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RESEARCH ARTICLE

ON r - DYNAMIC VERTEX COLORING OF SOME GRAPHS

C.S. Gomathi and N. Mohanapriya*
PG and Research Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore-641029, Tamil Nadu, India

ABSTRACT

An r - dynamic is an proper vertex k coloring with an function $a : V(G) \rightarrow T$ where $|T| = k$ and it is k – colorable. It can be defined as $|a(Neigh(v)| \geq \min\{r, deg_G(v)\}$, for each $v \in V(G)$. The r- dynamic chromatic number of a graph G is the minutest coloring *k* of G which is r-dynamic k -colorable and denoted by $\chi_r(G)$. In this paper, we have obtain the r- coloring results of some special graphs such as Flower graph F_n, Double cone graph $C_{p,q}$, Triangle snake graph TS_n, Helm graph H_m, Crossed prism graph CP_q.

Mathematics subject classification: 05C15

Keywords: r- dynamic coloring; Flower graph; Double cone graph; Triangle snake graph; Helm graph; Crossed prism graph.

1. INTRODUCTION

Let the graph G be undirected connected with m vertices and n edges. In this paper, the study is made on the r -dynamic chromatic number which was first introduced by Montgomery [6]. An r-dynamic coloring is an proper vertex coloring such that coloring of the adjacent vertices should not receives the similar color and each vertex $V(G)$ has neighbors atleast $min\{r, deg_G(v)\}$ different color classes. When $r = 1$ then it is equal to the chromatic number of the graph. The bounds of r - dynamic coloring was given minimum and maximum degree. Furthermore, some of references are given for r - dynamic coloring in the following paper [1], [7], [4], [5] and the bounds are studied from [2] [3]. The most familiar lower bound was given in the following lemma.

Lemma 1. $\chi_r(G) \geq \min\{\mathrm{r}, \Delta(\mathrm{G})\} + 1$

The *Flower graph* are derived from the Helm graph by adding pendent edge from pendent vertex to the hub vertex.

The *Double cone graph* is an special case of cone graph $C_m + \overline{K}_n$, $\overline{K} = 2$.

The *Triangle snake graph* is derived from the path graph p_{n-1}, and addi- tionally adding edges (v_{2i-1}, v_{2i+1}) for $i = 1, 2, \ldots, n-1$. It is denoted as TS_n, *n* is odd.

The *Helm graph* is obtained from the wheel graph by adding pendent edge to each vertex. So that each pendent edge has additionally a vertex. [7]

The *Crossed prism graph* is an even graph and it is obtained by taking two disjoint cycle and adding an edges (v_i, v_{2i+1}) and (v_{i+1}, v_{2i}) (v_{i+1}, v_{2i}) for $i = 1, 3, \cdots, n-1$.

2. Results on *r*- dynamic coloring on some graphs

Lemma 2. Let F_a be the flower graph. The lower bound for r -dynamic chromatic number of flower graph is

$$\chi_r[F_a] \geq \begin{cases} \delta + 1, & 1 \leq r \leq \delta \\ \Delta + 1, & \delta + 1 \leq r \leq \Delta \end{cases}$$

Proof : Let $V(F_a) = \{u, u_i, t_j : 1 \leq i \leq a, 1 \leq j \leq a\}$ where *u* is the center vertex which joins the vertices u_i and t_j for $1 \leq i \leq a$ and $1 \leq j \leq a$. The minimum degree of F_a is 2 and the maximum degree is 2a. For $1 \leq r \leq \delta$, the

*Correspondence: N. Mohanapriya, PG and Research Department of Mathematics, Kongunadu Arts and Science College, Coimbatore – 641 029, Tamil Nadu, India. E.mail: n.mohanamaths@gmail.com

vertices $V = u, u_1, u_2$ persuade a clique of order $\delta + 1$ in F_a. Thus, $\chi_r[F_a] \geq \delta + 1$. Thus for $\delta + 1 \leq r \leq \Delta$ based on Lemma 1, we have $\chi_r[F_a] \geq \min\{r, \Delta[F_a]\} = \Delta + 1$. Thus, it completes the proof.

