DOI:10.26524/krj.2020.30

Kong. Res. J. 7(2): 114-117, 2020
Publisher: Kongunadu Arts and Science College, Coimbatore.

https://www.krjournal.com

RESEARCH ARTICLE

b-CHROMATIC NUMBER OF EXTENDED CORONA OF SOME GRAPHS

Kiruthika, S. and Mohanapriya, N. *
PG and Research Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore – 641029, Tamil Nadu, India

ABSTRACT

In this paper we find out the b-chromatic number for the extended corona of path with complete on the same order $P_n \bullet K_n$ path on order n with star graph on order n+1 say $P_n \bullet K_{n+1}$, cycle with complete on the same order $C_n \bullet K_n$, cycle on order n with star graph on order n+1 say $C_n \bullet K_{n+1}$, star graph on order n+1 with complete on order n say $K_{n+1} \bullet K_n$,complete on order n with star graph on order n+1 say $K_n \bullet K_{n+1}$ respectively.

Keywords: b-coloring, b-chromatic number, extended corona.

AMS(2010): 05C15.

1. INTRODUCTION

A b-coloring of a graph G is a proper coloring of the vertices of G such that there exists a vertex in each color class joined to atleast a vertex in each other color class, such a vertex is called a dominating vertex. The b-chromatic number of a graph G, denoted by φ(G) [6], is the maximal integer k such that G may have a b-coloring by k-colors. This parameter has been derived by Irving and Manlove [3] in the year 1999 and gave an introduction about the concept of b-coloring and showed that the problem of determining φ(G) is NP-hard for general graphs but it is polynomial-time solvable for trees.Since every b-coloring is a proper coloring, we obtain that the chromatic number χ(G) is a lower bound for φ(G) . For the upper bound notice that every color class must have a b-vertex and moreover a b-vertex can have at most Δ(G) different colors in its neighborhood. The only additional color which is possible, is the color of a b-vertex itself. Therefore the trivial upper bound for φ(G) is Δ(G)+1 . Hence, we have the following bounds $\chi(G) \leq \varphi(G) \leq \Delta(G)+1$ [4]. Here a graph is considered as an undirected, connected graph with no loops and multiple edges. This paper investigate the b-chromatic number of neighborhood corona of some graphs.

Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices respectively. The Corona $G_1 \circ G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and then joining the i^{th} vertex of G_1 to each and every vertex in the i^{th} copy of G_2. The extended corona [1], $G_1 \bullet G_2$ is the graph obtained by taking the corona $G_1 \circ G_2$ and joining each vertex of i^{th} copy G_2 of to every vertex of j^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

2. b-Coloring of extended corona of graphs

Theorem 2.1

For $n \geq 5$, the b-chromatic number of extended corona of P_n with K_n is 3n

i.e.,φ $(P_n \bullet K_n)$ = 3n.

Proof

Let $\{a_1, a_2, \ldots, a_n\}$ be the vertices of Path graph P_n and $\{b_1, b_2, \ldots, b_n\}$ be the vertices of Complete graph K_n

i.e., V(P_n) = $\{a_1, a_2, \ldots, a_n\}$ and V(K_n) = $\{b_1, b_2, \ldots, b_n\}$

By the definition of extended corona each vertex of P_n is adjacent to corresponding copy of K_n and each copies of K_n is adjacent to their neighborhood copies of K_n.

V($P_n \bullet K_n$) = $\{a_i : 1 \leq i \leq n\}$ ∪ $\{b_{ij} : 1 \leq i \leq n, 1 \leq j \leq n\}$.

Order of the graph $|V(P_n \bullet K_n)|$ is n(n + 1), Size of the graph $|E(P_n \bullet K_n)|$ is $\frac{1}{2}(3n^3 - n^2 + 2n - 2)$, maximum degree $\Delta(P_n \bullet K_n)$ is 3n and minimum degree $\delta(P_n \bullet K_n)$ is n + 1.

