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ON $\alpha\delta$-REGULAR SPACES

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ABSTRACT

The concept of $\alpha\delta$-closed sets was introduced by Devi *et al.,* 2012. The aim of this paper is to consider and characterize $\alpha\delta$-regular spaces and study some of their properties.

Keywords: αδ-regular spaces.

1. INTRODUCTION

The importance of general topological spaces rapidly increases in many fields of applications such as data mining. Information systems are basic tools for producing knowledge from data in any real-life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones.

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of $\alpha\delta$-open (Devi *et al.,* 2012) sets introduced by R.Devi, V.Kokilavani and P.Basker. In this paper, we will continue the study of related functions with $\alpha\delta$-open and $\alpha\delta$-closed sets. We introduce and characterize the concept of $\alpha\delta$-regular spaces and study some of their properties.

2. PRELIMINARIES

Throughout the present paper, spaces X and Y always mean topological spaces. Let X be a topological space and A a subset of $X.$ The closure of A and the interior of $A$are denoted by $Cl(A)$ and I$nt(A)$, respectively. A subset A is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$, The δ-interior (Velico, 1968) of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $Int_\delta(A)$.The subset A is called δ-open (Velico, 1968) if $A = Int_\delta(A)$, i.e., a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ -closed. Alternatively,a set $A \subset X, \tau$ is called δ-closed (Velico, 1968) if $A = cl_\delta(A)$,where $cl_\delta\ A =$ $x/x \in U \in \tau \Rightarrow int(cl(A)) \cap A \neq \varphi$. The family of all δ-open (resp. δ-closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called semiopen (Noiri, 1998) (resp. α-open (Devi *et al.,* 2012), δ-semiopen[2]] if $A \subset cl(int(A))$ (resp.$A \subset int(cl(int\ A\ ))$, $A \subset cl(Int_\delta\ (A))$) and the complement of a semiopen(resp. α-open, δ-semiopen) are called semiclosed(resp. α-closed, δ-semiclosed).The intersection of all semiclosed (resp. α-closed, δ-semiclosed) sets containing A is called the semi-closure(resp. α-closure, δ-semiclosure) of A and is denoted by $scl(A)$ (resp.$\alpha cl(A)$, δ-$scl(A)$). Dually, semi-interior(resp. α-interior, δ-semi-nterior) of A is defined to be the union of all semiopen (resp. α-open,δ-semiopen) sets contained in A and is denoted by $sint(A)$ (resp.$\alpha int\ (A)$, δ-$sint(A)$). Note that δ-$scl\ A\ = A \cup int(cl_\delta(A))$ and δ-$sint(A) = A \cup cl(Int_\delta(A))$.

We recall the following definition used in sequel.

2.1. Definition (Devi *et al.,* 2012) A subset A of a space X is said to be

(a) An α-generalized closed (αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

(b) A $\alpha\delta$-closed set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X, τ).

2.2. Definition A function $f: (X, \tau) \to (Y, \sigma)$ is called

(a) $\alpha\delta$-continuous (Kokilavani and Basker, 2006) if $f^{-1}(V)$ is $\alpha\delta$-closed in (X, τ) for every closed set V of (Y, σ).

(b) $\alpha\delta$-irresolute (Kokilavani and Basker, 2006) if $f^{-1}(V)$ is $\alpha\delta$-closed in (X, τ) for every $\alpha\delta$-closed set V of (Y, σ).

(c) semi-closed (Noiri, 1998) if $f(V\ )$ is semiclosed in Y for every closed set V in X.

(d) pre-closed (Noiri, 1998) if $f(V)$ is closed in Y for every semi-closed set V in X.

2.3. Definition (Kokilavani Varadharajan and Basker Palaniswamy, 2013) A topological space X, τ is called

(a) $T_0^{\#\alpha\delta}$ if for any distinct pair of points in X, there is a $\alpha\delta$-*open* set containing one of the points but not the other.

