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RESEARCH ARTICLE

GAME THEORY PROBLEMS USING INTERVAL PARAMETERS

Radhakrishnan, S.* and Saikeerthana, D.

Department of Mathematics, D.G. Vaishnav College (Autonomous), Chennai-600106, TamilNadu, India.

ABSTRACT

In this paper, we consider a game with imprecise values in the payoff matrix. All the imprecise values are taken as intervals. Here, we solve anintervalgame problem based on pure strategy, mixed strategy and using dominance property. A graphical method is also given for solving 2 x n and m x 2interval game problem. To illustrate this, numerical examples are provided.

Keywords: Pay-off matrix, Intervals, Saddle point, Strategies.

1. INTRODUCTION

Operations Research is relatively a new discipline which originated during World War II, and later became very popular throughout the world. It is used successfully in almost all the fields. Operations Research helps us to make better decisions in complex scenarios. It also includes the application of scientific tools for finding the optimum solution to a problem involving the operations of a system.

Game Theory is one of the applications of Operations Research. It was developed in twentieth century and later it turns into a wider application in 1944 by publishing an article in the name of "Theory of games and Economic Behavior" by John von Neumann and Oscar Morgenstern [2].Game theory is the study of conflict and competition. It attempts to enable decision making in the situations where there are two or more opponents involved under the conditions of conflict and competition. The concept of game theory is to formulate a structure, to analyse, and to understand strategies. Game theory assist in making decisions in the fields like economics, management etc.

In classical game theory, it is assumed that the player share aware of all the data of a game exactly. However, in real situation, it is hard to know the exact values of pay off. In such situation, it is useful to consider the game with payoff as intervals. In an interval game problem, all the parameters are intervals.

Interval computation was suggested by Dwyer [1]. Suparna Das and Chakraverty [3] have proposed the new methods for solving the linear simultaneous equations with interval and fuzzy parameters (triangular and trapezoidal).

Therest of this paper is organized as follows:

In section 2, basic definition of interval and its arithmetic, ordering of intervals are given as preliminaries. In section 3, the concept of game theory is discussed. In section 4, numerical examples are given. Finally, the conclusion.

2. PRELIMINARIES

2.1 Interval Number

An interval number A is defined as A = $[\beta_1, \beta_2]$ =$\{x: \beta_1 \leq x \leq \beta_2, x \in \mathbb{R}\}$. Here, $\beta_1, \beta_2 \in \mathbb{R}$ are the lower and upper bounds of the interval.

2.1.1 Arithmetic operations of interval:

Let A = $[\beta_1, \beta_2]$ and B= $[\gamma_1, \gamma_2]$ be two intervals. Then

Addition: A+B =$[\beta_1 + \gamma_1, \beta_2 + \gamma_2]$

Subtraction: A-B = $[\beta_1 - \gamma_2, \beta_2 - \gamma_1]$

Multiplication: A*B = $\left[\min(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2), \max(\beta_1\gamma_1, \beta_1\gamma_2, \beta_2\gamma_1, \beta_2\gamma_2)\right]$

Division: $\frac{A}{B} = \frac{[\beta_1,\beta_2]}{[\gamma_1,\gamma_2]} = [\beta_1, \beta_2].\frac{1}{[\gamma_1,\gamma_2]}$

where $\frac{1}{[\gamma_1,\gamma_2]} = \left[\frac{1}{\gamma_2}, \frac{1}{\gamma_1}\right]$, $0 \notin [\gamma_1, \gamma_2]$

$\frac{1}{[\gamma_1,0]} = \left[-\infty, \frac{1}{\gamma_1}\right]$

$\frac{1}{[0,\gamma_2]} = \left[\frac{1}{\gamma_2}, \infty\right]$ and

$\frac{1}{[\gamma_1,\gamma_2]} = \left[-\infty, \frac{1}{\gamma_1}\right] \cup \left[\frac{1}{\gamma_2}, \infty\right] = [-\infty, \infty]$, $0 \in [\gamma_1, \gamma_2]$

Scalar Multiplication*:*

Let A= $[\beta_1, \beta_2]$ then uA= $[u\beta_1, u\beta_2]$, u≥ 0 and uA= $[u\beta_2, u\beta_1]$, u ≤ 0.

