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B-CHROMATIC NUMBER OF CENTRAL GRAPH OF LADDER GRAPH AND COMPLETE GRAPH

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ABSTRACT

In this paper, we discuss about the b-colouring andb-chromatic number of Central graph of Ladder graph and Central graph of Complete graph denoted as $C(L_n)$ and $C(K_n)$ respectively. Also we discuss about the structural properties of $C(L_n)$ and $C(K_n)$.

Keywords: Central graph, b-colouringand b-chromatic number.

1. INTRODUCTION AND PRELIMINARIES

All graphs considered here are finite and simple. Notations and terminology not defined here will conform to those in (Bondy and Murty, 1976). For a graph G, let $V(G)$, $E(G)$, $p(G)$, $q(G)$ and $\Delta(G)$,$\delta(G)$, respectively, be the set of vertices, the set of edges, the order, the size, the maximum and minimum degree of G.

Let G be a graph without loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$. The smallest number k for which G admits a colouring with k colours is the chromatic number $\chi(G)$ of G. Many graph invariants related to colourings have been defined. Most of them try to minimize the number of colours used to colour the vertices under some constraints. For some invariants, it is meaningful to try to maximize this number. The b-chromatic number is one such example.

A *b-colouring* (Jakovac and Klavzar, 2010; Jakovac and Peterin, 2012; Kouider, 2002; Kouider and Zaker, 2006) of a graph G is a proper colouring of the vertices of G such that there exist a vertex in each colour class joined to at least a vertex in each other colour class; such a vertex is called a dominating vertex. The *b-chromatic number* of a graph G, denoted by $\varphi(G)$, is the maximal integer k such that G may have a b-colouring by k colours. This parameter has been derived by Irving and Manlove in the year 1999.

The central graph (Thilagavathi *et al.,* 2010; VernoldVivin *et al.,* 2008; Vijayalakshmi and Thilagavathi, 2010) of any graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacentvertices of G. By the definition pC(G) = p + q. For any (p, q) graph there exactly p vertices of degree p -1 and q vertices of degree 2 in C (G).

2. THE B-CHROMATIC NUMBER OFCENTRAL GRAPH OF LADDER GRAPH

2.1. Theorem

For any integer $1<n<20$, $\varphi[C(L_n)] = n+\frac{n}{2}$

Proof

Let $L_n$be any ladder with vertices [12] $v_1,v_2...v_n$ labeled in anticlockwise direction. Let v_{ij} be the newly introduced vertex in the edge connecting v_i and v_j, $1<i, j< 2n$ in $C(L_n)$. Now in $C(L_n)$, we see that each v_i is adjacent with all the vertices except v_{i+1} and $v_{2n-(i-1)}$ for $i=1,2,3...2n$. Let $S=\{v_{ij}/1<i,j<2n\}$.

Now assign a proper colouring to these vertices as follows. Consider a colour class $C=\{c_1,c_2,c_3..c_n\}$.For $i=1,2,3...2n$ assign the colour c_i to the vertex v_i. Due to the above mentioned non adjacency this will not produce a b-chromatic colouring.

To overcome this, assign a proper colouring to v_{ij}'s. consider an arbitrary vertex v_i, but v_i is not adjacent with v_{i+1} and v_{i-1},thus the vertex v_i to realize the colourc_i, we should colour$v_{i,i+1}$as c_{i-1} and $v_{i,i-1}$ as c_{i+1}. Now v_i will realize the colour c_i. Next consider the vertex v_{i+1} which is coloured as c_{i+1}. In order to realize the colour $c_{i+1,}$ colour the two neighbors of v_{i+1} as c_{i+1} and c_i but by previous colouring v_i had left out only one vertex to be coloured. Thus realization of v_{i+1} is not possible. Proceeding in the same manner this will not be possible for remaining vertices. This implies that assigning different colours to v_i is not possible otherwise there should be repetition of colours. A close examination will reveal that there should be minimum of $\frac{n}{2}$ repetitions.

Now assign the colour $c_{i-[i/6]}$ to the vertex v_i for $i=1,2,3,...2n-1$ and assign the colour $c_{i-\{[i/6]+1\}}$ to the vertex v_{2n}.

To make the above colouring as b-chromatic, assign a proper colouring to the remaining $v_{ij}'s$. Suppose if we assign any new colour to any of the $v_{ij}'s$ as $c_{i-\{[i/6]+1\}}$ it contradicts the definition of b-chromatic colouring. Hence we should assign only the existing colours to $v_{ij}'s$ inorder that all the vertices $v_1, v_2 \ldots v_{2n}$ realizes its own colour. Thus by the colouring procedure and under observation the above said colouring is maximal and b-chromatic colouring.

Example

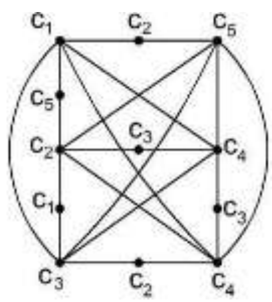

Figure:1 $\boldsymbol{\varphi[C(L_3)] = 5}$

2.2. Structural Properties of Central Graph of Ladder Graph

- Number of vertices in $C(L_n) = 5n\text{-}2$
- Maximum degree in $C(L_n)$ i.e. $\Delta = 2n\text{-}1$
- Minimum degree in $C(L_n)$ i.e. $\delta = 2$

3. B-CHROMATIC NUMBER OF CENTRAL GRAPH OF COMPLETE GRAPH

3.1. Theorem

For any integer $n>3$, $\varphi[C(K_n)] = n\text{-}1$

Example

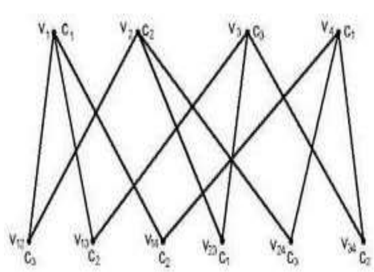

Figure :2 $\boldsymbol{\varphi[C(K_4)] = 3}$

3.2. Structural Properties of Central graph of Complete Graph

- Number of vertices in $C(K_n) = \frac{n(n+1)}{2}$
- Number of edges in $C(K_n) = n(n\text{-}1)$
- Maximum degree in $C(K_n) = (n\text{-}1)$
- Minimum degree in $C(K_n) = 2$
- n vertices with maximum degree $n\text{-}1$ and $\frac{(n-1)}{2}$ vertices of degree .

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