Consider the set of colors C = $\{c_1, c_2, \ldots, c_{3n}\}$

To make the coloring as b-chromatic consider the following procedure

$c(a_i)$= 1, if i ≡1 mod 3

*Correspondence: Mohanapriya, N., PG and Research Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore – 641029, Tamil Nadu, India E.mail: n.mohanamaths@gmail.com

$c(a_i)$= n+1, if i $\equiv$ 2 mod 3
$c(a_i)$= 2n+1, if i $\equiv$ 0 mod 3
$c(b_{ij})$=j, 1 $\leq$ j $\leq$n for each i $\equiv$ 2 mod 3
$c(b_{ij})$=j+n, 1 $\leq$ j $\leq$n for each i $\equiv$ 0 mod 3
$c(b_{ij})$=j+2n, 1 $\leq$ j $\leq$n for each i $\equiv$ 1 mod 3

The above coloring procedure gives that φ ($P_n \bullet K_n$) $\geq$ 3n.

If we introduce any new color c_{3n+1} to any vertex in the graph, that will not adjacent to all other color class, therefore b-coloring with c_{3n+1} colors is not possible. Thus we have φ ($P_n \bullet K_n$) $\leq$ 3n. Hence, φ ($P_n \bullet K_n$) = 3n.

Remark : φ ($P_n \bullet K_n$) = 2n, n $\leq$ 4.

Theorem 2.2

For n $\geq$5, the b-chromatic number of extended corona of P_n with $K_{1,n}$ is 6

i.e.,φ ($P_n \bullet K_{1,n}$) = 6.

Proof

Let $\{a_1, a_2, \dots, a_n\}$ be the vertices of Path graph P_n and $\{b_1, b_2, \dots, b_n\}$ and {w}be the vertices of Star graph $K_{1,n}$. Let w be the central vertex of Star, w is adjacent to each $\{b_i$: 1 $\leq$ i $\leq$ n}

i.e., V(P_n) = $\{a_1, a_2, \dots, a_n\}$ and V($K_{1,n}$) = $\{b_1, b_2, \dots, b_n\} \cup$ {w}

By the definition of extended corona each vertex of P_n is adjacent to corresponding copy of $K_{1,n}$ and each copies of $K_{1,n}$ is adjacent to their neighborhood copies of $K_{1,n}$.

V($P_n \bullet K_{1,n}$) = $\{a_i : 1 \leq i \leq n\} \cup \{ b_{ij} : 1 \leq i \leq n, 1 \leq j \leq n \} \cup \{w_i : 1 \leq i \leq n\}$

Order of the graph $|V(P_n \bullet K_{1,n})|$ is $n^2 + 2n$, Size of the graph $|E(P_n \bullet K_{1,n})|$ is $n^3 + 3n^2 + n - 2$), maximum degree $\Delta(P_n \bullet K_{1,n})$ is 3n+3 and minimum degree δ $(P_n \bullet K_{1,n})$ is n + 2.

Consider the set of colors C = $\{c_1, c_2, \dots, c_6\}$

To make the coloring as b-chromatic consider the following procedure

$c(w_1, w_2, \dots, w_n)$ = (6,1, 2, 3, 4, 5, 6,1, 2...)
$c(b_{ij})$=1, 1 $\leq$ i $\leq$n for each j $\equiv$ 5 mod 6
$c(b_{ij})$=2, 1 $\leq$ i $\leq$n for each j $\equiv$ 0 mod 6
$c(b_{ij})$=3, 1 $\leq$ i $\leq$n for each j $\equiv$ 1 mod 6
$c(b_{ij})$=4, 1 $\leq$ i $\leq$n for each j $\equiv$ 2 mod 6
$c(b_{ij})$=5, 1 $\leq$ i $\leq$n for each j $\equiv$ 3 mod 6
$c(b_{ij})$=6, 1 $\leq$ i $\leq$n for each j $\equiv$ 4 mod 6
For 1 $\leq$ i $\leq$ 6, $c(a_i)$ = i
For 7 $\leq$ i $\leq$ n, c($a_7, a_8, \dots, a_n$) = (1,2,3,4,5,6,1,2,...)

