DOI:10.26524/krj.2021.1

*Kong. Res. J.* 8(1): 1-4, 2021 ISSN 2349-2694, All Rights Reserved,
**Publisher**: Kongunadu Arts and Science College, Coimbatore. https://www.krjournal.com

**RESEARCH ARTICLE**

# NANO *gα-NORMAL SPACES AND ALMOST NANO *gα-NORMAL SPACES

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## ABSTRACT

The aim of this paper is to introduce a new class of different normal spaces, namely nano*gα-normal spaces, strongly nano*gα-closed, almost *gα-irresolute functions in nano topological spaces and their properties also studied.

**Keywords:** nano*gα-normal spaces, strongly nano*gα-closed functions, almost *gα-irresolute functions.

## 1. INTRODUCTION

In the year 1971, Vigiline [1] who defined semi normal spaces and it is an important topological property. The class of almost normal spaces is proved that a space is normal if and only if its both a semi normal spaces and an almost normal space by Singal and Arya [2]. In 2013, Lellis Thivagar [3] who introduced nano topology and defined in forms of lower approximations, upper approximations and boundary region of a subset of a universe using and equivalence relation on it. The author also defined the basis, nano interior operator and nano closure operator in nano topological spaces. The concepts of nano continuous and nano pre-continuous and their properties were also studied by Lellis Thivagar [3]. The Characterizations of mildly nano gb-normal spaces were introduced by Dhanis Arul Mary and Arockiarani [4].

## 2. Nano*gα-normal spaces

**Definition 2.1** Let $(U,\tau_R(X))$ be a Nano topological spaces. A subset A of$(U,\tau_R(X))$ is called Nano*gα-closed set if $Ncl(A)\subseteq V$ Where $A\subseteq V$ and V is Nano gα-open.

**Definition 2.2** A nano topological space $(U, \tau_R(X))$ is said to nano*gα-normal if for any pair of disjoint nano-closed sets A and B, there exist nano *gα-open sets M and N such that $A \subset M$ and $B \subset N$.

**Example 2.3** Let U = {x, y, z} with U/R = {{x}, {y, z}}. Then the nano topology $\tau_R(X)$ = {U, φ, {x}, {y, z}}. Hence the only pair of disjoint closed subsets of $(U, \tau_R(Y))$ is {y, z}, {x}. Also {x}, {y, z} are *gα-open sets such that $\{x\} \subset \{x\}, \{y,z\} \subset \{y,z\}$.

**Theorem 2.4** For a nano topological space f: $(U, \tau_R(X)) \to (V, \tau_R'(Y))$
the followings statements are equivalent:
(i) U is *gα-normal.
(ii) for every pair of nano open sets M and N whose union is U, there exist nano *gα-closed sets A and B such that $A\subset M$ and $B \subset N$ and $A\cup B = U$.
(iii) for every nano closed sets H and every nano open set K containing H, there exists nano *gα-open set M such that $H \subset M \subset N{*}g\alpha cl(M) \subset K$.

**Proof.** (i) ⇒ (ii) let M and N be a pair of nano open sets in a nano *gα-normal space U such that $U = M \cup N$. Then $U - M$, $U - N$ are disjoint nano closed sets since U is nano *gα-normal there exist nano *gα-open sets $M_1$ and such $N_1$ such that $U - M \subset M_1$ and $U - N \subset N_1$ Let $A = U - M_1$, $B = U - N_1$ Then A and B are nano *gα-closed sets such that $A \subset M$ and $B \subset N$ and $A \cup B = U$.
(ii)⇒ (iii) Let H be a nano closed set and K be a nano open set containing H. Then U − H and K are nano open sets whose union is U. Then by (ii), there exist nano *gα-closed sets $P_1$ and $P_2$ such that $P_1 \subset U - H$ and $P_2 \subset K$ and $P_1 \cup P_2 = U$. Then $H \subset U - P_1$ and $U - K \subset U - P_2$ and $(U - P_1) \cap (U - P_2) = \varphi$. Let $M = U - P_1$ and $N = U - P_2$. Then M and N are disjoint nano *gα-open sets such that $H \subset M \subset U - N \subset K$. As U − N is nano*gα-closed set, we have
$N{*}g\alpha\text{-}cl(M) \subset U - N$ and $H \subset M \subset N{*}g\alpha\text{-}cl(M) \subset K$.
(iii)⇒ (i) Let $H_1$ and $H_2$ are two disjoint nano closed sets of U. Put $K = U - H_2$. Then



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$H_2 \cap K = \varphi$. $H \subset K$ where K is a nano open set. Then by (iii), there exists a nano *gα-open sets M of U such that $H_1 \subset M \subset N$ *gα-cl (M) $\subset K$. It follows that $H_2 \subset U - N$ *gα-cl (M) = N. Say, then N is nano *gα-open and $M \cap N = \varphi$. Hence $H_1$ and $H_2$ are separated by nano *gα-open sets M and N. Therefore U is nano *gα-normal.