Theorem 1. For $a \geq 3$, the r- dynamic coloring of the flower graph F_a are

$$\chi_r[F_a] = \begin{cases} 3, for\ 1 \leq r \leq 2, a\ is\ even \\ 4, for\ 1 \leq r \leq 3, a\ is\ odd \\ 6, for\ 4 \leq r \leq 5, a = 5 \\ r + 1, \ otherwise \end{cases}$$

Proof : The upper bound for the theorem are illustrated in the following cases:

Case : 1 $1 \leq r \leq 2, a\ is\ even$

Based on the lemma 2, we have $\chi_r[F_a] \geq 3$. To find the upper bound cosider the following coloring: Color the middle vertex u with color 3, then color the vertices u_i with color 1 and 2 alternatively for $1 \leq i \leq a$ and next color the vertices t_j with color 2 and 1 orderly for $1 \leq j \leq a$ Thus, $\chi_r[F_a] \leq 3$. Hence, we have $\chi_r[F_a] = 3$.

Case: 2 $1 \leq r \leq 2, a\ is\ even$

Based on the lemma 2, we have $\chi_r[F_a] \geq 4$. To find the upper bound consider the following coloring: Color the vertices u_i with color 1 and 2 alter- natively for $1 \leq i \leq a - 1$, then color the vertex u_a with color 3. Next color the vertices t_j with color 2 and 1 in order for $1 \leq j \leq a$ and finally color the middle vertex u with color 4. Thus, $\chi_r[F_a] \leq 4$. Therefore, $\chi_r[F_a] = 4$.

Case: 3 $4 \leq r \leq 5, a = 5$

Based on the lemma 2, we have $\chi_r[F_a] \geq 6$. To find the upper bound consider the following coloring: At this particular case, we color the vertices u_i with separate colors such as $1,2,\cdots,5$ for $1 \leq i \leq a$ Then color the vertices t_j with color 3,4,5,1 and 2 for $1 \leq j \leq a$. At the last color the center vertex u with color 6. Thus, $\chi_r[F_a] \leq 6$. Hence, we have $\chi_r[F_a] \leq 6$.

Case: 4 otherwise

Based on the lemma 2, we have $\chi_r[F_a] \geq r + 1$. To find the upper bound consider the following coloring: Color the center vertices u_i and t_j with color $1,2,\cdots,r$ based on the r –adjacency condition. But, there is also a need of one more color to satisfy the condition so color the vertex u with color $r + 1$. Therefore, $\chi_r[F_a] \leq r + 1$. Hence, we have $\chi_r[F_a] = r + 1$.

Theorem 2. For $p = 2,\ q \geq 4$, the r - dynamic coloring of the double cone graph $C_{p,q}$ are

$$\chi_r[C_{p,q}] = \begin{cases} 3, for\ 1 \leq r \leq 2, q is\ even \\ 4, for\ 1 \leq r \leq 2\ and\ q\ is\ odd\ 4 \\ 4, for\ r = 3,\ m \equiv 0 (mod\ 3) \\ 5, for\ r \neq 3,\ m \not\equiv 0 (mod\ 3) \\ 7,\ for\ 4 \leq r \leq 5, m \equiv 2 (mod\ 3) \\ r + 2, otherwise \end{cases}$$

Proof: Let the vertices of $C_{p,q} = V(\overline{K}_2) \cup V(C_q)$ i.e., $V(C_{p,q}) = \{ p_1, p_2, u_i : 1 \leq i \leq q\}$. The vertices of $\overline{K}_2$ are adjacent to each vertices of C_q but p_1 is not adjacent to p_2 .The edges are $\{ u_i u_{i+1} : 1 \leq i \leq q - 1\} \cup \{u_q u_1\} \cup \{p_1 u_i : 1 \leq i \leq q\} \cup \{p_2 u_i : 1 \leq i \leq q\}$. The maximum and minimum degree of $C_{2,q}$ are q and 4.