*Correspondence: Radhakrishnan,S., Department of Mathematics, D.G. Vaishnav College (Autonomous), Chennai-600106, Tamil Nadu, India.E-mail: dgvcradhakrishnan@gmail.com

2.2 Types of intervals

Let A $= [\beta_1, \beta_2]$ and B$= [\gamma_1, \gamma_2]$ be two intervals. Therefore these can be classified into three types as follows:

Type I- Non overlapping intervals:

If two intervals are disjoint then they are known as non overlapping intervals.

Type II- Partially overlapping intervals:

If one interval contains the other interval partially then they are known as partially overlapping intervals.

Type III- Completely overlapping intervals:

If one interval completely contained in the other interval then they are known as completely overlapping intervals.

These three types of intervals are shown in Figure 1

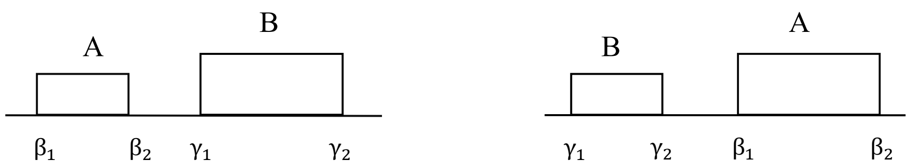


Fig. 1(a): Type – I intervals

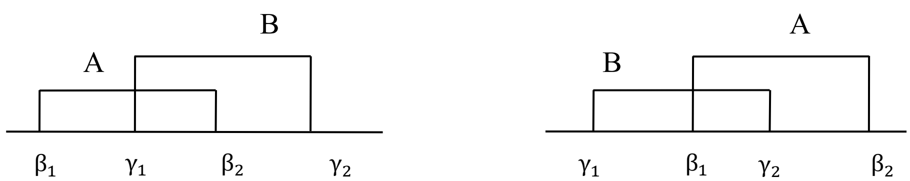


Fig. 1(b): Type – II intervals

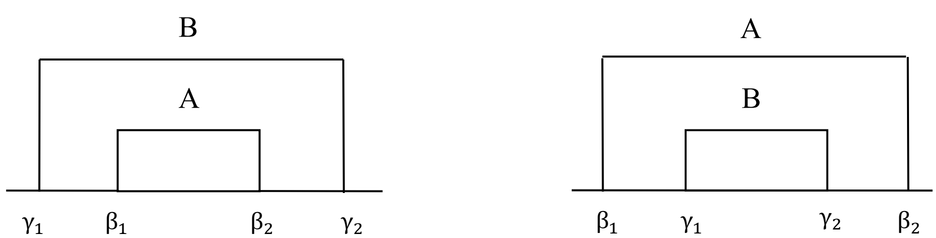


Fig. 1(c): Type – III intervals

Fig. 1. Different types of intervals

2.3. Ordering of intervals:

Let A$= [\beta_1, \beta_2]$ be the interval number. It can also be expressed by its centre and radius and it is denoted by $\langle a_C, a_W \rangle$, where $a_C = \frac{\beta_1+\beta_2}{2}$ and $a_W = \frac{\beta_2-\beta_1}{2}$ and they are known as centre and radius of the interval respectively.

Let A$= [\beta_1, \beta_2] = \langle a_C, a_W \rangle$ and B $=[\gamma_1, \gamma_2] = \langle b_C, b_W \rangle$. Then the relation on interval number is defined as

i. A < B iff $a_C < b_C$ whenever $a_C \neq b_C$.

ii. A > B iff $a_C > b_C$ whenever $a_C \neq b_C$.

iii. A < B iff $a_w < b_w$ whenever $a_C = b_C$.

iv. A > B iff $a_w > b_w$ whenever $a_C = b_C$.

v. A = B whenever $a_w = b_w$ and $a_C = b_C$.