The above coloring procedure gives that φ ($P_n \bullet K_{1,n}$) $\geq$ 6.

If we introduce any new color c_7 to any vertex in the graph that will not adjacent to all other colors in the color set, therefore b-coloring with 7 colors is not possible. Thus we have, φ ($P_n \bullet K_{1,n}$) $\leq$ 6. Hence φ ($P_n \bullet K_{1,n}$) = 6, n $\geq$5.

Theorem 2.3

For n $\geq$ 3 and n $\neq$ 4, the b-chromatic number of extended corona of C_n with K_n is 3n.

i.e.,φ ($C_n \bullet K_n$) = 3n.

Proof

Let $\{a_1, a_2, \dots, a_n\}$ be the vertices of Cycle graph C_n and $\{b_1, b_2, \dots, b_n\}$ be the vertices of Complete graph K_n

i.e., V(C_n) = $\{a_1, a_2, \dots, a_n\}$ and V(K_n) = $\{b_1, b_2, \dots, b_n\}$

By the definition of extended corona each vertex of C_n is adjacent to corresponding copy of K_n and each copies of K_n is adjacent to their neighborhood copies of K_n.

V($C_n \bullet K_n$) = $\{a_i : 1 \leq i \leq n\} \cup \{b_{ij} : 1 \leq i \leq n, 1 \leq j \leq n\}$.

Order of the graph $|V(C_n \bullet K_n)|$ is n(n + 1), Size of the graph $|E(C_n \bullet K_n)|$ is $\frac{1}{2}$ (3 $n^3 + n^2 + 2n$), maximum degree $\Delta(C_n \bullet K_n)$ is 3n and minimum degree δ $(C_n \bullet K_n)$ is n + 2.

Consider the set of colors C = $\{c_1, c_2, \dots, c_{3n}\}$

To make the coloring as b-chromatic consider the following procedure

Assign the color c_1 for a_3 and a_n
Assign the color c_{n+1} for a_1, a_4, a_6, a_8,...
Assign the color c_{2n+1} for a_2, a_5, a_7, a_9,...
For 1 $\leq$ i $\leq$ n, Assign the color c_{n+i} for b_{2i}, b_{5i}, b_{7i}, b_{9i},...
For 1 $\leq$ i $\leq$ n, Assign the color c_{2n+i} for b_{3i} and b_{ni}
For 1 $\leq$ i $\leq$ n, Assign the color c_i for b_{1i}, b_{4i}, b_{6i}, b_{8i},...

The above coloring procedure gives that φ ($C_n \bullet K_n$) $\geq$ 3n.

If we introduce any new color c_{3n+1} to any vertex in the graph that will not adjacent to all other colors in the color set, therefore b-coloring with 3n+1 colors is not possible. Thus we have, φ ($C_n \bullet K_n$) $\leq$ 3n+1. Hence φ ($C_n \bullet K_n$) = 3n, n $\geq$ 3 and n $\neq$ 4.

Remark : φ ($C_4 \bullet K_4$) = 8.

Theorem 2.4

For n $\geq$ 3 and n $\neq 4$, the b-chromatic number of extended corona of C_n with $K_{1,n}$ is 6

i.e.,φ ($C_n \bullet K_{1,n}$) = 6.

Proof

Let $\{a_1, a_2, \ldots, a_n\}$ be the vertices of cycle graph C_n and $\{b_1, b_2, \ldots, b_n\}$ and {w}be the vertices of Star graph $K_{1,n}$. Let w be the central vertex of Star, w is adjacent to each $\{b_i : 1 \leq i \leq n\}$

i.e., V(C_n) = $\{a_1, a_2, \ldots, a_n\}$ and V($K_{1,n}$) = $\{b_1, b_2, \ldots, b_n\} \cup \{w\}$

By the definition of extended corona each vertex of C_n is adjacent to their corresponding copy of $K_{1,n}$ and each copies of $K_{1,n}$ is adjacent to their neighborhood copies of $K_{1,n}$.