Case: 1 $1 \leq r \leq 2$

If q is *even*, color p_1 and p_2 with color 1 and the leftover vertices u_i with color 2 and 3 alternatively for $1 \leq i \leq q$. Hence, $\chi_r[C_{p,q}] \leq 3$. Based on the Lemma 1 we have $\chi_r[C_{p,q}] \geq \min\{r, \Delta[C_{p,q}]\} + 1 = 3$. Hence, $\chi_r[C_{p,q}] = 3$.

If q is *odd*, Color p_1 and p_2 with color 1. The remaining vertices u_i with color 2 and for $1 \leq i \leq q - 1$ and the leftover vertex u_q with color 4. Therefore, $\chi_r[C_{p,q}] \leq 4$. Based on the Lemma 1 we have $\chi_r[C_{p,q}] \geq \min\{r, \Delta[C_{p,q}]\} + 1 = 3$. Thus, $\chi_r[C_{p,q}] \geq 4$. Hence, $\chi_r[C_{p,q}] = 4$.

Case: 2 $r = 3$

If $m \equiv 0(mod\ 3)$, then color the vertices u_i with color 2,3 and 4 orderly for $1 \leq i \leq q$. Atlast color the vertices p_1 and p_2 with color 1. Thus, $\chi_r[C_{p,q}] \leq 4$. Based on the Lemma 1 the lower bound for $C_{p,q}$ are $\chi_r[C_{p,q}] \geq 4$. Therefore, $\chi_r[C_{p,q}] = 4$.

If $m \not\equiv 0(mod\ 3)$, then color the vertices p_1 and p_2 as in the case $m \equiv 0(mod\ 3)$ and color u_i with color 2,3 and 4 sequencingly for $1 \leq i \leq q-1$. But, to satisfy the r-adjacency condition we need of one more color so color the vertex u_q with color 5. Therefore, $\chi_r[C_{p,q}] \leq 5$. Based on the Lemma 1 the lower bound for $C_{p,q}$ are $\chi_r[C_{p,q}] \geq 5$. Therefore, $\chi_r[C_{p,q}] = 5$.

Case: 3 $4 \leq r \leq 5, m \equiv 2(mod\ 3)$

Based on the Lemma 1 the lower bound for $C_{p,q}$ are $\chi_r[C_{p,q}] \geq 7$. The upper bound can be calculated by the following coloring: Color the vertices p_1 and p_2 with color 1, then color the vertices u_i with color 2,3 and 4 alternatively for $1 \leq i \leq q-2$. Next color the vertices u_{q-1} with color 5 and u_q with color 6. But still there is an need of one color so change the color of vertex p_2 with color 7. Hence, $\chi_r[C_{p,q}] \leq 7$. Therefore, $\chi_r[C_{p,q}] = 7$.

Case: 4 Otherwise

Based on the Lemma 1 the lower bound for $C_{2,q}$ are $\chi_r[C_{p,q}] \geq r+2$. The upper bound can be calculated by the following coloring: Other than the about values of r, the result of double cone graph leads to $r+2$. So, color the vertex p_1 with color 1, then color the vertices u_i with color $2, 3, \cdots, r+1$ for $1 \leq i \leq q$ either randomly or sequencingly but the coloring should satisfy the r-adjacency condition. Finally, color the vertex p_2 with color $r+2$. Hence, $\chi_r(C_{p,q}) \leq r+2$. Therefore, $\chi_r(C_{p,q}) = r+2$.

Lemma 3. Let TS_n, be the Triangle snake graph. The lower bound for r- dynamic chromatic number of Triangle snake graph is

$$\chi_r[TS_n] \geq \begin{cases} \delta + 1, & 1 \leq r \leq \delta \\ \Delta + 1, & \delta + 1 \leq r \leq \Delta \end{cases}$$

Proof: Let $V(TS_n) = \{u_i : 1 \leq i \leq n-1\} \cup \{u_{ii+1} : 1 \leq i \leq n-2\}$ where u_i are the vertices of path P_{n-1} and u_{ii+1} are the vertices corresponding to the edges u_i and u_{i+1}. Thus the minimum degree of TS_n are 2 and the maximum degree is 4. For $1 \leq r \leq \delta$, the vertices $V = u_{12}, u_1, u_2$ persuade a clique of order $\delta + 1$ in (TS_n). Thus, $\chi_r[TS_n] \geq \delta + 1$. Thus for $\delta + 1 \leq r \leq \Delta$ based on Lemma 1, we have $\chi_r[TS_n] \geq \min\{r, \Delta[TS_n]\} = \Delta + 1$. Thus, it completes the proof.