3.CONCEPT OF GAME THEORY

3.1. Two person zero-sum game

In a game, if the algebraic sum of gains and losses of all the players are zero then it is called Zero-Sum game. A game with two players where one person's gain is a loss of the other is called two person zero-sum game.

3.2 Basic Terminology

i. **Player:** The participant in a game is called player.

ii. **Play:**It is the course of action chosen by each player in a game.

iii. **Payoff:** It is an outcome of the game. A payoff matrixindicates the amount (gain or loss) received by the player after all possible plays of the game.

iv. **Strategy:** A strategy for a player is a plan which specifies his action for every possible action of his opponent during the game. In a two person game if player 1 has 'm' strategies and player 2 has 'n' strategies then the game is said to be $m \times n$ game.

v. **Pure Strategy:** Pure strategy is a decision making rule to choose a particular course of action. In this case, the payoff matrix contains a saddle point.

vi. **Mixed Strategy:** In certain cases, pure strategy fails (i.e. saddle point does not exist). In such cases, players may adopt an optimal blend of strategies called mixed strategy. It is a decision in advance for all plays, to choose a course of action for each player with certain probabilistic fixation.

3.3. Saddle point:

It is a point where the maximin value equals the minimax value in a game.

3.3.1 Rules for findinga Saddle point:

i. Select the minimum element of each row of the payoff matrix using orderingof intervalsand choose the maximum value among them. It is called the maximin value.

ii. Select the maximum element of each column of the payoff matrix using orderingof intervals and choose the minimum value among them. It is called the minimax value.

iii. The position where the maximin value equals the minimax value in a game is called the saddle point and the corresponding strategies of the saddle point are called optimum strategies. The payoff at the saddle point is called the value of the game.

3.4. Solution of 2×2 Interval Game Matrix

Let $\begin{array}{cc} & \begin{array}{cc} \boldsymbol{B_1} & \mathbf{B_2} \end{array} \\ \begin{array}{c} \mathbf{A_1} \\ \mathbf{A_2} \end{array} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$ be the general 2 × 2interval game matrix.Test for a saddle point. If the game has a saddle point then go by the above rule. If thereis no saddle point,the optimum strategies and the value of the game can be computed as follows:

The optimum mixed strategies are given by

$S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$

where $p_1 = \frac{d-c}{(a+d)-(b+c)}$ and $p_2 = [1, 1] - p_1$

$q_1 = \frac{d-b}{(a+d)-(b+c)}$ and $q_2 = [1, 1] - q_1$

Value of the game $V = \frac{ad-bc}{(a+d)-(b+c)}$

3.5. Dominance Property

Dominance Property is applicable to both pure strategies and mixed strategies. Sometimes, one of the pure strategies of either player may be inferior to atleast one of the remaining ones. Hence, the inferior ones are said to be dominated by the superior strategies. In such cases, we canomit dominated strategies to reduce the size ofthe payoff matrixusing ordering of intervals and solve it by applying the following rules.

Rules:

i. In the payoff matrix, if all the elements of the i^{th} row are less than or equal to the corresponding elements of the k^{th} row then k^{th} row dominates the i^{th} row and hence omit the i^{th} row.

ii. In the payoff matrix, if all the elements of the i^{th} column are greater than or equal to the corresponding elements of the k^{th} column then k^{th} column dominates the i^{th} column and hence omit the i^{th} column.

3.6. Graphical Method

The graphical method is used to solve the games whose payoff matrix has

i. Two rows and n columns (2 x n)

ii. m rows and two columns (m x 2)

3.6.1 Algorithm for solving 2 x n game matrix

i. Reduce the size of the payoff matrix by applying the dominance property, if possible.

ii. Convert an interval game problem into a crisp game problem using centre of intervals.

iii. Draw the two vertical axes 1 unit apart. The two lines are x_1= 0, x_1 = 1.

iv. Plot the graph for crisp game problem by taking the points of the first row in the payoff matrix on the vertical line x_1= 1 and thepoints of the second row in the payoff matrix on the vertical line x_1= 0.

v. Form a straight line by joining the point a_{1j}on the axis x_1 = 1 to the point a_{2j} on the axis x_1 = 0. Draw 'n' straight lines for j=1, 2… n and determine the highest point of the lower boundary obtained. This will be the maximin point.

vi. The two lines passing through the maximin point gives the 2x2payoff matrix.

vii. Enter the corresponding entries of the reduced payoff matrix of a crisp game problem in terms of intervals and solve this matrix by finding the optimum strategies and the value of the game.

