) $\cup$ $m-int(m-cl*(X-A)))$

$\subset$cl(m-int(X-A)) $\cup$m-int(m-cl(X-A)))

=[X-cl(m-int(A)] $\cup$[X-(m-int(m-cl(A)))]

=X-[cl(m-int(A) $\cap$(m-int(m-cl(A)))]. Therefore, m-int(m0cl*(A)) $\cap$m-cl*(m-int(A)) $\subset$A.

3.9. Corollary

Let A be a subset of (X, τ, $\mathcal{M}$) such that X-[m-int(m-cl*(A))]=m-cl*(m-int(X-A)) and X-[m-cl*m-int(A))]=m-int(m-cl*(X-A)). Then A is b-mI-closed if and only if m-int(m-cl*(A)) $\cap$m-cl*(m-int(A)) $\subset$A.

Proof. Necessity:

This is an immediate consequence of Theorem 3.8.

Sufficiency:

Let m-int(m-cl*(A)) $\cap$m-cl*(m-int(A)) $\subset$A. Then X-A$\subset$X-[cl(m-int(A)] $\cap$(m-int(m-cl(A)))] $\subset$[X-m-cl*(m-int(A))] $\cup$[X-m-int(m-cl*(A))]=[cl*(m-int(X-A) $\cup$m-int(m-cl*(X-A))). Thus X-A is b-mI-open and so A is b-mI-closed.

4. DECOMPOSITION OF CONTINUITY VIA MINIMAL IDEALS

4.1.Definition

A function f: (X,$\mathcal{M}$, I)$\rightarrow$(Y,σ) is said to be b-mI-continuous if for every V$\in \sigma$,f^{-1}(V) is an b-mI-open set of (X,$\mathcal{M}$, I).

4.2. Definition

A function f: (X,$\mathcal{M}$)$\rightarrow$(Y,σ) is said to be b-m-continuous if for every V$\in \sigma$,f^{-1}(V) is an b-m-open set of (X,$\mathcal{M}$).

4.3. Theorem

If a function f: (X,$\mathcal{M}$, I)$\rightarrow$(Y,σ) is said to be b-mI-continuous then f is b-m-continuous.

Proof. The proof is obvious.

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