Theorem 3. For $n \geq 3$, n is odd the r- dynamic coloring of the Triangle snake graph TS_n are

$$\chi_r[TS_n] \geq \begin{cases} 3, & 1 \leq r \leq \delta \\ \Delta + 1, & \delta + 1 \leq r \leq \Delta \end{cases}$$

Proof: The upper bound for Triangle snake graph are illustrated in following cases:

Case : 1 $1 \leq r \leq \delta$

Based on the Lemma 3 the lower bound of TS_n are $\chi_r(TS_n) \geq 3$. To find the upper bound we consider the following coloring: color the vertices u_i with color 1 and 2 alternatively for $1 \leq i \leq n-1$. Then color the remaining vertices u_{ii+1} with single color 3 for $1 \leq i \leq n-2$. Thus we have $\chi_r(TS_n) \leq 3$. Therefore, $\chi_r(TS_n) = 3$.

Case: 2 $\delta + 1 \leq r \leq \Delta$

- Based on the Lemma 3 the lower bound of TS_n are $\chi_r(TS_n) \geq 4$. To find the upper bound we consider the following coloring: when $r = 3$, color the vertices u_i with color 1,2 and 3 orderly for $1 \leq i \leq n-1$ and finally color the last set of vertices u_{ii+1} with color 4 for $1 \leq i \leq n-2$. Therefore, $\chi_r(TS_n) \leq 4$. $\chi_r(TS_n) = 4$.
- Based on the Lemma 3 the lower bound of TS_n are $\chi_r(TS_n) \geq r+1$. To find the upper bound we consider the following coloring: when $r = 4$, color the vertices u_i with the colors as given in the $r = 3$.

Next, color the vertices u_{ii+1} with color 4 and 5 alternatively for $1 \leq i \leq n-2$. Therefore, $\chi_r(TS_n) \leq 5$. Thus we have obtained $\mathrm{r}+1$ colors. Hence, we have $\chi_r(TS_n) \leq \mathrm{r}+1$. $\chi_r(TS_n) = \mathrm{r}+1$.

Theorem 4. For $\mathrm{q} \geq 4$, q is even the r - dynamic coloring of the Crossed prism graph CP_q are

$$\chi_r[CP_q] = \begin{cases} 2, for\ r = 1 \\ 4, for\ 2 \leq r \leq \Delta, q \equiv 0(mod\ 4) \\ 3, for\ r = 2, q \not\equiv 1(mod\ 3) \\ 4, for\ r = 2, q \equiv 1(mod\ 3) \\ 5, for\ r \geq \Delta, q \not\equiv 0(mod\ 4) \\ 6,\ for\ r \geq \Delta, q = 60L+6, L \in W \end{cases}$$

Proof: The vertex set $V(CP_q) = \{ t_i : 1 \leq i \leq q\} \cup \{x_j : 1 \leq j \leq q \}$ where t_i are the inner cycle of crossed prism and x_j is the outer cycle. The edges are crossed between the vertices t_i and x_{j+1} for i is odd, and the next set of edges are crosses between t_i and x_{j-1} for i is even. The maximum and minimum degree of CP_q are $\delta = \Delta = 3$. Since q is even, we get $q/2$ set of crossed vertices. Here t_1 is the second vertex of first crossed prism and t_2 is the first vertex of second crossed prism. It continues upto q. Similarly, it is same as for x_j.

Case: 1 $r = 1$

Color the vertices t_i and x_j with color 1 and 2 alternatively for $1 \leq i \leq q$. Thus $\chi_r(CP_q) \leq 2$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 2$. Therefore, $\chi_r(CP_q) = 2$.