3.6.2 Algorithm for solving m x 2 game matrix

i. Reduce the size of the payoff matrix by applying the dominance property, if possible.

ii. Convert an interval game problem into a crisp game problem using centre of intervals.

iii. Draw the two vertical axes 1 unit apart. The two lines arex$_1$ = 0, x_1 = 1.

iv. Plot the graph for crisp game problem by taking the points of the first column in the payoff matrix on the vertical line x_1 = 1 and the points of the second column in the payoff matrix on the vertical line x_1 = 0.

v. Form a straight line by joining the point a_{j1} on the axis x_1 = 1 to the point a_{j2} on the axis x_1 = 0. Draw 'n' straight lines for j=1, 2… n and determine the lowest point ofthe upper boundary obtained. This will be the minimax point.

vi. The two lines passing through the minimax point gives the 2 x2payoff matrix.

vii. Enterthe corresponding entries of the reduced payoff matrix of a crisp game problem in terms of intervals and solve this matrix by finding the optimum strategies and the value of the game.

4. NUMERICAL EXAMPLES

1. Consider the following 3× 3 game with payoff as intervals. We solve it by existing method.

Player B

$$\begin{array}{cc} & \begin{array}{ccc} \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} \end{array} \\ \textbf{Player A}\ \begin{array}{c} \mathbf{A_1} \\ \mathbf{A_2} \\ \mathbf{A_3} \end{array} & \begin{pmatrix} [0,2] & [2,4] & [0,2] \\ [-1,1] & [-5,-3] & [-4,-2] \\ [0,2] & [4,6] & [-2,0] \end{pmatrix} \end{array}$$

To find the saddle point

Player B

$$\begin{array}{ccc} & \begin{array}{ccc} \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} \end{array} & \textbf{Row Minima} \\ \textbf{Player} & & \\ \begin{array}{c} \mathbf{A_1} \\ \mathbf{AA_2} \\ \mathbf{A_3} \end{array} & \begin{pmatrix} [0,2] & [2,4] & [0,2] \\ [-1,1] & [-5,-3] & [-4,-2] \\ [0,2] & [4,6] & [-2,0] \end{pmatrix} & \begin{array}{c} [\mathbf{0},\mathbf{2}] \\ [-\mathbf{5},-\mathbf{3}] \\ [-\mathbf{2},\mathbf{0}] \end{array} \\ \text{Column} & \begin{array}{ccc} [0,2] & [4,6] & [0,2] \end{array} & \end{array}$$

Maxima

Minimax = [0,2]; Maximin = [0,2]

Saddle point = (A_1,B_1) or (A_1,B_3)

Value of the game = [0, 2]

2. Consider the following 2×2 game with payoff as intervals. We solve it by existing method.

$$\begin{array}{cc} & \mathbf{B} \\ \mathbf{A} & \begin{pmatrix} [1,3] & [4,6] \\ [6,8] & [2,4] \end{pmatrix} \end{array}$$

Step 1: Test for a saddle point.

$$\begin{array}{ccc} & \mathbf{B} & \textbf{Row Minima} \\ \mathbf{A} & \begin{pmatrix} [1,3] & [4,6] \\ [6,8] & [2,4] \end{pmatrix} & \begin{array}{c} [1,3] \\ [2,4] \end{array} \\ \text{Column} & \begin{array}{cc} [6,8] & [4,6] \end{array} & \end{array}$$

Maxima

Minimum of column Maxima = [4,6]

Maximum of row minima = [2,4]

Minimax≠ Maximin. Therefore, no saddle point.

