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RESEARCH ARTICLE

INTERVAL NETWORK ANALYSIS IN PROJECT MANAGEMENT

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ABSTRACT

This paper deals with an analysis of Critical Path Method (CPM) and Programme Evaluation Review Technique (PERT) in Project Network. Here, we solve the PERT and CPM methodology using intervals and we determine the critical path and project duration of the network. We can also convert the fuzzy parameters (triangular and trapezoidal fuzzy numbers) into intervals using $\alpha -$ cuts. After which, we calculate the project duration and critical path. To illustrate this, numerical examples are provided.

Keywords: PERT, CPM, Triangular fuzzy numbers, Trapezoidal fuzzy numbers, $\alpha -$ cut interval number, activity, event, network, path.

INTRODUCTION

Operations Research is relatively a new discipline which originated during World War II, and later became very popular throughout the world. It is used successfully in almost all the fields. Operations Research helps us to make better decisions in complex scenarios. It also includes the application of scientific tools for finding the optimum solution to a problem involving the operations of a system.

Network Analysis is considered to be one of the important applications of Operations research. Network is a graphical representation of a project operation, and it is composed of activities and events that must be completed in order to attain their objectives. Network scheduling is used for planning and scheduling large projects in the field of construction, software development, research development and designs, etc. It is considered as a complex decision making problem because of conflicting goals, limited resources and the difficulties involved in modelling real world problems accurately. Critical Path Method (CPM) and Programme Evaluation Review Technique (PERT) are widely recognized as valuable tools for network analysis. A deterministic data for activity time is used in CPM whereas random time data are employed in PERT. However in reality, it is often difficult to obtain the exact activity time estimates of all activities due to factors such as uncertainty of information as well as the variations in management scenario. Thus, the conventional approaches, both deterministic and random process become less effective in conveying the imprecision or vagueness of the linguistic values. In order to handle this uncertainty, the interval concept and fuzzy techniques can be used as an important decision making tool.

Interval computation has been developed by Dwyer [1] and Moore. The concept of fuzzy sets were proposed by Zadeh [3].Suparna Das and Chakraverty [2] have proposed the new methods for solving the linear simultaneous equations with interval and fuzzy parameters (triangular and trapezoidal).

The rest of this paper is organized as follows:

In section 2, basic preliminaries of interval and its arithmetic, types of intervals, ordering of intervals, fuzzy number, $\alpha -$ cut of a fuzzy number, various types of fuzzy numbers and its arithmetic are given. In section 3, we explain the network diagram representation, few basic terminologies involved in PERT and CPM Technique and the rules for constructing project networks. In section 4, PERT and CPM procedures are given at intervals and conversion of fuzzy numbers in intervals. In section 5, numerical examples are given. Finally, the conclusion.

2. PRELIMINARIES

In this section we will discuss about interval and its arithmetic, types of intervals, ordering of intervals, Fuzzy number, $\alpha -$ cut of a fuzzy number, various types of fuzzy numbers and its arithmetic.

2.1.Interval Number

An interval number A is defined as A = [β_1,β_2] ={x: β_1≤x≤β_2,x∈R}. Here, β_1,β_2∈R are the lower and upper bounds of the interval.

2.1.1. Arithmetic operations of interval

Let A $= [\beta_1, \beta_2]$ and B= $[\gamma_1, \gamma_2]$ be two intervals. Then

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Addition: A+B =$[\beta_1+\gamma_1, \beta_2+\gamma_2]$

Subtraction: A-B = $[\beta_1-\gamma_2, \beta_2-\gamma_1]$

Multiplication:

A*B= $[\min(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2), \max(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2)]$

Division: $\frac{A}{B}=\frac{[\beta_1,\beta_2]}{[\gamma_1,\gamma_2]}=[\beta_1,\beta_2].\frac{1}{[\gamma_1,\gamma_2]}$

where $\frac{1}{[\gamma_1,\gamma_2]}=\left[\frac{1}{\gamma_2},\frac{1}{\gamma_1}\right]$, $0 \notin [\gamma_1,\gamma_2]$

$\frac{1}{[\gamma_1,0]}=\left[-\infty,\frac{1}{\gamma_1}\right]$ $\frac{1}{[0,\gamma_2]}=\left[\frac{1}{\gamma_2},\infty\right]$ and $\frac{1}{[\gamma_1,\gamma_2]}=\left[-\infty,\frac{1}{\gamma_1}\right]\cup\left[\frac{1}{\gamma_2},\infty\right]$= $[-\infty, \infty]$, $0 \in [\gamma_1,\gamma_2]$

Scalar Multiplication:

Let A= $[\beta_1,\beta_2]$ then uA= $[u\beta_1, u\beta_2]$, u≥ 0 and uA= $[u\beta_2, u\beta_1]$, u ≤ 0.

2.2 Types of intervals

Let A = $[\beta_1,\beta_2]$ and B= $[\gamma_1,\gamma_2]$ be two intervals. Therefore these can be classified into three types as follows:

Type I- Non overlapping intervals:

If two intervals are disjoint then they are known as non overlapping intervals.

Type II- Partially overlapping intervals:

If one interval contains the other interval partially then they are known as partially overlapping intervals.

Type III- Completely overlapping intervals:

If one interval is completely contained in the other interval then they are known as completely overlapping intervals.

These three types of intervals are shown in Fig. 1

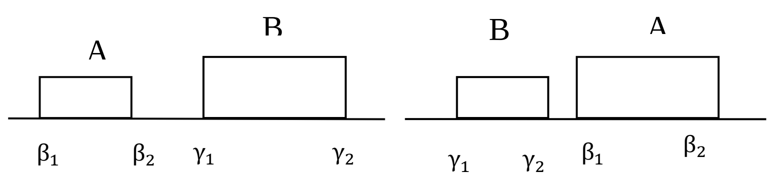

Fig. 1(a): Type – I intervals

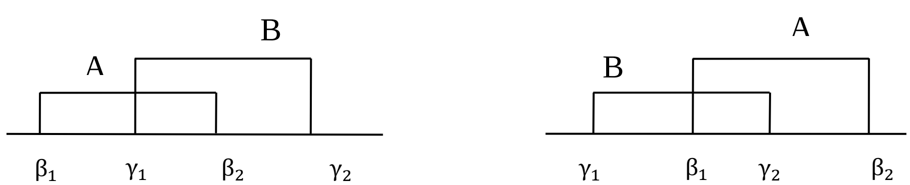

Fig. 1(b): Type – II intervals

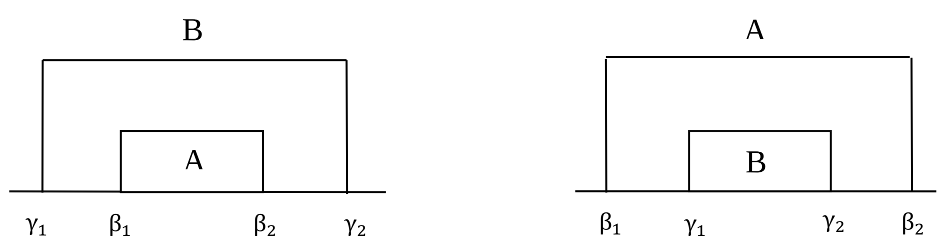

Fig. 1(c): Type – III intervals

Fig. 1: Different types of intervals

2.3. Ordering of intervals

Let A= $[\beta_1, \beta_2]$ be the interval number. It can also be expressed by its centre and radius and is denoted by $\langle a_C, a_W \rangle$, where $a_C = \frac{\beta_1+\beta_2}{2}$ and $a_W = \frac{\beta_2-\beta_1}{2}$ and they are known as centre and radius of the interval respectively.

