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RESEARCH ARTICLE

ON DOMINATOR CHROMATIC NUMBER OF RADIAL GRAPH OF SOME GRAPHS

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ABSTRACT

A dominator coloring is a coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class. In this paper, we obtain the Dominator Chromatic number of the Radial graph for the Central graph of Star graph, Super-radial graph for Middle graph of Cycle and Central graph of Path.

Keywords: Domination, Dominator Coloring, Radial Graph, Distance, Radius and Diameter.

1. INTRODUCTION

All graphs considered here are finite, undirected, simple graphs. For graph theoretic terminology refer to D. B. West [1]. Let G be a graph, with vertex set

V (G) and edge set E (G).

In a graph G, the distance d (u, v) between a pair of vertices u and v is the length of a shortest path joining them.

A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. Graph coloring and domination are two major areas in graph theory. A set is a dominating set if every vertex of V(G) \ D has a neighbor in D. An excellent detail of domination is given in the book by Haynes et .al, [2].

A dominator coloring of a graph G is a proper coloring of graph such that every vertex of V dominates all vertices of at least one color class (possibly its own class). i.e., it is coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class. Dominator chromatic number is the minimum number of color classes in a dominator coloring of G, and this concept was introduced by Ralucca Michelle Gera in 2006 [3]. The dominator coloring was studied in [4]. The dominator coloring of Trees, Bipartite graph, Central, Middle graph of Path and Cycle graph were also studied in various papers [5 -10].

The Middle graph of G, denoted by M(G) is defined as follows.

The vertex set of M(G) is . Two vertices x, y in the vertex set of M(G)

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i. x, y are in E(G) and x, y are adjacent in G.
ii. x is in V(G), y is in E(G), and x, y are incident in G.

The Central graph [11] C(G) of a graph G denoted by C(G) is formed by adding a new vertex on each edge of G and joining each pair of vertices of the original graph which were previously non-adjacent.

2. DOMINATOR CHROMATIC NUMBER OF RADIAL GRAPHS

Theorem 2.1. [6] Let Pn be any path on n ≥ 5 vertices, then

$$R(P_n) = \begin{cases} (n/2)K_2 \ if \ \ n \ is \ even \\ P_3 \cup ((n-3)/2)K_3 \ \ if \ \ n \ is \ odd \end{cases}$$

Theorem 2.2. For any n ≥ 2, the dominator chromatic number of radial graph of C(K1,n) = n + 1 i.e.,

$$\chi_d R(C(K_{1,n})) = n+1$$

Proof.

By the definition of the Central graph, the edge vvi, 1 ≤ i ≤ n be subdivided by the vertices u'j, in C(K1,n). Clearly

$$V(C(K_{1,n})) = \{v\} \cup \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\}.$$

Now we define a coloring

$$c : V(R(C(K_{1,n}))) \rightarrow \{1,2,3,\ldots.n+1\}$$

for

$$c_i = \begin{cases} u_i & for\ 1 \le i \le n \\ v_i & for\ 1 \le i \le n \end{cases}$$

It is not hard to see that above assignment is a proper coloring and also a dominator coloring of the graph R(C(K1,n)). Every vertex in dominates any one color class of ci and the root vertex v dominate itself. Therefore

To Prove

Let us assume that

$$\chi_d R(C(K_{1,n})) < n+1$$

. i.e., $\chi_d R(C(K_{1,n})) = n$.

We assign n colors $\{v_i, u_i : 1 \le i \le n\}$ for proper dominator coloring. Assign n colors for ui because ui forms a clique. If we assign any one the colors from the vertices vi to v then vertices vvi are bicolored, so we have to assign the color n + 1 to the root vertex v. Therefore dominator coloring with n colors is not possible. Hence, $\chi_d R(C(K_{1,n})) \ge n+1$. An easy check shows that $\chi_d R(C(K_{1,n})) = n+1$.

Theorem 2.3. For any n ≥ 6, the dominator chromatic number of radial graph of Path is,

$$R(P_n) = \begin{cases} \lceil n/2 \rceil + 1, & if\ \ n\ is\ even \\ \lceil n/2 \rceil, & if\ \ n\ is\ odd \end{cases}$$

Proof.

Let G = R(Pn) and its dominator chromatic number be $\chi_d(G)$. Let D be the minimal dominating set of the given graph G and $V(R(P_n) = \{v_i : 1 \le i \le n\}$. The dominator coloring of G is given in the following cases.

Case 1: If n is even

Now we define a coloring $c : V(R(C(P_n))) \rightarrow \{1,2,3,\ldots.\lceil n/2 \rceil + 1\}$ for

$$c_i = \begin{cases} v_i & for\ 1 \le i \le n/2 \\ \lfloor n/2 \rfloor + 1 & otherwise \end{cases}$$

It is easy to see that above assignment is a dominator coloring of *R(Pn)* as the set $D = v_i,\ 2 \le i \le \lceil n/2 \rceil$ dominates itself and the remaining vertices of *V - D* are dominates atleast any one of the color classes in *D*.

Case 2: If n is odd.

Now we de ne a coloring $c : V(R(C(P_n))) \rightarrow \{1,2,3,\ldots.\lceil n/2 \rceil\}$ for

$$c_i = \begin{cases} v_{i-1} & for\ 2 \le i \le \lceil n/2 \rceil \\ \lfloor n/2 \rfloor & otherwise \end{cases}$$

Above assignment is a dominator coloring of *G* as the set $D = v_i,\ 2 \le i \le \lceil n/2 \rceil$ dominate itself and the remaining vertices of *V - D* dominate any color class in *D*. This completes the proof of the theorem.

Theorem 2.4. For every *n ≥ 6* the dominator chromatic $\chi_d R^*(C(P_n)) = n$ number of super radial graph of *C(Pn) = n*. ie.,

Proof. Let *G =* *R *(C(Pn))* and its dominator chromatic number be $\chi_d(G)$. Let *D* be the minimal dominating set of given graph *G*, and $V(R^*(C(P_n))) = \{v_i, u_i : 1 \le i \le n\}$ be the vertices of radial graph of central graph of path.

$$c_i = \begin{cases} v_i & for\ 1 \le i \le n \\ u_i & for\ 1 \le i \le n \end{cases}$$

Next we define a coloring $c : V(R(C(P_n))) \rightarrow \{1,2,3,\ldots.n\}$ for

It is not hard to see that above coloring is proper coloring. Next we have to show that it is a dominator coloring of given graph *G*. A procedure to obtain a dominator coloring as follows. Here $u_i : 1 \le i \le n$ forms a clique, so that we must assign *n* colors to u_i. Every vertex in *G* dominates at least any one color classes of v_i, u_i. Therefore this proper coloring gives rise to a dominator coloring for the respective graphs. Hence $\chi_d R^*(C(P_n)) = n$.

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