Step 2: We compute the optimum strategies and the value of the game as follows:

Here $a = [1,3]; b = [4,6]; c = [6,8]; d = [2,4]$

$p_1 = \frac{[2,4]-[6,8]}{[3,7]-[10,14]} = \left[\frac{2}{11}, 2\right]$ and $p_2 = [1, 1] - \left[\frac{2}{11}, 2\right] = \left[-1, \frac{9}{11}\right]$

$q_1 = \frac{[2,4]-[4,6]}{[3,7]-[10,14]} = \left[0, \frac{4}{3}\right]$ and $q_2 = [1,1] - \left[0, \frac{4}{3}\right] = \left[\frac{-1}{3}, 1\right]$

The optimum mixed strategies are

$S_A = \begin{bmatrix} A_1 & A_2 \\ \left[\frac{2}{11}, 2\right] & \left[-1, \frac{9}{11}\right] \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ \left[0, \frac{4}{3}\right] & \left[\frac{-1}{3}, 1\right] \end{bmatrix}$

Value of the game $= \frac{([1,3]*[2,4])-([4,6]*[6,8])}{[3,7]-[10,14]}$

$= \left[\frac{12}{11}, \frac{46}{3}\right]$

3. Consider the following game with payoff as intervals. We reduce this matrixusing dominance property and solve it by existing method.

$$\begin{array}{cc|ccc} & & & \mathbf{B} & \\ & & \mathbf{I} & \mathbf{II} & \mathbf{III} \\ \mathbf{A} & \begin{array}{c}\mathbf{I}\\ \mathbf{II}\\ \mathbf{III}\end{array} & \multicolumn{3}{c}{\begin{pmatrix} [0,2] & [6,8] & [1,3] \\ [5,7] & [1,3] & [6,8] \\ [5,7] & [0,2] & [5,7] \end{pmatrix}} \end{array}$$

We use ordering of intervals to omit dominated rows or columns.

Step 1: Row III is dominated by Row II and Column III is dominated by Column I. Therefore, we omit Row III and column III.

Hence we have

$$\begin{array}{cc} & \mathbf{B} \\ & \begin{array}{cc}\mathbf{I} & \mathbf{II}\end{array} \\ \mathbf{A}\begin{array}{c}\mathbf{I}\\ \mathbf{II}\end{array} & \begin{pmatrix} [0,2] & [6,8] \\ [5,7] & [1,3] \end{pmatrix} \end{array}$$

Step 2: We compute the optimum strategies and the value of the game as follows:

Here $a = [0,2]; b = [6,8]; c = [5,7]; d = [1,3]$

$p_1 = \frac{[1,3]-[5,7]}{[1,5]-[11,15]} = \left[\frac{2}{14}, 1\right]$ and $p_2 = [1, 1] - \left[\frac{2}{14}, 1\right] = \left[0, \frac{12}{14}\right]$

$q_1 = \frac{[1,3]-[6,8]}{[1,5]-[11,15]} = \left[\frac{3}{14}, \frac{7}{6}\right]$ and $q_2 = [1, 1] - \left[\frac{3}{14}, \frac{7}{6}\right] = \left[\frac{-1}{6}, \frac{11}{14}\right]$

The optimum mixed strategies are

$S_A = \begin{bmatrix} \mathrm{I} & \mathrm{II} & \mathrm{III} \\ \left[\frac{2}{14}, 1\right] & \left[0, \frac{12}{14}\right] & [0,0] \end{bmatrix}$ and

$S_B = \begin{bmatrix} \mathrm{I} & \mathrm{II} & \mathrm{III} \\ \left[\frac{3}{14}, \frac{7}{6}\right] & \left[\frac{-1}{6}, \frac{11}{14}\right] & [0,0] \end{bmatrix}$