Let A= $[\beta_1, \beta_2] = \langle a_C, a_W \rangle$ and B $=[\gamma_1, \gamma_2] = \langle b_C, b_W \rangle$. Then the relation on interval number is defined as

i. A < B iff $a_C < b_C$ whenever $a_C \neq b_C$.

ii. A > B iff $a_C > b_C$ whenever $a_C \neq b_C$.

iii. A < B iff $a_w < b_w$ whenever $a_C = b_C$.

iv. A > B iff $a_w > b_w$ whenever $a_C = b_C$.

2.4 Definition

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$: $X \rightarrow [0,1]$. The assigned value indicates the membership grade or degree of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\widetilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

2.5. Fuzzy Number

A fuzzy set $\tilde{A}$ defined on a set of real number R is said to be a fuzzy number, if its membership function $\mu_{\tilde{A}}(x): R \rightarrow [0, 1]$ that satisfies the following properties.

a. $\widetilde{A}$ is convex.

   i.e., $\mu_{\tilde{A}}\{\lambda x_1 + (1-\lambda)x_2\} > min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in R$ and $\lambda \in [0,1]$.

b. $\widetilde{A}$ is normal there exists an element $x_0 \in \widetilde{A}$ such that $\mu_{\tilde{A}}(x_0) = 1$.

c. $\mu_{\tilde{A}}(x)$ is piecewise continuous.

2.6. Types of Fuzzy Number

Here we will discuss two types of fuzzy numbers, namely triangular and trapezoidal fuzzy numbers.

2.6.1. Triangular fuzzy number

A fuzzy number $\tilde{A} = (\beta_1, \beta_2, \beta_3)$ is said to be triangular fuzzy number if its membership function is given by, where β_1, β_2 and β_3 are real numbers

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \quad , & x < \beta_1 \\ \dfrac{x-\beta_1}{\beta_2-\beta_1}, & \beta_1 \leq x \leq \beta_2 \\ \dfrac{\beta_3-x}{\beta_3-\beta_2}, & \beta_2 \leq x \leq \beta_3 \\ 0 \quad , & x > \beta_3 \end{cases}$$

2.6.2. Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (\beta_1, \beta_2, \beta_3, \beta_4)$ is said to be trapezoidal fuzzy number if its membership function is given by, where $\beta_1, \beta_2, \beta_3$ and β_4 are real numbers

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \quad , & x < \beta_1 \\ \dfrac{x-\beta_1}{\beta_2-\beta_1}, & \beta_1 \leq x \leq \beta_2 \\ 1 \quad , & \beta_2 \leq x \leq \beta_3 \\ \dfrac{\beta_4-x}{\beta_4-\beta_3}, & \beta_3 \leq x \leq \beta_4 \\ 0 \quad , & x > \beta_4 \end{cases}$$

2.7. Definition of Alpha Cut

The crisp set of elements that belong to the fuzzy set $\widetilde{A}$ at least to the degree α is called the $\alpha -$ level set.

$\widetilde{A}_\alpha = \{x \in X \in : \mu_{\tilde{A}}(x) \geq \alpha\}$

2.8. Conversion from Fuzzy Number to Interval Using Alpha Cut

2.8.1. Triangular Fuzzy Number to Interval

Let $\tilde{A} = (\beta_1, \beta_2, \beta_3)$ be the triangular fuzzy number then to find α -cut of $\tilde{A}$ we first set α equal to the left and right membership function of $\tilde{A}$. That is,

$\alpha = \frac{x-\beta_1}{\beta_2-\beta_1}$ and $\alpha = \frac{\beta_3-x}{\beta_3-\beta_2}$

Expressing x in terms of α we have, $x = \alpha(\beta_2 - \beta_1) + \beta_1$ and $x = -\alpha(\beta_3 - \beta_2) + \beta_3$.

Therefore we can write the fuzzy interval in terms of α –cut interval as:

$$\widetilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_3 - \beta_2) + \beta_3]$$

2.8.2. Trapezoidal Fuzzy Number to Interval

Let $\tilde{A} = (\beta_1, \beta_2, \beta_3, \beta_4)$ be the trapezoidal fuzzy number then to find α -cut of $\tilde{A}$. We follow the similar procedure as above, we can write the fuzzy interval in terms of α –cut interval as:

$$\widetilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$$

2.9. Fuzzy Arithmetic

As $\tilde{A}_\alpha$ is now interval, so fuzzy addition, subtraction, multiplication and division are the same as interval arithmetic.

3. NETWORK TECHNIQUES – PERT AND CPM

3.1. Network

A graphical representation of all the activities and events arranged in a logical and sequential order. It is also called an arrow diagram. It plays an important role in project management. Managing a project involves three main functions. They are planning, scheduling and controlling.

Planning: It involves setting the objectives, identifying the various tasks or jobs to be performed and determining the requirements of resources such as men, materials, machines, etc. of the project. It estimates the cost and duration of various activities of the project.

Scheduling: It is the determination of the time that should be required to perform each activity at each stage of the project.

Controlling: It consists of reviewing the progress of the project and if there is any difference between the planned schedule and actual performance, remedial action is taken to correct this difference wherever possible.

3.2. Network Diagram representation

3.2.1. Activity

It is a task of work to be done in a project. An activity is represented by an arrow with a node at the beginning indicating the start of the activity and a node at the end indicating the termination of the activity.It is classified into four categories

a) Predecessor Activity:

It is an activity that must be completed immediately before another activity starts.

b) Successor activity:

Activity which starts only after one or more of other activities are completed and immediately succeeds at them is called successor activity.

c) Concurrent activity:

If more than one activity is accomplished at the same time then it is known as concurrent activity.

d) Dummy Activity:

It is an imaginary activity which does not consume any kind of resource and which serves the purpose of indicating the predecessor or successor relationship during the construction of the project.

3.2.2. Event

An event denotes a point in time signifying the start and finish of an activity. This is usually represented by a circle in a network which is also known as a node. It contains a number that helps to identify its location. The events are classified into three categories. They are:

a) Merge event:

When one or more activity comes and joins an event, such event is called merge event

b) Burst event:

When one or more activitiesleave an event, such an event is called a burst event.

c) Merge and burst event:

An activity may be merged and burst at the same time.