(b) $T_1^{\#\alpha\delta}$ if each pair of distinct points x and y in X there exists a $\alpha\delta$-*open* set U in X such that $x \in U$ and $y \notin U$ and a $\alpha\delta$-*open* set V in X such that $y \in V$ and $x \notin V$.

(c) $T_2^{\#\alpha\delta}$ if for each pair of distinct points x and y in X there exist $\alpha\delta$-*open* sets U and V such that $U \cap V = \varphi$ and $x \in U$, $y \in V$.

3. *$\alpha\delta$-REGULAR SPACES*

In this section, we introduce and study $\alpha\delta$-regular spaces and some of their properties.

3.1. Definition A topological space X is said to be $\alpha\delta$-regular if for each closed set F and each point $x \notin F$, there exist disjoint $\alpha\delta$-open sets U and V such that $x \in U$ and $F \subset V$.

3.2. Definition A function $f: (X, \tau) \to (Y, \sigma)$ is called

(a) semi-$\alpha\delta$-continuous if $f^{-1}(V)$ is $\alpha\delta$-closed in (X, τ) for every semi-closed set V of (Y, σ).

(b) $\alpha\delta$-open if $f(V)$ is an $\alpha\delta$-open in (Y, σ) for every open set V in (X, τ).

(c) $p\alpha\delta$-open if $f(V)$ is an $\alpha\delta$-open in (Y, σ) for every semi-open set V in (X, τ).

(d) $(Q, \alpha\delta)$-open if $f(V)$ is an open in (Y, σ) for every $\alpha\delta$-open set V in (X, τ).

(e) Strongly $\alpha\delta$-open if $f(V)$ is an $\alpha\delta$-open in (Y, σ) for every $\alpha\delta$-open set V in (X, τ).

3.3. Theorem Every $\alpha\delta$-regular $T0$-space is $T_2^{\#\alpha\delta}$.

Proof. Let $x, y \in X$ such that $x \neq y$. Let X be a $T0$-space and V be an open set which contains x but not y. Then $X - V$ is a closed set containing y but not x. Now by $\alpha\delta$-regularity of X there exist disjoint $\alpha\delta$-open sets U and W such that $x \in U$ and $X - V \subset W$. Since $y \in X - V$, $y \in W$. Thus for $x, y \in X$ with $x \neq y$, there exist disjoint open sets U and W such that $x \in U$ and $y \in W$. Hence X is $T_2^{\#\alpha\delta}$.

3.4. Theorem In a topological space X, the following conditions are equivalent:

(a) X is $\alpha\delta$-regular.

(b) For every point $x \in X$ and open set V containing x there exists a $\alpha\delta$-open set U such that $x \in U \subset \alpha\delta_{Cl}(U) \subset V$,

(c) For every closed set F, $F = \cap\ \alpha\delta_{Cl}\ \ V : F \subset V \ and\ V\ is\ \alpha\delta - open\ set\ of\ X$,

(d) For every set A and an open set B such that $A \cap B \neq \varphi$, there exists $\alpha\delta$-open set O such that

$A \cap O \neq \varphi$ and $\alpha\delta_{Cl}(O) \subset B$,

(e) For every non empty set A and closed set B such that $A \cap B \neq \varphi$, there exist disjoint $\alpha\delta$-open

sets L and M such that $A \cap L \neq \varphi$ and $B \subset M$.

Proof. $(a) \Rightarrow (b)$: Let V be an open set containing x. Then $X - V$ is closed set not containing x. Since X is $\alpha\delta$-regular, there exist $\alpha\delta$-open sets L and U such that $x \in U$, $X - V \subset L$ and $U \cap L = \varphi$. This implies $U \subset X - L$. Therefore, $\alpha\delta_{Cl}(U) \subset \alpha\delta_{Cl}(X - L) = X - L$, because $X - L$ is $\alpha\delta$-closed. Hence $\in U \subset \alpha\delta_{Cl}(U) \subset X - L \subset V$. That is $\in U \subset \alpha\delta_{Cl}(U) \subset V$.