Value of the game $= \frac{([0,2]*[1,3])-([6,8]*[5,7])}{[1,5]-[11,15]}$

$= \left[\frac{24}{14}, \frac{56}{6}\right]$

4. Consider the following game with payoff as intervals. We solve it by dominance property.

$$\begin{array}{c|cccc} & \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \mathbf{B_4} \\ \begin{array}{c}\mathbf{A_1}\\ \mathbf{A_2}\\ \mathbf{A_3}\end{array} & \multicolumn{4}{c}{\begin{bmatrix} [7,9] & [8,12] & [6,12] & [11,17] \\ [8,12] & [10,12] & [7,9] & [10,14] \\ [10,16] & [10,14] & [11,17] & [10,16] \end{bmatrix}} \end{array}$$

We use ordering of intervals to omit dominated rows or columns.

Step 1: Fourth column is dominated by first column. Therefore, omit Fourth column. Then the game is reduced to

$$\begin{array}{c|ccc} & \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} \\ \begin{array}{c}A_1\\ A_2\\ A_3\end{array} & \multicolumn{3}{c}{\begin{bmatrix} [7,9] & [8,12] & [6,12] \\ [8,12] & [10,12] & [7,9] \\ [10,16] & [10,14] & [11,17] \end{bmatrix}} \end{array}$$

Step 2: Third row dominates the First and second row. Therefore, omit first and second row. Then the game is reduced to

$$\begin{array}{cccc} & \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} \\ \mathbf{A_3} & [10, 16] & [10, 14] & [11, 17] \end{array}$$

Step 3: Now second column dominates the first and third column. Therefore, omit first and third column.

$$\begin{array}{cc} & \mathbf{B_2} \\ \mathbf{A_3} & [[10,14]] \end{array}$$

The value of the game is [10, 14]

The Saddle point of the game is (A_3, B_2)

5. Consider the following 2×5 game with payoff as intervals.We reduce this matrix using dominance property and solve it graphically.

$$\begin{array}{c|ccccc} & \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \mathbf{B_4} & \mathbf{B_5} \\ \begin{array}{c}\mathbf{A_1}\\ \mathbf{A_2}\end{array} & \multicolumn{5}{c}{\begin{pmatrix} [1,3] & [-3,1] & [1,9] & [-3,-1] & [4,8] \\ [-3,-1] & [2,6] & [-4,-2] & [-2,4] & [-1,1] \end{pmatrix}} \end{array}$$

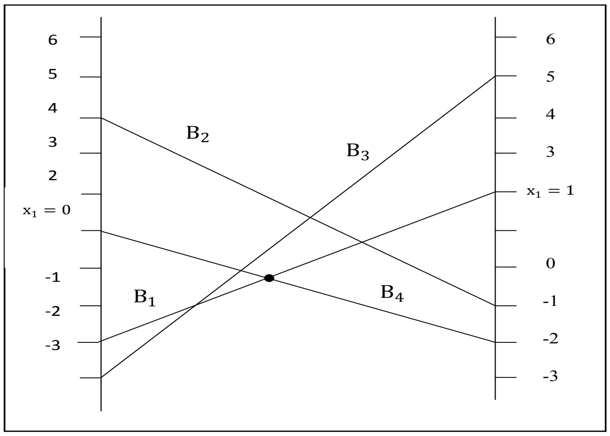
Step 1: We use ordering of intervals to omit dominated rows or columns. First column dominates fifth column. Therefore, we omit fifth column. We have the following matrix.

$$\begin{array}{c|cccc} & \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \mathbf{B_4} \\ \begin{array}{c}\mathbf{A_1}\\ \mathbf{A_2}\end{array} & \multicolumn{4}{c}{\begin{pmatrix} [1,3] & [-3,1] & [1,9] & [-3,-1] \\ [-3,-1] & [2,6] & [-4,-2] & [-2,4] \end{pmatrix}} \end{array}$$

Step 2: We convert an interval game problem into a crisp game problem by using centre of intervals. Hence, we have

$$\begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \mathbf{B_4} \end{array} \\ \begin{pmatrix} 2 & -1 & 5 & -2 \\ -2 & 4 & -3 & 1 \end{pmatrix} \end{array}$$

Step 3: We plot the graph for crisp game problem.