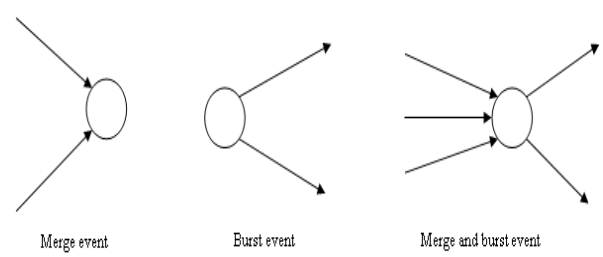

Figure 2: Categories of event

3.3. Rules for constructing a project network

Two nodes should get connected by only one activity.

No two activities can be identified by the same end events.

The network should have only one start event and one end event.

Arrows should not cross each other.

Nodes are numbered uniquely to identify an activity. Tail (start) node should be smaller than the head (end) node of an activity.

3.4. Errors to be avoided in constructing a project network

i. **Looping:** It represents the performance of activities in a cyclic manner.

ii. **Dangling:** Disconnecting an activity before the completion of all activities in a network diagram.

iii. **Redundancy:** Inserting a dummy activity unnecessarily in a network.

3.5. Critical Path Method

The critical path method is a project modeling technique developed by Morgan R. Walker and James E. Kelley. Initially, a critical path method was used for managing plant maintenance projects.

It is a technique for planning, scheduling and controlling projects whose activities are not subjected to any uncertainty and the performance times are fixed. Hence it is a deterministic model. It is an activity oriented system. It is used in projects where overall cost is of primarily important, therefore resources are utilized efficiently andis also suitable for civil constructions. The Path and critical path are referred as follows:

Path: It is a series of adjacent activities from one event to another.

Critical path:

It is the sequence of activities which gives the longest duration in any project network. It is the shortest time possible to complete the project.

3.6. Programme Evaluation Review Technique

PERT was developed in the late 1950's for the U. S. Navy's Polaris ballistic missile system project having thousands of contractors. It has the potential to reduce both time and cost required to complete a project.

It is a technique for planning, scheduling and controlling of projects whose activities are subjected to uncertainty in the performance time. Hence it is a probabilistic model. It is an Event oriented system. It is used in projects where resources (men, materials, money) are always available when required. It is suitable for Research and Development projects where time cannot be determined.

This technique takes into account the uncertainty of project duration.

PERT depends upon the three time estimates. They are:

3.6.1. Optimistic time estimate

It is the shortest possible time in which an activity can be completed when everything goes on very well during the project. It is expressed as t_o or a. It is also known as the least time estimate.

3.6.2. Pessimistic time estimate

An activity would take the longest possible time to complete the project. It is the estimate when almost everything goes against our will and unexpected problems erupt while doing a project. It is expressed as t_p or b. It is also known as greatest time estimates.

3.6.3. Most likely time estimate

Itis the duration of any activity, when sometimes things go perfectly well and at times things go on very bad while doing the project. It is expressed as t_m or m.

Taking all these estimates of time into consideration, the expected time of an activity is calculated.

Expected duration of each activity is given by$t_e = \frac{t_0+4t_m+t_p}{6}$

Expected variance is given by $\sigma^2 = \left(\frac{t_p-t_0}{6}\right)^2$

Project duration: It is the total sum of the duration of each activity in a critical path.

4. PERT AND CPM PROCEDURE

4.1. PERT and CPM Procedure for intervals

CPM:

i. Construct the project network.

ii. Determine the various paths in the project network.

iii. Identify the path with the longest duration using the centre of intervals which is known as critical path. If intervals have the same centre then use ordering of intervals to identify the longest duration.

iv. Calculate the project duration of the network.

PERT:

i. Construct the project network.

ii. Compute the expected duration of each activity using time estimates.

iii. Compute the expected variance of each activity.

iv. Determine the various paths in the project network.

v. Identify the path with the longest duration using the centre of intervals which is known as critical path. If intervals have the same centre then use ordering of intervals to identify the longest duration.

vi. Calculate the project duration of the network and compute the expected variance of the project length which is the sum of the variances of all activities in the critical path.

4.2. PERT and CPM Procedure for conversion of fuzzy numbers into intervals

CPM:

i. Convert the triangular or trapezoidal fuzzy numbers into intervals using $\alpha-$ cuts.

ii. Construct the project network.

iii. Determine the various paths in the project network.

iv. Identify the path with the longest duration using the centre of intervals which is known as critical path. If intervals have the same centre then use ordering of intervals to identify the longest duration.

v. Calculate the project duration of the network.

PERT:

i. Convert the triangular or trapezoidal fuzzy numbers into intervals using $\alpha-$ cuts.

ii. Construct the project network.

iii. Compute the expected duration of each activity using time estimates.

iv. Determine the various paths in the project network.

v. Identify the path with the longest duration using the centre of intervals which is known as critical path. If intervals have the same centre then use ordering of intervals to identify the longest duration.

vi. Calculate the project duration of the network.

5. NUMERICAL EXAMPLES

1. Consider a project consisting of 6 activities A, B, C, D, E and F with their duration given in the following Table 1. Construct a network diagram and determine the critical path and project duration of the network.

Table 1. Duration of each activity in the network.

Activity	A	B	C	D	E	F
Preceding Activity	-	-	A	A	B	C, E
Duration	[6, 10]	[1, 7]	[9, 11]	[1, 3]	[3, 7]	[2, 4]

The network diagram is constructed using Table 1 and shown in Figure 3.

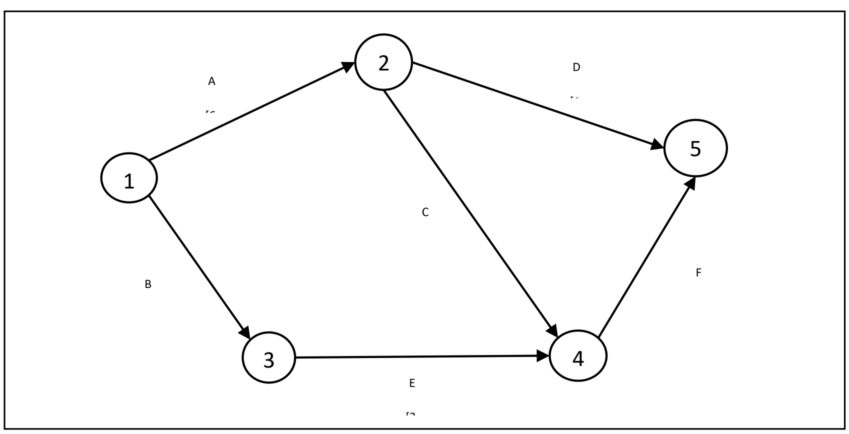

Fig. 3. Network Diagram

Various paths are given below:

A-D: [6, 10] + [1, 3] = [7, 13]

A-C-F: [6, 10] + [9, 11] + [2, 4] = [17, 25]

B-E-F: [1, 7] + [3, 7] + [2, 4] = [6, 18]

Critical path: A-C-F

Project duration: [17, 25]

2. Consider a project consisting of 7 activities A, B, C, D, E, F and G whose time estimates are given in the following Table 2. Construct a network diagram and also determine the critical path and project duration. Calculate the variance of the project length.