Case: 2 $2 \leq r \leq \Delta, q \equiv 0(mod\ 4)$

Color the vertices t_i with color 1,2,3 and 4 alternatively for $1 \leq i \leq q$. Next we need to color the verices x_j which is quite different since, it does not follow any order or sequencing. The coloring of the vertices x_j is dependent on the coloring of the vertices t_i. Since it is an even graph, color the vertex x_1 with the color of the vertex t_2 and color the vertex x_2 with the color of the vertex t_1. Then, color the vertex x_3 with the color of t_4 and the vertex x_4 with the color of t_3. By continuing this way, color the vertex x_q with color of the vertex t_{q-1} and the vertex x_{q-1} with the color of t_q. Thus the coloring are interchanged between every pair of vertices. Thence, $\chi_r(CP_q) \leq 4$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 4$. Therefore, $\chi_r(CP_q) = 4$.

Case: 3 $r = 2$

Sub case: 1 $q \not\equiv 1(mod\ 3)$, in this subcase there are two subdivisions which are as follows:

If $q \equiv 0(mod\ 3)$, color the vertices t_i with color 1,2 and 3 alternatively for $1 \leq i \leq q$. Similarly, color the vertices x_j with the colors have used in t_i $1 \leq i \leq q$. Therefore, $\chi_2(CP_q) \leq 4$.

If $q \equiv 2(mod\ 3)$, color the vertices t_i with color 1, 2 and 3 orderly for $1 \leq i \leq q-2$. Next, color the vertex t_{q-1} with color 1_2 and color the vertex t_q with color 2. Finally, color the vertices x_j from the color 1, 2 and 3 either sequencingly or unorderly for $1 \leq j \leq q$ but with an r - adjacency condition. Thus, $\chi_2(CP_q) \leq 4$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 4$. Therefore, $\chi_r(CP_q) = 4$.

Sub case : 2 $q \equiv 1(mod\ 3)$

Color the vertices t_i with colors 1,2,3 and 4 orderly for $1 \leq i \leq q-2$ and color the vertex t_{q-1} with color 2 and color the vertex t_q with color 3. Next, color the vertices x_j with the same colors as given in t_i unorderly with an 2-adjacency condition. Thus, $\chi_2(CP_q) \leq 4$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 4$. Therefore, $\chi_r(CP_q) = 4$.

Case: 4 $r \geq \Delta$

- If $q \equiv 0(mod\ 4)$, then color the vertices t_i with color 1,2,3,4 and 5 for $1 \leq i \leq q$. Then the coloring of the vertices x_j is dependent on the coloring of the vertices t_i. The coloring of x_j follows the similar way as given in case-2 but with five colors. Thence, $\chi_\mathrm{r}(CP_q) \leq 5$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 5$. Therefore, $\chi_r(CP_q) = 5$.

- If $q = 60L + 6$, it is an special case of crossed prism graph, since we need to give six different colors to satisfy the Δ -adjacency condition. So color the vertices t_i with color 1, 2 and 3 sequencingly for $1 \leq i \leq q$ and color the vertices x_j with color 4, 5 and 6 orderly for $1 \leq i \leq q$. Therefore, $\chi_{\mathrm{r}}(CP_q) \leq 6$. Based on the Lemma 1 the lower bound for CP_q are $\chi_r(CP_q) \geq 6$. Therefore, $\chi_r(CP_q) = 6$.