The reduced payoff matrix of a crisp game problem is

$$\begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{B_1} & \mathbf{B_4} \end{array} \\ \begin{pmatrix} 2 & -2 \\ -2 & 1 \end{pmatrix} \end{array}$$

Step 4: We enter the corresponding entries of the reduced payoff matrix of a crisp game problem in terms of intervals.

The reduced payoff matrix in terms of intervals is given by

$$\begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{B_1} & \mathbf{B_4} \end{array} \\ \begin{pmatrix} [1,3] & [-3,-1] \\ [-3,-1] & [-2,4] \end{pmatrix} \end{array}$$

Step 5: We compute the optimum strategies and the value of the game as follows:

Here $a = [1,3]; b = [-3,-1]; c = [-3,-1]; d = [-2,4]$

$p_1 = \frac{[-2,4]-[-3,-1]}{[1,13]}$ = [-1, 7] and p_2 =[1, 1] – [-1, 7] = [-6, 2]

$q_1 = \frac{[-2,4]-[-3,-1]}{[1,13]}$ = [-1, 7] and q_2 =[1, 1] – [-1, 7] = [-6, 2]

The optimum strategies are

$S_A = \begin{bmatrix} A_1 & A_2 \\ [-1,7] & [-6,\ 2] \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 & B_5 \\ [-1,7] & [0,0] & [0,0] & [-6,2] & [0,0] \end{bmatrix}$

Value of the game = $\frac{([1,3]*[-2,4])-([-3,-1]*[-3,-1])}{([1,3]+[2,4])-([-3,-1]+[-3,-1])}$

$= \frac{[-15,11]}{[1,13]}$

= [-15, 11]

6. Consider the following game with payoff as intervals.We reduce this matrix using dominance propertyand solve it graphically.

Player B

$$\textbf{Player A} \begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \\ \mathbf{A_3} \\ \mathbf{A_4} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \mathbf{B_4} \end{array} \\ \begin{bmatrix} [16,20] & [3,5] & [5,7] & [3,5] \\ [4,8] & [1,3] & [12,14] & [6,8] \\ [10,12] & [2,8] & [14,20] & [2,4] \\ [6,8] & [5,7] & [7,17] & [1,3] \end{bmatrix} \end{array}$$

We use ordering of intervals to omit dominated rows or columns.

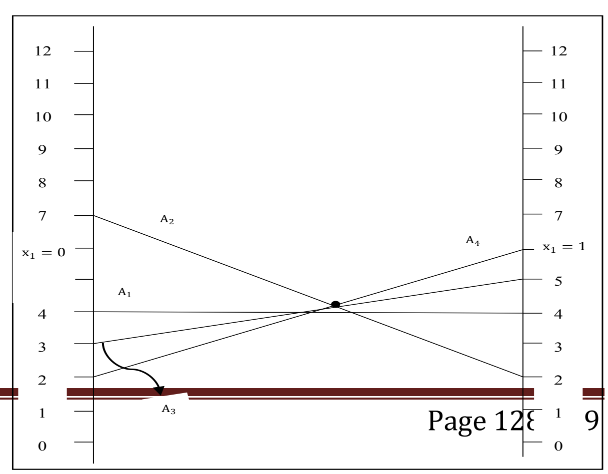
Step 1: Column I is dominated by column II and Column III is dominated by Column IV. Therefore, omit Column I and Column III. We have the following matrix.

$$\begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \\ \mathbf{A_3} \\ \mathbf{A_4} \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{B_2} & \mathbf{B_4} \end{array} \\ \begin{bmatrix} [3,5] & [3,5] \\ [1,3] & [6,8] \\ [2,8] & [2,4] \\ [5,7] & [1,3] \end{bmatrix} \end{array}$$

Step 2: We convert an interval game problem into a crisp game problem by using centre of intervals. Hence, we have

$$\begin{array}{c} \\ \mathbf{A_1} \\ \mathbf{A_2} \\ \mathbf{A_3} \\ \mathbf{A_4} \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{B_2} & \mathbf{B_4} \end{array} \\ \begin{bmatrix} 4 & 4 \\ 2 & 7 \\ 5 & 3 \\ 6 & 2 \end{bmatrix} \end{array}$$

Step 3: We plot the graph for a crisp game problem.