$(b) \Rightarrow (c)$: Let F be a closed set and $x \notin F$. Then $X - F$ is an open set containing x. By (b), there

is a $\alpha\delta$-open set U such that $x \in U \subset \alpha\delta_{Cl}(U) \subset X - F$. And so, $F \subset X - \alpha\delta_{Cl}(U) \subset X - U$. Consequently $X - U$ is $\alpha\delta$-closed set not containing x. Put $V = X - \alpha\delta_{Cl}(U)$. This implies $F \subset V$ and V is $\alpha\delta$-open set of X and $x \notin \alpha\delta_{Cl}(V)$, implies

$$\cap\ \ \alpha\delta_{Cl}\ \ V : F \subset V\ and\ V\ is\ \ \alpha\delta - open\ \ set\ \ of\ X \subset F \qquad 1$$

But F is closed and every closed set is $\alpha\delta$ closed. Therefore

$$F \subset \cap \{\ \alpha\delta_{Cl}(V) : F \subset V\ and\ V\ is\ \alpha\delta - open\ \ set\ \ of\ X\} \qquad (2)$$

is always true. From (1) and (2),

$$F = \cap \{\ \alpha\delta_{Cl}(V) : F \subset V\ and\ V\ is\ \alpha\delta - open\ \ set\ \ of\ X\}.$$

$(c) \Rightarrow (d)$: Let $A \cap B \neq \varphi$ and B is open. Let $x \in A \cap B$. Then $X - B$ is a closed set not containing x. By (c), there exists a $\alpha\delta$-open set V of X such that $X - B \subset V$ and $x \neq \alpha\delta_{Cl}(V)$. Put $O = X - \alpha\delta_{Cl}(V)$, then O is $\alpha\delta$-open set of X, $x \in A \cap O$ and $\alpha\delta_{Cl}(O) \subset \alpha\delta_{Cl}(X - V) = X - V \subset B$. Hence $\alpha\delta_{Cl}(O) \subset B$.

$(d) \Rightarrow (e)$: If $A \cap B = \varphi$, where A is non empty and B is closed, then $A \cap (X - B) \neq \varphi$ and $X - B$ is open. Therefore by (d), there exists $\alpha\delta$-open set L such that $A \cap L \neq \varphi$ and $L \subset \alpha\delta_{Cl}(L) \subset X - B$.

$(e) \Rightarrow (a)$: Let F be a closed set such that $x \notin F$, then $\{x\} \cap F = \varphi$. By (e), there exist disjoint open sets L and M such that $\{x\} \cap L \neq \varphi$ and $F \subset M$, which implies $x \in L$ and $F \subset M$. Hence, X is $\alpha\delta$-regular.

3.5. Theorem If $f : X \to Y$ is continuous bijective, $\alpha\delta$-open (resp. $p\alpha\delta$-open) function and X is a regular (resp. s-regular) space, then Y is $\alpha\delta$-regular.

Proof. Let F be a closed set in Y and $y \notin F$. Take $y = f(x)$ for some $x \in X$. Since f is continuous $f^{-1}(F)$ is closed set in X such that $x \notin f^{-1}(F)$. Now X is regular (resp. s-regular), there exist disjoint open (resp. semi-open) sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. That is, $y = f(x) \in f(U)$ and $F \subset f(V)$. Since f is $\alpha\delta$-open (resp. $p\alpha\delta$-open) function $f(U)$ and $f(V)$ are $\alpha\delta$-open sets in Y and f is bijective $f(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Therefore, Y is $\alpha\delta$-regular.

3.6. Theorem If $f : X \to Y$ is semi continuous bijective, $p\alpha\delta$-open function and X is semi regular space, then Y is $\alpha\delta$-regular.