**Definition 2.5** A function f: (U, $\tau_R(X)$) → (V, $\tau_R'$ (Y)) is called strongly nano *gα-open if f (M) ∈ N *gαO (V) for each M ∈N*gαO(U).

**Definition 2.6** A function f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is called strongly nano*gα-closed if f (M) ∈ N *gαC (V) for each M ∈N*gαC(U).

**Theorem 2.7** A function f : (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is called strongly nano *gα-closed if and only if for each subset B in V and for each nano *gα-open set M in U containing $f^{-1}(B)$, there exist a nano *gα-open set N containing B such that $f^{-1}(N) \subset M$.

**Proof.** Suppose that f is strongly nano *gα-closed. Let B be a subset of V and M ∈ N *gαO (U) containing $f^{-1}$ (B). Put N = V – f (U – M), then N is a nano*gα-open set of V such that $B \subset N$ and $f^{-1}$ (N) $\subset$ M. Conversely let K be any nano *gα-closed set of U. Then $f^{-1}$ (V – f (K)) $\subset$ U – K and U – K ∈ N *gαO (U). There exists a nano *gα-open set N of V such that V – f (K) $\subset$ N and $f^{-1}$ (N) $\subset$ U – K. Therefore we have f (K) ⊃ V – N and K $\subset f^{-1}$ (V – N). Hence we obtain f (K) = V – N and f (K) is nano *gα-closed in V. This shows that f is strongly nano *gα-closed.

**Theorem 2.8** If f : (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is a strongly nano *gα-closed continuous function from a nano *gα-normal space U on to a space V, then V is nano *gα-normal.

**Proof.** Let $K_1$ and $K_2$ are disjoint nano closed sets in V. Then $f^{-1}$ ($K_1$) and $f^{-1}$ ($K_2$) are nano closed sets in V. Since U is nano *gα-normal then there exist disjoint nano *gα-open sets M and N such that $f^{-1}$ ($K_1$) $\subset$ M and $f^{-1}$ ($K_2$) $\subset$ N. Then there exist nano *gα-open sets A and B such that $K_1 \subset A$, $K_2 \subset B$, $f^{-1}$ (A) $\subset$ M and $f^{-1}$ (B) $\subset$ N. Also A and B are disjoint. Thus V is nano *gα-normal.

**Definition 2.9** A function f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is called almost nano*gα-irresolute if for each u in U and each nano *gα-neighbourhood N of f (u), N *gα – cl ($f^{-1}$(N)) is a nano *gα-neighbourhood of u.

**Lemma 2.10** For a function f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) the following statements are equivalent.

(i)f is almost nano *gα-irresolute.

**(ii)**$f^{-1}$(N) ⊂ N*gα-int(N*gα-cl($f^{-1}$(N))) for every N ∈ N*gαO(V).

**Theorem 2.11** A function f :(U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) almost nano*gα-irresolute if and only if f (N*gα – cl(M)) ⊂ N*gα – cl(f(M)) for every M ∈ N *gαO(U).

**Proof.** Let M ∈ N*gαO(U). Suppose V ∉ N*gα – cl(f (M). Then there exists N ∈ N *gαO(V, v)) such that N ∩ f (M) = φ. Hence, $f^{-1}$(N) ∩ M = φ .Since M ∈ N*gαO(U). We have N*gα – int(N*gα – cl($f^{-1}$(N))) ∩ N *gα – cl(M) = φ. Then by lemma [2.10], $f^{-1}$(N) ∩ N *gα – cl(M) = φ and hence N ∩ f(N*gα – cl(M)) = φ. This implies that V∉f (N*gα – cl(M)).

Conversely if N ∈ N *gαO(V) then P = U/N *gα – cl($f^{-1}$(N)) ∈ N *gαO(U). By hypothesis, f (N *gα – cl(P)) ⊂ N *gα – cl(f(P)) and hence, U/N*gα – int(N*gα – cl($f^{-1}$(N))) = N*gα – cl(P)⊂ $f^{-1}$(N *gα – cl(f (P))) ⊂ $f^{-1}$N *gα – cl(f (U)/ f $^{-1}$(N))⊂ $f^{-1}$N *gαcl(V/N)⊂ $f^{-1}$(V/N) = U/$f^{-1}$(N).

Therefore $f^{-1}$(N) ⊂ N *gα – int(N *gα – cl($f^{-1}$(N)). By Lemma [3.10], f is almost nano *gα-irresolute.

**Theorem 2.11** If f: (U, $\tau_R(X)$) → (V, $\tau_R'$ (Y)) is a strongly nano*gα-open continuous almost nano*gα-irresolute function from a nano*gα-normal space U onto a space V. Then V is nano*gα-normal space.