V($C_n \bullet K_{1,n}$) = $\{a_i : 1 \leq i \leq n\} \cup \{ b_{ij} : 1 \leq i \leq n, 1 \leq j \leq n \} \cup \{w_i : 1 \leq i \leq n\}$

Order of the graph $|V(C_n \bullet K_{1,n})|$ is $n^2 + 2n$, Size of the graph $|E(C_n \bullet K_{1,n})|$ is $n^3 + 4n^2 + 3n$), maximum degree $\Delta(C_n \bullet K_{1,n})$ is 3n and minimum degree δ ($C_n \bullet K_{1,n}$) is n + 3.

Consider the set of colors C = $\{c_1, c_2, \ldots, c_6\}$

To make the coloring as b-chromatic consider the following procedure

For 1 $\leq$ i $\leq$ 6, assign the color c_i to a_i

For 7 $\leq$ i $\leq$ n, assign the colors 4,2,4,2,... to consecutive vertices of a_i's

For 1 $\leq$ i $\leq$ 6, assign the color c_i to w_{i+1}, for 8 $\leq$ i $\leq$ n assign the colors 1,2,1,2,... to consecutive vertices of w_i's and c_6 to w_1

For 1 $\leq$ i $\leq$ n c(3,4,5,6,1,2,3,4,5,6,1,2,...) = $(b_{1i}, b_{2i}, b_{3i}, \ldots, b_{ni})$

The above coloring procedure gives that φ ($C_n \bullet K_{1,n}$) $\geq$ 6.

If we introduce any new color c_7 to any vertex in the graph that will not adjacent to all other colors in the color set, therefore b-coloring with 7 colors is not possible. Thus we have, φ ($C_n \bullet K_{1,n}$) $\leq$ 6. Hence φ ($C_n \bullet K_{1,n}$) = 6, n $\geq$ 3 and n $\neq 4$.

Remark : φ ($C_4 \bullet K_{1,4}$) = 4.

Theorem 2.5

For n $\geq$ 3, the b-chromatic number of extended corona of K_n with $K_{1,n}$ is 2n

i.e.,φ ($K_n \bullet K_{1,n}$) = 2n.

Proof

Let $\{a_1, a_2, \ldots, a_n\}$ be the vertices of complete graph K_n and $\{b_1, b_2, \ldots, b_n\}$ and {w}be the vertices of Star graph $K_{1,n}$. Let w be the central vertex of Star, w is adjacent to each $\{b_i : 1 \leq i \leq n\}$

i.e., V(K_n) = $\{a_1, a_2, \ldots, a_n\}$ and V($K_{1,n}$) = $\{b_1, b_2, \ldots, b_n\} \cup \{w\}$

By the definition of extended corona each vertex of K_n is adjacent to their corresponding copy of $K_{1,n}$ and each copies of $K_{1,n}$ is adjacent to their neighborhood copies of $K_{1,n}$.

V($K_n \bullet K_{1,n}$) = $\{a_i : 1 \leq i \leq n\} \cup \{ b_{ij} : 1 \leq i \leq n, 1 \leq j \leq n \} \cup \{w_i : 1 \leq i \leq n\}$

Order of the graph $|V(K_n \bullet K_{1,n})|$ is $n^2 + 2n$, Size of the graph $|E(K_n \bullet K_{1,n})|$ is $\frac{1}{2}$ ($n^4 + 2n^3 + 3n^2$), maximum degree $\Delta(K_n \bullet K_{1,n})$ is $n^2 + n$ and minimum degree δ ($K_n \bullet K_{1,n}$) is 2n.

Consider the set of colors C = $\{c_1, c_2, \ldots, c_{2n}\}$

To make the coloring as b-chromatic consider the following procedure

For 1 $\leq$ i $\leq$ n-1, assign the color c_{i+1} to a_i and c_1 to a_n

For 1 $\leq$ i $\leq$ n, assign the color c_i to w_i

For 1 $\leq$ i $\leq$ n, assign the color c_{n+i} to bij , for each 1 $\leq$ j $\leq$ n

The above coloring procedure gives that φ ($K_n \bullet K_{1,n}$) $\geq$ 2n.