The reduced payoff matrix of a crisp game problem is

$$\begin{array}{cc} & \begin{array}{cc} \boldsymbol{B_2} & \boldsymbol{B_4} \end{array} \\ \begin{array}{c} \boldsymbol{A_2} \\ \boldsymbol{A_4} \end{array} & \begin{pmatrix} 2 & 7 \\ 6 & 2 \end{pmatrix}. \end{array}$$

Step 4: We enter the corresponding entries of the reduced payoff matrix of a crisp game problem in terms of intervals.

The reduced payoff matrix in terms of intervals is given by

$$\begin{array}{cc} & \begin{array}{cc} \boldsymbol{B_2} & \boldsymbol{B_4} \end{array} \\ \begin{array}{c} \boldsymbol{A_2} \\ \boldsymbol{A_4} \end{array} & \begin{pmatrix} [1,3] & [6,8] \\ [5,7] & [1,3] \end{pmatrix} \end{array}$$

Step 5: We compute the optimum strategies and the value of the game as follows:

Here $a = [1,3]$; $\mathrm{b} = [6,8]$; $\mathrm{c} = [5,7]$; $\mathrm{d} = [1,3]$

$\mathrm{p}_1 = \frac{[1,3]-[5,7]}{[-13,-5]} = \left[\frac{2}{13}, \frac{6}{5}\right]$ and $\mathrm{p}_2 = [1, 1] - \left[\frac{2}{13}, \frac{6}{5}\right] = \left[\frac{-1}{5}, \frac{11}{13}\right]$

$\mathrm{q}_1 = \frac{[1,3]-[6,8]}{[-13,-5]} = \left[\frac{3}{13}, \frac{7}{5}\right]$ and $\mathrm{q}_2 = [1, \quad 1] - \left[\frac{3}{13}, \frac{7}{5}\right] = \left[\frac{-2}{5}, \frac{10}{13}\right]$

The optimum strategies are

$$\mathrm{S_A} = \begin{bmatrix} \mathrm{A_1} & \mathrm{A_2} & \mathrm{A_3} & \mathrm{A_4} \\ [0, 0] & \left[\frac{2}{13}, \frac{6}{5}\right] & [0, 0] & \left[\frac{-1}{5}, \frac{11}{13}\right] \end{bmatrix} \text{ and}$$

$$\mathrm{S_B} = \begin{bmatrix} \mathrm{B_1} & \mathrm{B_2} & \mathrm{B_3} & \mathrm{B_4} \\ [0, 0] & \left[\frac{3}{13}, \frac{7}{5}\right] & [0, 0] & \left[\frac{-2}{5}, \frac{10}{13}\right] \end{bmatrix}$$

$$\text{Value of the game} = \frac{([1,3]*[1,3])-([6,8]*[5,7])}{([1,3]+[1,3])-([6,8]+[5,7])}$$

$$= \frac{[-55,-21]}{[-13,-5]}$$

$$= \left[\frac{21}{13}, 11\right]$$

5. CONCLUSION

In this paper, we solved game problems with payoffs as intervals. An interval game problem based on pure strategy, mixed strategy and using dominance property are discussed after which we obtained the optimal strategies and the value of the game. We also solved $2 \times n$ and $m \times 2$ game matrix using graphical method and obtained the optimum strategies and the value of the game.

REFERENCES

1. Dwyer, P.S., (1951). *Linear Computations*, John Wiley and Sons, New York.
2. John Von Newmann and O.Morgenstern,(1947).*Theory of Games and Economic Behavior. Princeton,* New Jersey: Princeton University press.
3. Suparna Das and S. Chakraverty, (2012). *Numerical Solution of Interval and Fuzzy System of Linear Equations*. 7:334-356.

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