Table 2. Time estimates for the project

Activity	Preceding Activity	Optimistic time (t_0)	Most likely time (t_m)	Pessimistic time (t_p)
A	-	[1,3]	[2,6]	[4,6]
B	-	[1,5]	[2,6]	[3,9]
C	-	[2,6]	[4,6]	[3,9]
D	A	[6,10]	[8,10]	[10,12]
E	A	[3,9]	[6,10]	[11,13]
F	B	[1,3]	[1,5]	[2,6]
G	C, F, D	[1,3]	[4,6]	[6,8]

The expected duration and variance for each activity are calculated in Table 3.

Table 3.t_e andσ^2 calculated

Activity	(t_0)	(t_m)	(t_p)	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
A	[1,3]	[2,6]	[4,6]	[2.16, 5.5]	$\left[\frac{1}{36}, \frac{25}{36}\right]$
B	[1,5]	[2,6]	[3,9]	[2, 6.33]	$\left[\frac{-4}{9}, \frac{16}{9}\right]$
C	[2,6]	[4,6]	[3,9]	[3.5 6.5]	$\left[\frac{-7}{12}, \frac{49}{36}\right]$
D	[6,10]	[8,10]	[10,12]	[8, 10.33]	[0,1]
E	[3,9]	[6,10]	[11,13]	[6.33, 10.33]	$\left[\frac{1}{9}, \frac{25}{9}\right]$
F	[1,3]	[1,5]	[2,6]	[1.17, 4.83]	$\left[\frac{-5}{36}, \frac{25}{36}\right]$
G	[1,3]	[4,6]	[6,8]	[3.83, 5.83]	$\left[\frac{1}{4}, \frac{49}{36}\right]$

The network diagram is constructed using Table 3 and shown in Fig. 4.

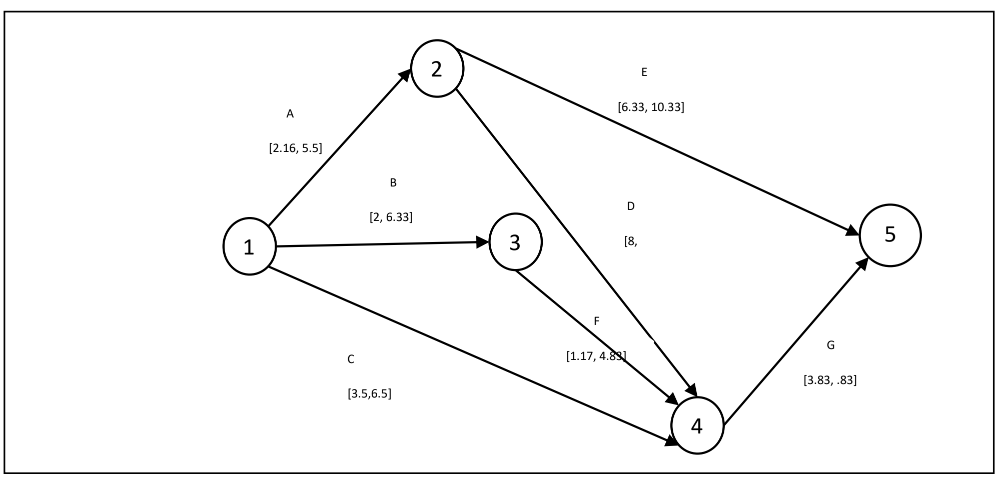

Fig. 4. Network Diagram

Various paths are given below:

A-E = [2.16, 5.5] + [6.33, 10.33] = [8.49, 15.83]

A-D-G = [2.16, 5.5] + [8, 10.33] + [3.83, 5.83] = [13.99, 21.66]

B-F-G = [2, 6.33] + [1.17, 4.83] + [3.83, 5.83] = [7, 16.99]

C-G = [3.5, 6.5] + [3.83, 5.83] = [7.33, 12.33]

Critical path: A-D-G

Project duration = [13.99, 21.66]

Expected variance of the project length

= [1/36, 25/36] + [0, 1] + [1/4, 49/36]

= [0.027, 0.694] + [0, 1] + [0.25, 1.361]

= [0.277, 3.055]

3. Consider a project consisting of 10 activities A, B, C, D, E, F, G, H, I and J with their duration given in the following Table 4. Construct a network diagram and also determine the critical path and project duration.

Table 4: Duration of each activity in network problems.

Activity	A	B	C	D	E	F	G	H	I	J
Preceding Activity	-	-	-	A	A	B,D	B,D	B,D	C,G	E,F
Duration	(7,8,9)	(5,7,9)	(10,12,14)	(3,4,5)	(7,10,13)	(2,3,4)	(3,5,7)	(7,10,13)	(2,4,6)	(5,7,9)

Convert the triangular fuzzy numbers into intervals using α-cut

$$\widetilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_3 - \beta_2) + \beta_3]$$

Table 5: Conversion of triangular numbers into intervals.

Activity	Preceding activity	Duration	Duration in intervals
A	-	(7,8,9)	$[\alpha + 7, -\alpha + 9]$
B	-	(5,7,9)	$[2\alpha + 5, -2\alpha + 9]$
C	-	(10,12,14)	$[2\alpha + 10, -2\alpha + 14]$
D	A	(3,4,5)	$[\alpha + 3, -\alpha + 5]$
E	A	(7,10,13)	$[3\alpha + 7, -3\alpha + 13]$
F	B,D	(2,3,4)	$[\alpha + 2, -\alpha + 4]$
G	B,D	(3,5,7)	$[2\alpha + 3, -2\alpha + 7]$
H	B,D	(7,10,13)	$[3\alpha + 7, -3\alpha + 13]$
I	C,G	(2,4,6)	$[2\alpha + 2, -2\alpha + 6]$
J	E,F	(5,7,9)	$[2\alpha + 5, -2\alpha + 9]$

The network diagram is constructed using Table 5 and shown in Fig. 5.

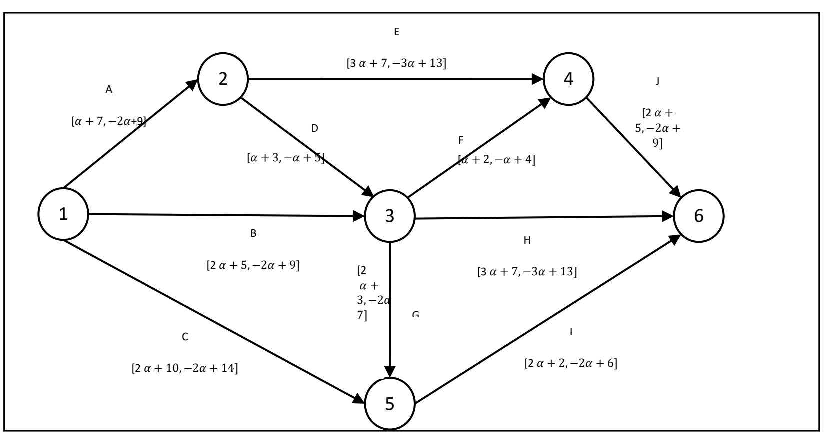

Fig. 5: Network Diagram

Various paths are given below:

A-E-J:$[\alpha+7,-\alpha+9]+[3\alpha+7,-3\alpha+13]+[2\alpha+5,-2\alpha+9]=[6\alpha+19,-6\alpha+31]$

A-D-F-J:$[\alpha+7,-\alpha+9]+[\alpha+3,-\alpha+5]+[\alpha+2,-\alpha+4]+[2\alpha+5,-2\alpha+9]$

$=[5\alpha+17,-5\alpha+27]$

B-F-J: $[2\alpha+5,-2\alpha+9]+[\alpha+2,-\alpha+4]+[2\alpha+5,-2\alpha+9]=[5\alpha+12,-5\alpha+22]$

B-H:$[2\alpha+5,-2\alpha+9]+[3\alpha+7,-3\alpha+13]=[5\alpha+12,-5\alpha+22]$

A-D-H:$[\alpha+7,-\alpha+9]+[\alpha+3,-\alpha+5]+[3\alpha+7,-3\alpha+13]=[5\alpha+17,-5\alpha+27]$

A-D-G-I:$[\alpha+7,-\alpha+9]+[\alpha+3,-\alpha+5]+[2\alpha+3,-2\alpha+7]+[2\alpha+2,-2\alpha+6]$

$=[6\alpha+15,-6\alpha+27]$

B-G-I:$[2\alpha+5,-2\alpha+9]+[2\alpha+3,-2\alpha+7]+[2\alpha+2,-2\alpha+6]=[6\alpha+10,-6\alpha+22]$

C-I:$[2\alpha+10,-2\alpha+14]+[2\alpha+2,-2\alpha+6]=[4\alpha+12,-4\alpha+20]$

Critical Path: A-E-J

Project duration:$[6\alpha+19,-6\alpha+31]$

Corresponding plot of the project duration is given in Figure 6.

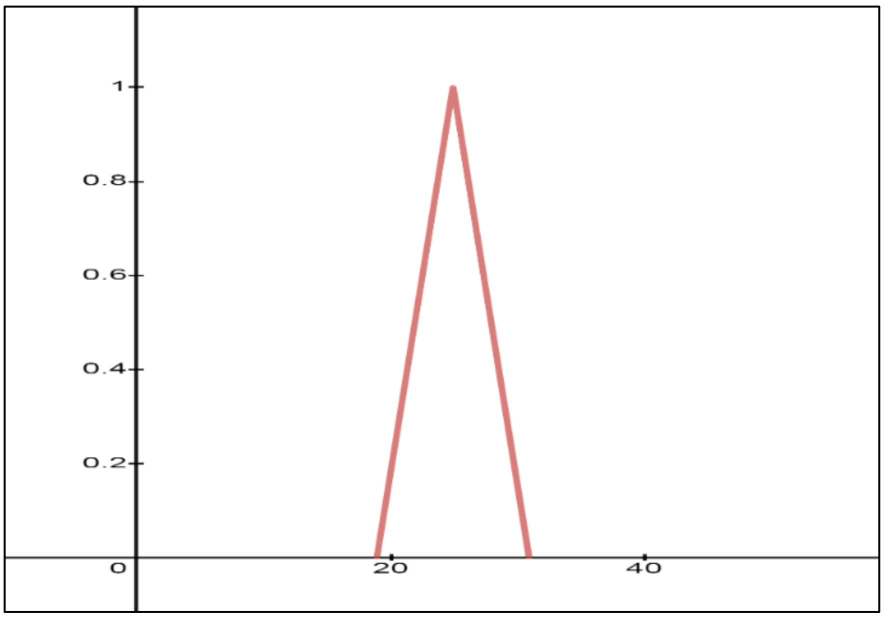

Fig.6. Solution of Project duration

The project duration in terms of triangular fuzzy number is (19, 25, 31)

4. Consider a project consisting of 9 activities A, B, C, D, E, F, G, H and I whose time estimates are given in the following Table 6. Construct a network diagram and also determine the critical path and project duration. Calculate the variance of the completion time.

Table 6. Time estimates for the project

Activity	Preceding Activity	Optimistic time (t_0)	Most likely time (t_m)	Pessimistic time (t_p)
A	-	(2,3,4)	(5,6,7)	(14,15,16)
B	-	(1,2,3)	(3,5,7)	(12,14,16)
C	-	(3,6,9)	(9,12,15)	(29,30,31)
D	B	(2,3,4)	(3,6,9)	(14,15,16)
E	A	(3,5,7)	(10,11,12)	(15,17,19)
F	A	(1,2,3)	(3,5,7)	(7,8,9)

G	C	(2,3,4)	(7,9,11)	(26,27,28)
H	D,E	(1,2,3)	(3,5,7)	(7,8,9)
I	F	(0,1,2)	(3,4,5)	(6,7,8)

Convert the triangular fuzzy numbers into intervals using α-cut

$$\widetilde{A}_{\alpha} = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_3 - \beta_2) + \beta_3]$$

Table 7. Conversion of triangular numbers into intervals.

Activity	(t_0)	(t_m)	(t_p)
A	$[\alpha + 2, -\alpha + 4]$	$[\alpha + 5, -\alpha + 7]$	$[\alpha + 14, -\alpha + 16]$
B	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[2\alpha + 12, -2\alpha + 16]$
C	$[3\alpha + 3, -3\alpha + 9]$	$[3\alpha + 9, -3\alpha + 15]$	$[\alpha + 29, -\alpha + 31]$
D	$[\alpha + 2, -\alpha + 4]$	$[3\alpha + 3, -3\alpha + 9]$	$[\alpha + 14, -\alpha + 16]$
E	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 10, -\alpha + 12]$	$[2\alpha + 15, -2\alpha + 19]$
F	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 7, -\alpha + 9]$
G	$[\alpha + 2, -\alpha + 4]$	$[2\alpha + 7, -2\alpha + 11]$	$[\alpha + 26, -\alpha + 28]$
H	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 7, -\alpha + 9]$
I	$[\alpha + 0, -\alpha + 2]$	$[\alpha + 3, -\alpha + 5]$	$[\alpha + 6, -\alpha + 8]$

The expected duration of each activity is calculated as in Table 8:

Table 8.t_e Calculated

Activity	(t_0)	(t_m)	(t_p)	$t_e = \frac{t_0 + 4t_m + t_p}{6}$
A	$[\alpha + 2, -\alpha + 4]$	$[\alpha + 5, -\alpha + 7]$	$[\alpha + 14, -\alpha + 16]$	$[\alpha + 6, -\alpha + 8]$
B	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[2\alpha + 12, -2\alpha + 16]$	$[1.83\alpha + 4.17, -1.83\alpha + 7.83]$
C	$[3\alpha + 3, -3\alpha + 9]$	$[3\alpha + 9, -3\alpha + 15]$	$[\alpha + 29, -\alpha + 31]$	$[2.67\alpha + 11.33, -2.67\alpha + 16.67]$
D	$[\alpha + 2, -\alpha + 4]$	$[3\alpha + 3, -3\alpha + 9]$	$[\alpha + 14, -\alpha + 16]$	$[2.33\alpha + 4.67, -2.33\alpha + 9.33]$
E	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 10, -\alpha + 12]$	$[2\alpha + 15, -2\alpha + 19]$	$[1.33\alpha + 9.67, -1.33\alpha + 12.33]$
F	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 7, -\alpha + 9]$	$[1.67\alpha + 3.33, -1.67\alpha + 6.67]$
G	$[\alpha + 2, -\alpha + 4]$	$[2\alpha + 7, -2\alpha + 11]$	$[\alpha + 26, -\alpha + 28]$	$[1.67\alpha + 9.33, -1.67\alpha + 12.67]$
H	$[\alpha + 1, -\alpha + 3]$	$[2\alpha + 3, -2\alpha + 7]$	$[\alpha + 7, -\alpha + 9]$	$[1.67\alpha + 3.33, -1.67\alpha + 6.67]$
I	$[\alpha + 0, -\alpha + 2]$	$[\alpha + 3, -\alpha + 5]$	$[\alpha + 6, -\alpha + 8]$	$[\alpha + 3, -\alpha + 5]$

The network diagram is constructed using Table 8 and shown in Fig. 7.

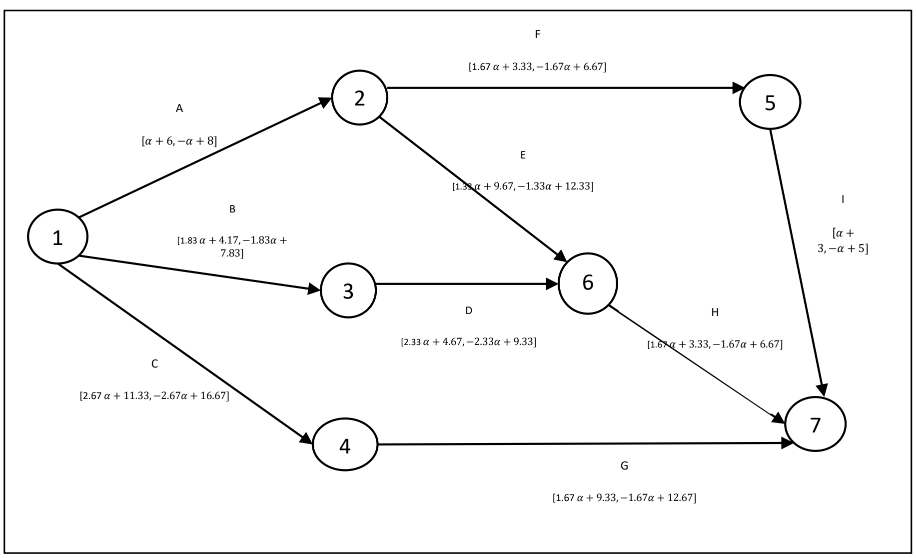

Fig. 7. Network Diagram

Various paths are given below:

A-F-I: $[\alpha + 6, -\alpha + 8] + [1.67\alpha + 3.33, -1.67\alpha + 6.67] + [\alpha + 3, -\alpha + 5]$

$= [3.67\alpha + 12.33, -3.67\alpha + 19.67]$

A-E-H $= [\alpha + 6, -\alpha + 8] + [1.33\alpha + 9.67, -1.33\alpha + 12.33] + [1.67\alpha + 3.33, -1.67\alpha + 6.67]$

$= [4\alpha + 19, -4\alpha + 27]$

B-D-H: $[1.83\alpha + 4.17, -1.83\alpha + 7.83] + [2.33\alpha + 4.67, -2.33\alpha + 9.33][1.67\alpha + 3.33, -1.67\alpha + 6.67]$

$= [5.83\alpha + 12.17, -5.83\alpha + 23.83]$

C-G: $[2.67\alpha + 11.33, -2.67\alpha + 16.67] + [1.67\alpha + 9.33, -1.67\alpha + 12.67]$

$= [4.34\alpha + 20.66, -4.34\alpha + 29.34]$

Critical Path: C-G

Project Duration: $[4.34\alpha + 20.66, -4.34\alpha + 29.34]$

Corresponding plot of the project duration is given in Figure 8.

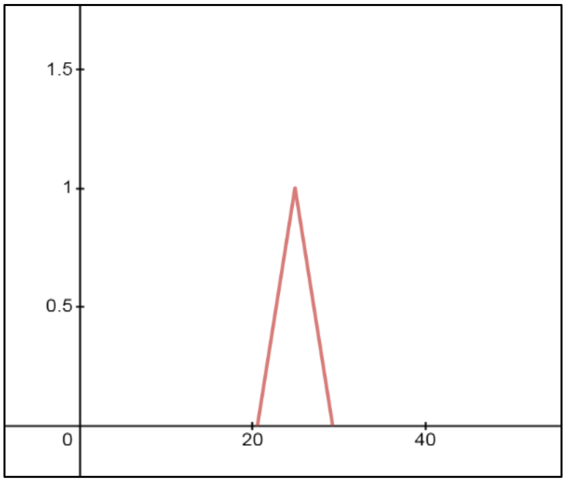

Fig. 8. Solution of Project duration

The project duration in terms of triangular fuzzy number is (20.66, 25, 29.34)

5. Consider a project consisting of 6 activities A, B, C, D, E and F with their duration given in the following Table 9. Construct a network diagram and also determine the critical path and project duration.

Table 9. Duration of each activity in network problem.

Activity	A	B	C	D	E	F
Preceding Activity	-	-	A	A	B	C, E
Duration	(2,6,10,14)	(2,3,5,6)	(6,8,12,14)	(0,1,3,4)	(2,4,6,8)	(-1,2,4,7)

Convert the trapezoidal fuzzy numbers into intervals using α-cut

$$\widetilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$$

Table 10. Conversion of Trapezoidal fuzzy numbers into intervals

Activity	Preceding activity	Duration	Duration in intervals
A	-	(2,6,10,14)	$[4\alpha + 2, -4\alpha + 14]$
B	-	(2,3,5,6)	$[\alpha + 2, -\alpha + 6]$
C	A	(6,8,12,14)	$[2\alpha + 6, -2\alpha + 14]$
D	A	(0,1,3,4)	$[\alpha + 0, -\alpha + 4]$
E	B	(2,4,6,8)	$[2\alpha + 2, -2\alpha + 8]$
F	C,E	(-1,2,4,7)	$[3\alpha - 1, -3\alpha + 7]$

The network diagram is constructed using Table 10 and shown in Fig. 9.