Proof. Let F be a closed set in Y and $y \notin F$. Take $y = f(x)$ for some $x \in X$. Since f is semi continuous $f^{-1}(F)$ is semiclosed set in X and $x \notin f^{-1}(F)$. Now X is semi regular, there exist disjoint semiopen sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. That is, $y = f(x) \in f(U)$ and $F \subset f(V)$. Since f is $p\alpha\delta$-open function $f(U)$ and $f(V)$ are $\alpha\delta$-open sets in Y and f is bijective $(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Therefore, Y is $\alpha\delta$-regular.

3.7. Theorem If $f : X \to Y$ is continuous surjective, strongly $\alpha\delta$-open (resp. $(Q, \alpha\delta)$-open) function and X is $\alpha\delta$-regular space, then Y is $\alpha\delta$-regular (resp. regular).

Proof. Let F be a closed set in Y and $y \notin F$. Take $y = f(x)$ for some $x \in X$. Since f is continuous surjective $f^{-1}(F)$ is closed set in X and $x \notin f^{-1}(F)$. Now since X is $\alpha\delta$-regular, there exist disjoint $\alpha\delta$-open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. That is, $y = f(x) \in f(U)$ and $F \subset f(V)$. Since f is strongly $\alpha\delta$-open (resp. $(Q, \alpha\delta)$-open) and bijective, $f(U)$ and $f(V)$ are disjoint $\alpha\delta$-open (resp. open) sets in . Therefore, Y is $\alpha\delta$-regular (resp. regular).

3.8. Theorem If $f : X \to Y$ is $\alpha\delta$-continuous, closed, injection and Y is regular, then X is $\alpha\delta$-regular.

Proof. Let F be a closed set in X and $x \notin F$. Since f is closed injection $f(F)$ is closed set in Y such that $f(x) \notin f(F)$. Now Y is regular, there exist disjoint open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\alpha\delta$-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are $\alpha\delta$-open sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \varphi$. Hence X is $\alpha\delta$-regular.

3.9. Theorem If $f : X \to Y$ is semi $\alpha\delta$-continuous, closed (resp. semi-closed), injection and Y is s-regular (resp. semi regular) then X is $\alpha\delta$-regular.

Proof. Let F be a closed set in X and $x \notin F$. Since f is closed (resp. semi-closed) injection $f(F)$ is closed (resp. semi-closed) set in Y such that $f(x) \notin f(F)$. Now Y is s-regular (resp. semi regular), there exist disjoint semi-open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is semi $\alpha\delta$-continuous $f^{-1}(G)$ and $f^{-1}(H)$ are $\alpha\delta$-open sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \varphi$. Hence X is $\alpha\delta$-regular.

3.10. Theorem If $f : X \to Y$ is strongly $\alpha\delta$-continuous, closed (resp. pre-closed), injection and Y is $\alpha\delta$-regular then X is s-regular (resp. semi regular).

Proof. Let F be a closed (semi-closed) set in X and $x \notin F$. Since f is closed (resp. pre-closed) injection $f(F)$ is closed set in Y such that $f(x) \notin f(F)$. Now Y is $\alpha\delta$-regular, there exist disjoint $\alpha\delta$-open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is strongly $\alpha\delta$-continuous $f^{-1}(G)$ and $f^{-1}(H)$ are open (hence semi-open) sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \varphi$. Hence X is regular (resp. semi regular).

3.11. Theorem If $f : X \to Y$ is $\alpha\delta$-irresolute, closed, injection and Y is $\alpha\delta$-regular then X is $\alpha\delta$- regular.

Proof. Let F be a closed set in X and $x \notin F$. Since f is closed injection $f(F)$ is closed set in Y such that $f(x) \notin f(F)$. Now Y is $\alpha\delta$-regular, there exist disjoint $\alpha\delta$-open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $\alpha\delta$-irresolute $f^{-1}(G)$ and $f^{-1}(H)$ are $\alpha\delta$-open sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \varphi$. Hence X is $\alpha\delta$-regular.

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