**Proof.** Let A be a nano closed set of K and B be a nano open set containing A.

Then by continuity of f , $f^{-1}$(A) is nano closed and $f^{-1}$(B) is nano open set of U such that $f^{-1}$(A) ⊂ $f^{-1}$(B). As U is nano*gα-normal, there exists a nano *gα-open set M in U such that $f^{-1}$(A) ⊂ M ⊂ N*gα – cl(M) ⊂ $f^{-1}$(B) by Theorem 2.4. Then f($f^{-1}$(A)) ⊂ f(M) ⊂ f(N*gαcl(M)) ⊂ f ($f^{-1}$(B)). Since f is strongly nano *gα-open almost nano *gα-irresolute surjection, we obtain A ⊂ f (M) ⊂ N *gα – cl(f (M) ⊂ B. Then again Theorem 3.4 the space V is a nano *gα-normal.

## 3. Almost nano *gα-normal spaces

**Definition 3.1** A nano topological spaces $(U, \tau_R(X))$ is said to be almost nano *gα-normal if for each nano *gα-closed set A and nano regular closed set B such that $A \cap B = \varphi$, there exist disjoint nano *gα-open sets M and N such that $A \subset M$ and $B \subset N$.

**Definition 3.2** A nano topological spaces $(U, \tau_R(X))$ is said to be quasi nano*gα-closed if f (A) is nano *gα-closed in V for each $A \in N$ *gαC(U ).

**Theorem 3.3** For a nano topological spaces $(U, \tau_R(X))$ the following statements are equivalent:

(i) U is almost nano *gα-normal.

(ii) For every pair of nano sets A and B, one of which is nano *gα-open and the other is nano regular open whose union is U, there exist nano nano *gα-closed sets H and K such that $H \subset A$ and $K \subset B$ and $H \cup K = U$.

(iii) For every nano *gα-closed set H and nano regular open set K containing H, there exists a nano *gα-open set N such that $H \subset B \subset N$ *gαcl(B) $\subset K$.

**Proof.** (i) ⇒ (ii) Let A and B be a pair of nano open sets in a nano *gα-normal space U such that $U = A \cup B$. Then $U - A$ and $U - B$ are two disjoint nano *gα-closed sets. Since U is nano *gα-normal there exist nano*gα-open sets $A_1$ and such $B_1$, such that $U - A \subset A_1$ and $U - B \subset B_1$. Let $H = U - A_1$, $K = U - B_1$. Then H and K are nano *gα-closed sets such that $H \subset A$ and $K \subset B$ and $H \cup K = U$.

(ii)⇒ (iii) Let A be a nano *gα-closed set and B be a nano *gα-open set containing A. The $U - A$ and B are nano *gα-open sets whose union is U. Then by (ii), there existnano *gα-closed sets $W_1$ and $W_2$ such that $W_1 \subset U - A$ and $W_2 \subset B$ and $W_1 \cup W_2 = U$. Then $A \subset U - W_1$ and $U - B \subset W_2$ and $(U - W_1) \cap (U - W_2) = \varphi$. Let $X = U - W_1$ and $Y = U - W_2$. Then A and B are disjoint nano *gα-open sets such that $A \subset U - Y \subset B$. As $U - Y$ is nano
*gα-closed set, we have N *gαcl(X) $\subset U - Y$ and $A \subset X \subset N$ *gαcl(X) $\subset B$.

(iii)⇒ (i): Let $A_1$ and $A_2$ be any two disjoint nano *gα-closed sets of U.Put $B = U - A_2$, then $A_2 \cap B = \varphi$. $A_1 \subset B$ where B is a nano*gα-open sets. Then by (iii), there exists a nano *gα-open set X of U such that $A_1 \subset U - N$ *gαcl(X) = Y, then Y is nano *gα-open and $X \cap Y = \varphi$. Hence $A_1$ and $A_2$ are seperated by nano *gα-open sets X and Y. Therefore U is nano *gα-normal.

**Theorem 3.4** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))$ is a nano *gα-continuous, quasi nano *gα-closed surjection and U is nano *gα-normal, then V is normal.

**Proof** Let $W_1$ and $W_2$ be any disjoint nano *gα-closed sets of V. Since f is nano *gα-continuous, $f^{-1}(W_1)$ and $f^{-1}(W_2)$ are disjoint nano *gα-closed sets of U. Since U is nano *gα-normal, there exist disjoint $N_1$ and $N_2 \in N$ *gαO(V ), such that $f^{-1}(W_1) \subset N_i$ for $i = 1, 2$. Put $Q_i = V - f(U - N_i$ then $Q_i$ is nano*gα-open in V, $W_2 \subset Q_i$ and $f^{-1}(Q_i) \subset N_i$ for $i = 1, 2$. Since $N_1 \cap N_2 = \varphi$ and f is surjective. We have $Q_1 \cap Q_2 = \varphi$. This shows that V is nano *gα-normal.

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