Lemma 4. Let H_m be the Helm graph. The lower bound for r-dynamic chromatic number of Helm graph is

$$\chi_r[H_m] \geq \begin{cases} \delta + 1, & 1 \leq r \leq \delta \\ \Delta + 1, & \delta + 1 \leq r \leq \Delta \end{cases}$$

Proof: The vertices of H_m are $\{ x, x_i, y_i \colon 1 \leq i \leq m \}$. x is the hub vertex which is connected to vertex of cycle x_i and a pendent edge is add to each vertex of x_i. The vertices at the pendent edge are named as y. The minimum and maximum degree of H_m are $\delta = 1$ and $\Delta = \mathrm{m}$. For $1 \leq r \leq \delta$, the vertices $V = x, x_1, x_2$ persuade a clique of order $\delta + 1$ in (H_m). Thus, $\chi_r[H_m] \geq \delta + 1$. Thus for $\delta + 1 \leq r \leq \Delta$ based on Lemma 1, we have $\chi_r[H_m] \geq \min\{r, \Delta[H_m]\} = \Delta + 1$. Thus, it completes the proof.

Theorem 5. For $m \geq 3$, the r- dynamic coloring of the Helm graph H_m are

$$\chi_r[H_m] = \begin{cases} 4, 1 \leq r \leq 3 \; and \; m \; is \; odd \\ 3, 1 \leq r \leq 2 \text{ and m is even} \\ \mathrm{r} + 1, \text{otherwise} \end{cases}$$

Proof : The upper bound for the Helm graph are obtained from the following cases:

Case: 1 $1 \leq r \leq 3$

Based on the Lemma 4, we have $\chi_r(H_m) \geq 4$ To find the upper bound consider the following coloring: color the vertices x_i with color 1,2 orderly for $1 \leq i \leq m - 1$ and color the vertex x_m with color 3. Next color the vertices y_i with color 3 for $1 \leq i \leq m - 1$ and the vertex y_m with color 1. Finally, the last vertex x with color 4. Hence, $\chi_r(H_m) \leq 4$. Therefore, $\chi_r(H_m) = 4$.

Case : 2 $1 \leq r \leq 2$

Based on the Lemma 4, we have $\chi_r(H_m) \geq 3$. To find the upper bound consider the following coloring: color the vertices x_i with color 1,2 alternatively for $1 \leq i \leq m$ and color the vertex y_i with color 2 and 1 orderly for $1 \leq i \leq m$. The color the vertex x with color 3. Hence, $\chi_r(H_m) \leq 3$. Thus, $\chi_r(H_m) = 3$.

Case: 3 Otherwise

Based on the Lemma 4, we have $\chi_r(H_m) \geq 6$. To find the upper bound consider the following coloring: in this case, there is an special case that is $m = 5$, for $r = 4,5$. At this case we receives six colors. i.e., color the vertices x_i with five different colors for $1 \leq i \leq m$. Similarly, color the vertices y_i with the same colors as given in x_i but in different order with 4, 5-adjacency condition and color the vertex x with color 6. Hence, $\chi_r(H_m) \leq 6$. Thus, $\chi_r(H_m) = 6$.

Based on the Lemma 4, we have $\chi_r(H_m) \geq 5$. To find the upper bound consider the following coloring: next is to receive $r + 1$ colors, at $r = 4$ and $m \neq 5$ color the vertices x_i and y_i for $1 \leq i \leq m$ with colors 1,2,3,4 either orderly or unorderly but with 4- adjacency condition. Even though there is four different colors we also need one more color to satisfy 4- adjacency condition. So color the vertex x with color 5. Therefore, $\chi_r(H_m) \leq 5$. Thus, $\chi_r(H_m) = 5$.

Based on the Lemma 4, we have $\chi_r(H_m) \geq \mathrm{r} + 1$. To find the upper bound consider the following coloring: next at $r = 5$, color the vertices x_i and y_i for $1 \leq i \leq m$ with colors 1,2,3,4 either orderly or unorderly but with 5-adjacency condition. Here also we need one more color, so color the vertex x with color 6. Therefore, $\chi_r(H_m) \leq 6$. Thus, proceeding by this way, at $r = m$ color the vertices x_i and y_i for $1 \leq i \leq m$ with colors $1,2, \cdots, m$ but with r -adjacency condition. Atlast color the hub vertex x with color m+1. Thus, $\chi_r(H_m) \leq \mathrm{m} + 1$. Therefore, $\chi_r(H_m) \leq \mathrm{r} + 1$. Thus, $\chi_r(H_m) = \mathrm{r} + 1$.

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