If we introduce any new color c_{2n+1} to any vertex in the graph that will not adjacent to all other colors in the color set, therefore b-coloring with 2n+1 colors is not possible. Thus we have, φ ($K_n \bullet K_{1,n}$) $\leq$ 2n. Hence φ ($K_n \bullet K_{1,n}$) = 2n, n $\geq$ 3.

Theorem 2.6

For n $\geq$ 3, the b-chromatic number of extended corona of $K_{1,n}$ with K_n is 2n.

i.e.,φ ($K_{1,n} \bullet K_n$) = 2n.

Proof

Let $\{a_1, a_2, \ldots, a_n\}$ and {w} be the vertices of star graph $K_{1,n}$. Let w be the central vertex of Star, w is adjacent to each $\{ a_i : 1 \leq i \leq n\}$ and $\{b_1, b_2, \ldots, b_n\}$ be the vertices of Complete graph K_n

i.e., V($K_{1,n}$) = $\{a_1, a_2, \ldots, a_n\} \cup \{w\}$and V($K_n$) = $\{b_1, b_2, \ldots, b_n\}$

By the definition of extended corona each vertex of $K_{1,n}$ is adjacent to their corresponding copies of

K_n and corresponding copy K_n of the vertex w is adjacent to all other copies of K_n.

V($K_n \bullet K_n$) = $\{a_i: 1 \leq i \leq n\} \cup \{ b_{ij}: 1 \leq i \leq n, 1 \leq j \leq n \} \cup \{w_i: 1 \leq i \leq n\}$.

Order of the graph $|V(K_{1,n} \bullet K_n)|$ is $n^2 + 2n + 1$, Size of the graph $|E(K_{1,n} \bullet K_n)|$ is $\frac{1}{2}$ (3 $n^3 + n^2 + 3n$), maximum degree $\Delta(K_{1,n} \bullet K_n)$ is $n^2 + 1$ and minimum degree $\delta(P_n \bullet K_n)$ is 2n.

Consider the set of colors C = $\{c_1, c_2, \ldots, c_{2n}\}$

To make the coloring as b-chromatic consider the following procedure

Assign the color c_1 to w

For 1 ≤i ≤n assign the color c_{i+1} to a_i

For 1 ≤i ≤n assign the color c_1 to b_{i1}

For 1 ≤i ≤n assign the color c_{n+j} to b_{ij} for each fixed j=1,2,...,n

The above coloring procedure gives that φ ($K_{1,n} \bullet K_n$) ≥ 2n.

If we introduce any new color c_{2n+1} to any vertex in the graph that will not adjacent to all other colors in the color set, therefore b-coloring with 2n+1 colors is not possible. Thus we have, φ ($K_{1,n} \bullet K_n$) ≤ 2n. Hence φ ($K_{1,n} \bullet K_n$) = 2n, n ≥ 3.

REFERENCES

1. Chandrashekar Adiga, Rakshith B.R and K.N Subha Krishna (2016). Spectra of extended neighbourhood corona and extended corona of two graphs. *Electronic Journal of Graph Theory and its Applications* 101-110.
2. Indulal, G. (2011). The spectrum of neighbourhood corona of graphs. *Kragujevac Journal of Mathematics* 493-500
3. Irving, R.W. and Manlove, D.F. (1999). The b-chromatic number of a graph. *Discrete Applied Mathematics*
4. Kouider, M. and Maheo, M. (2002). Some Bounds for the b-chromatic number of Graph. *Disc. Math.* 267.
5. Lisna, P.C. and Sunitha, M.S. (2015). A Note on the b-Chromatic Number of Corona of Graphs, Journal of Interconnection Networks 15 (1 and 2).
6. Marko Jakovac and Sandi Klavzar, (2010). The b-chromatic number of cubic graphs. *Graphs and Combinatorics* 107-118.
7. Vernold Vivin, J. and Venkatachalam, M. (2012). The b-chromatic number of corona graphs. *Utilitas Math*. 299-307.