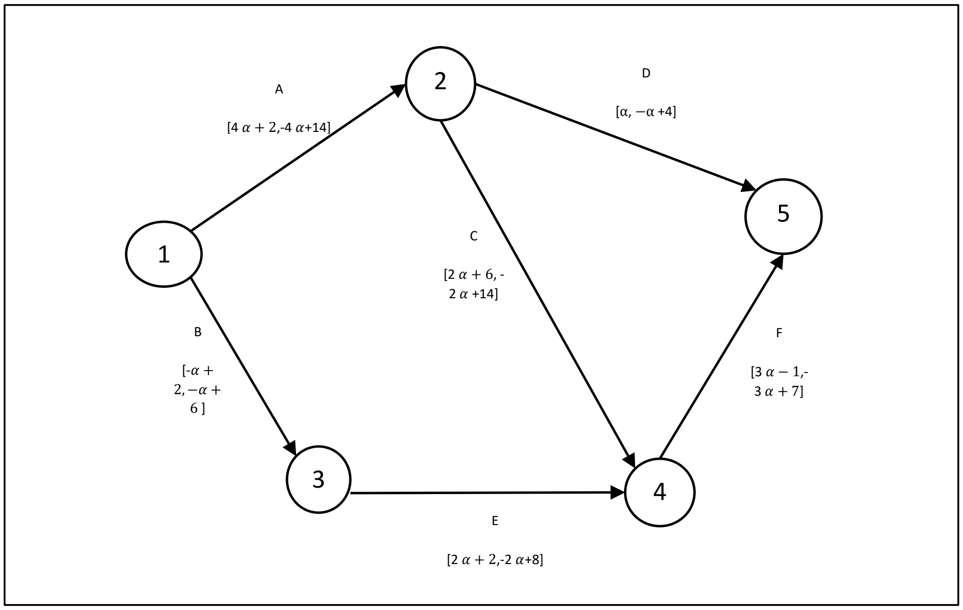

Fig. 9. Network Diagram

Various paths are given below:

A-D: $[4\alpha + 2, -4\alpha + 14] + [\alpha + 0, -\alpha + 4] = [5\alpha + 2, -5\alpha + 18]$

A-C-F: $[4\alpha + 2, -4\alpha + 14] + [2\alpha + 6, -2\alpha + 14] + [3\alpha - 1, -3\alpha + 7] = [9\alpha + 7, -9\alpha + 35]$

B-E-F: $[\alpha + 2, -\alpha + 6] + [2\alpha + 2, -2\alpha + 8] + [3\alpha - 1, -3\alpha + 7] = [6\alpha + 3, -6\alpha + 21]$

Critical Path: A-C-F

Project duration: $[9\alpha + 7, -9\alpha + 35]$

Corresponding plot of the project duration is given in Fig. 10.

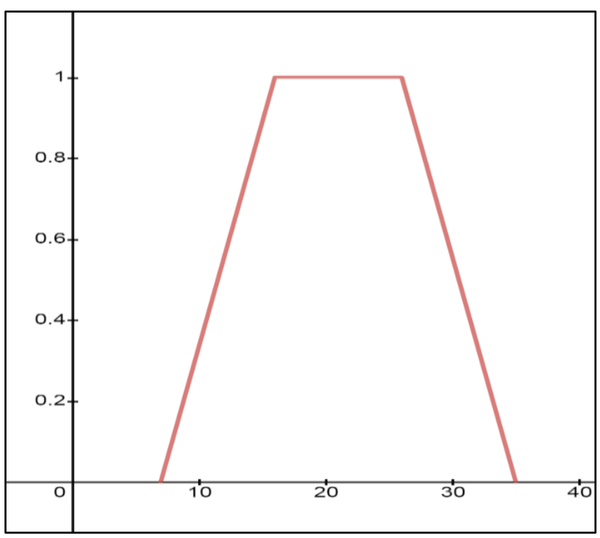

Fig. 10. Solution of Project duration

The project duration in terms of trapezoidal fuzzy number is (7, 16, 26, 35)

6. Consider a project consisting of 7 activities A, B, C, D, E, F and G whose time estimates are given in the following Table 11. Construct a network diagram and also determine the critical path and project duration. Calculate the variance of the completion time.

Table 11. Time estimates for the project

Activity	Preceding Activity	Optimistic time (t_0)	Most likely time (t_m)	Pessimistic time (t_p)
A	-	(0,1,3,4)	(2,3,5,6)	(1,4,6,9)
B	-	(-1,2,4,7)	(2,3,5,6)	(4,5,7,8)
C	-	(2,3,5,6)	(1,4,6,9)	(4,5,7,8)
D	A	(2,6,10,14)	(7,8,10,11)	(9,10,12,13)
E	A	(4,5,7,8)	(2,6,10,14)	(8,10,14,16)
F	B	(0,1,3,4)	(-1,2,4,7)	(2,3,5,6)
G	C,F,D	(0,1,3,4)	(1,4,6,9)	(2,6,8,12)

Convert the trapezoidal fuzzy numbers into intervals using α-cut

$$\widetilde{A}_\alpha = [\alpha(\beta_2 - \beta_1) + \beta_1, -\alpha(\beta_4 - \beta_3) + \beta_4]$$

Table 12. Conversion of trapezoidal Fuzzy numbers into intervals

Activity	(t_0)	(t_m)	(t_p)
A	$[\alpha + 0, -\alpha + 4]$	$[\alpha + 2, -\alpha + 6]$	$[3\alpha + 1, -3\alpha + 9]$
B	$[3\alpha - 1, -3\alpha + 7]$	$[\alpha + 2, -\alpha + 6]$	$[\alpha + 4, -\alpha + 8]$
C	$[\alpha + 2, -\alpha + 6]$	$[3\alpha + 1, -3\alpha + 9]$	$[\alpha + 4, -\alpha + 8]$
D	$[4\alpha + 2, -4\alpha + 14]$	$[\alpha + 7, -\alpha + 11]$	$[\alpha + 9, -\alpha + 13]$
E	$[\alpha + 4, -\alpha + 8]$	$[4\alpha + 2, -4\alpha + 14]$	$[2\alpha + 8, -2\alpha + 16]$
F	$[\alpha, -\alpha + 4]$	$[3\alpha - 1, -3\alpha + 7]$	$[\alpha + 2, -\alpha + 6]$
G	$[\alpha, -\alpha + 4]$	$[3\alpha + 1, -3\alpha + 9]$	$[4\alpha + 2, -4\alpha + 12]$

The expected duration of each activity is calculated as in Table 13:

Table 13.t_e Calculated

Activity	(t_0)	(t_m)	(t_p)	$t_e = \frac{t_0 + 4t_m + t_p}{6}$
A	$[\alpha, -\alpha + 4]$	$[\alpha + 2, -\alpha + 6]$	$[3\alpha + 1, -3\alpha + 9]$	$[1.33\alpha + 1.5, -1.33\alpha + 6.2]$
B	$[3\alpha - 1, -3\alpha + 7]$	$[\alpha + 2, -\alpha + 6]$	$[\alpha + 4, -\alpha + 8]$	$[1.33\alpha + 1.83, -1.33\alpha + 6.5]$
C	$[\alpha + 2, -\alpha + 6]$	$[3\alpha + 1, -3\alpha + 9]$	$[\alpha + 4, -\alpha + 8]$	$[2.33\alpha + 1.67, -2.33\alpha + 8.33]$
D	$[4\alpha + 2, -4\alpha + 14]$	$[\alpha + 7, -\alpha + 11]$	$[\alpha + 9, -\alpha + 13]$	$[1.5\alpha + 6.5, -1.5\alpha + 11.83]$
E	$[\alpha + 4, -\alpha + 8]$	$[4\alpha + 2, -4\alpha + 14]$	$[2\alpha + 8, -2\alpha + 16]$	$[3.17\alpha + 3.33, -3.17\alpha + 13.33]$
F	$[\alpha, -\alpha + 4]$	$[3\alpha - 1, -3\alpha + 7]$	$[\alpha + 2, -\alpha + 6]$	$[2.33\alpha - 0.33, -2.33\alpha + 6.33]$
G	$[\alpha, -\alpha + 4]$	$[3\alpha + 1, -3\alpha + 9]$	$[4\alpha + 2, -4\alpha + 12]$	$[2.83\alpha + 1, -2.83\alpha + 8.67]$

The network diagram is constructed using Table 13 and shown in Fig. 11.

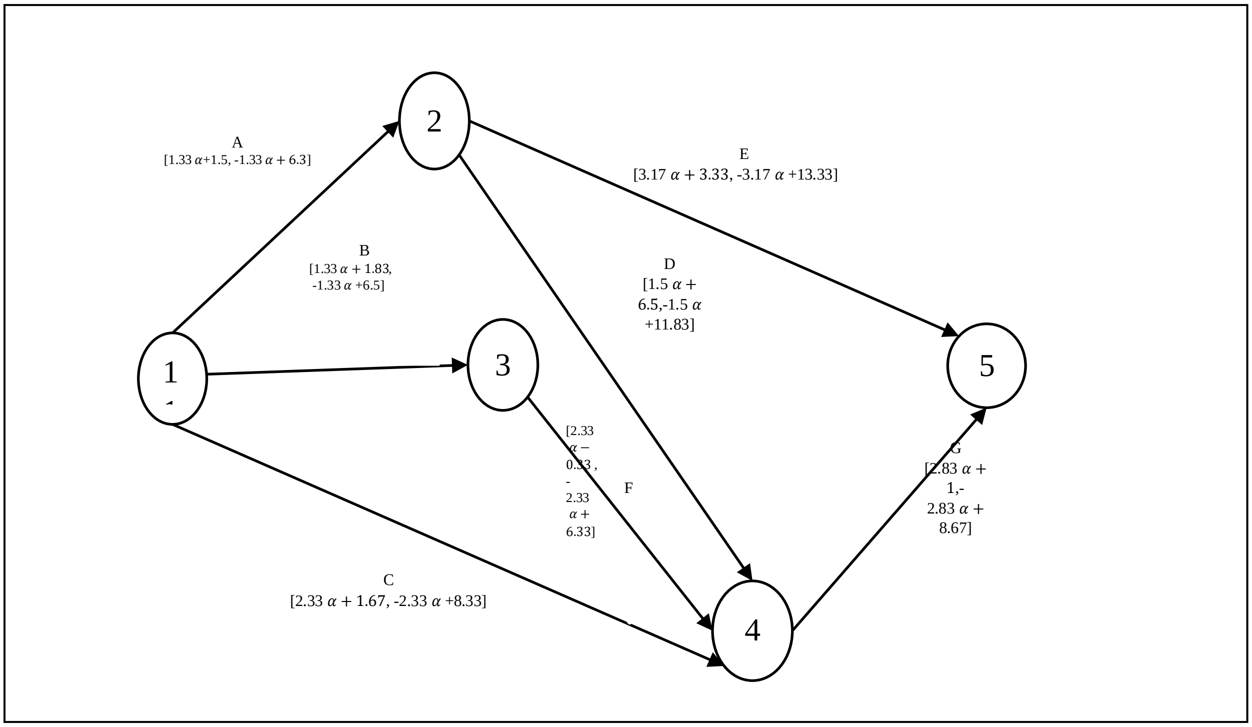

Fig. 11. Network Diagram

Various paths are given below:

A-E: $[1.33\alpha + 1.5, -1.33\alpha + 6.2]$+ $[3.17\alpha + 3.33, -3.17\alpha + 13.33]$

$=[4.5\alpha + 4.83, -4.5\alpha + 9.53]$

A-D-G: $[1.33\alpha + 1.5, -1.33\alpha + 6.2] + [1.5\alpha + 6.5, -1.5\alpha + 11.83] + [2.83\alpha + 1, -2.83\alpha + 8.67]$

$= [5.66\alpha + 9, -5.66\alpha + 26.7]$

B-F-G: $[1.33\alpha + 1.83, -1.33\alpha + 6.5] + [2.33\alpha - 0.33, -2.33\alpha + 6.33]$+$[2.83\alpha + 1, -2.83\alpha + 8.67]$

$= [6.49\alpha + 2.5, -6.49\alpha + 21.5]$

C-G: $[2.33\alpha + 1.67, -2.33\alpha + 8.33] + [2.83\alpha + 1, -2.83\alpha + 8.67] = [5.16\alpha + 2.67, -5.16\alpha + 17]$

Critical Path: A-D-G

Project duration: $[5.66\alpha + 9, -5.66\alpha + 26.7]$

Corresponding plot of the project duration is given in Figure 12.

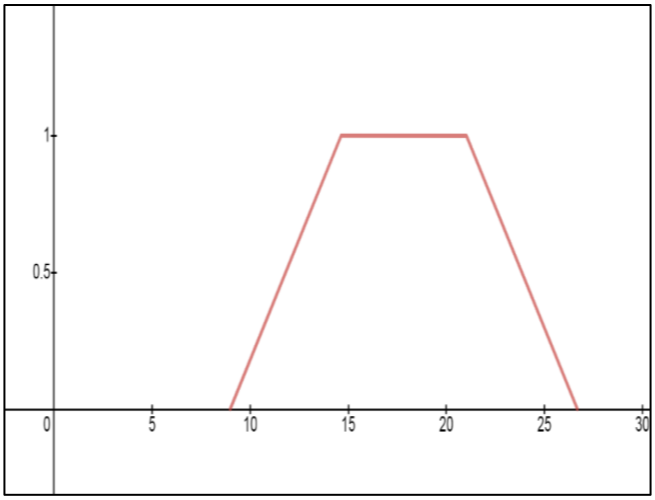

Fig. 12. Solution of the Project duration

The project duration in terms of trapezoidal fuzzy number is (9, 14.66, 21.04, 26.7)

CONCLUSION

In this paper, we have seen the Critical Path Method (CPM) and Programme Evaluation Review Technique (PERT) in Project Network. We solved problems related to CPM and PERT with intervals and also we discussed the conversion of fuzzy parameters (triangular and trapezoidal numbers) into intervals using $\alpha -$ cuts and we determined a critical path to complete the project and calculated the project duration of the network. The numerical illustrations are more efficient to complete the project network. It helps to formulate uncertainty in the actual environment and is also applicable for decision making in real life situations.

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