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RESEARCH ARTICLE

TRIANGULAR INTUITIONISTIC FUZZY SEQUENCING PROBLEM

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ABSTRACT

In this paper, we discuss different types of fuzzy sequencing problem with Triangular Intuitionistic Fuzzy Number. Algorithm is given for different types of fuzzy sequencing problem to obtain an optimal sequence, minimum total elapsed time and idle time for machines. To illustrate this, numerical examples are provided.

Keywords: Triangular Intuitionistic Fuzzy Number, Optimal sequence, Total elapsed time, Idle time, Score function and Ranking.

1. INTRODUCTION

Sequencing Problem is the problem of determining an appropriate order for a series of tasks to be performed on a finite number of service facilities so as to minimise the total time taken for finishing all the jobs. Johnson [1] has given a method of scheduling jobs in two machines. Its primary objective is to determine an optimal sequence of jobs and to reduce the total amount of time it takes to complete all the jobs. It also reduces the amount of idle time between the two machines. Furthermore, Johnson's method has been extended to 'm' machines problem with an objective to complete all the jobs in a minimum duration. But it is difficult to apply those ordinary approaches to real life situations. In reality, it is observed that the information available is of imprecise nature and there is an uncertainty in the problem. In order to handle this uncertainties, we use fuzzy sets which was introduced by Zadeh [2]. Here, we represent these uncertainties in terms of triangular intuitionistic fuzzy number. Atanassov [3] introduced the concept of Intuitionistic Fuzzy Sets, which is a generalization of the concept of fuzzy sets. Nagoorgani and Ponnalagu [4] have proposed a new algorithm to solve an Intuitionistic Fuzzy Linear Programming Problem. Nagoorgani and Ponnalagu [5] defined a new operation on triangular fuzzy number for solving fuzzy linear programming problem. Radhakrishnan and Saikeerthana [6] have solved problems on Game Theory using interval parameters. Radhakrishnan and Saikeerthana [7] have discussed and solved problems related to Critical Path Method and Programme Evaluation Review Technique with intervals and also with Conversion of fuzzy parameters(triangular and trapezoidal) into intervals using $\alpha - \text{cut}$ s. Radhakrishnan and Saikeerthana [8] solved fuzzy sequencing problem with triangular fuzzy numbers. Radhakrishnan and Saikeerthana [9] have discussed single machine sequencing problem using fuzzy parameters.

The rest of this paper is framed as follows:

In section 2, basic definitions, arithmetic operations of triangular intuitionistic fuzzy numbers, ranking and score functions are given as preliminaries. In section 3, algorithm for solving different types of fuzzy sequencing problem is provided. In section 4, numerical examples illustrating the algorithm are given. Finally, the conclusion.

2. PRELIMINARIES

2.1. Fuzzy set

A fuzzy set $\tilde{A}$ in X is a set of ordered pair defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$ where $\mu_{\tilde{A}}(x)$ is a membership function.

2.2. Fuzzy Number

A fuzzy set Ã defined on a set of real number R is said to be a fuzzy number, if its membership function $\mu_{\tilde{A}}(x)$: R→ [0, 1] that satisfies the following properties.

a. $\tilde{A}$ is convex.

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i.e.,
$\mu_{\tilde{A}}\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in R$ and $\lambda \in [0,1]$.

b. $\tilde{A}$ is normal i.e., there exists an element $x_0 \in \tilde{A}$ such that $\mu_{\tilde{A}}(x_0) = 1$.

c. $\mu_{\tilde{A}}(x)$ is piecewise continuous.

2.3. Intuitionistic Fuzzy Set

Let X is a non-empty set. An Intuitionistic Fuzzy Set is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x)); x \in X\}$ which assigns to each element x, a membership degree $\mu_{\tilde{A}}(x)$ and a non- membership degree $\vartheta_{\tilde{A}}(x)$ under the condition $0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1$, for all $x \in X$.

2.4. Intuitionistic Fuzzy Number

An Intuitionistic Fuzzy Number $\tilde{A}^I$ is defined as follows:

i. an intuitionistic fuzzy subset of the real line,
ii. normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1$, $\vartheta_{\tilde{A}^I}(x_0) = 0$,
iii. A convex set for the membership function $\mu_{\tilde{A}^I}(x)$,
i.e.,
$\mu_{\tilde{A}^I}\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\} \forall x_1, x_2 \in R$ and $\lambda \in [0,1]$.
iv. A concave set for the non-membership function $\vartheta_{\tilde{A}^I}(x)$,
i.e.,
$\vartheta_{\tilde{A}^I}\{\lambda x_1 + (1-\lambda)x_2\} \leq \max\{\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2)\} \forall x_1, x_2 \in R$ and $\lambda \in [0,1]$.

2.5. Triangular Intuitionistic Fuzzy Number

A Triangular Intuitionistic Fuzzy Number $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\vartheta_{\tilde{A}^I}(x)$,

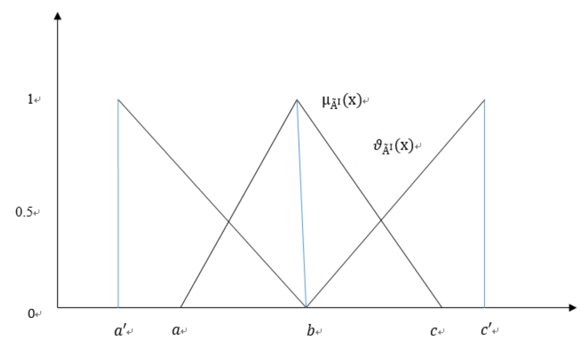
$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{x-c}{b-c}, & b \leq x \leq c \\ 0, & otherwise \end{cases}$$

and

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \frac{b-x}{b-a'}, & a' \leq x \leq b \\ \frac{x-b}{c'-b}, & b \leq x \leq c' \\ 1, & otherwise \end{cases}$$

Where $a' \leq a \leq b \leq c \leq c'$ and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \leq 1$ or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x) \; \forall x \in R$

This Triangular Intuitionistic Fuzzy Number is denoted by
$\tilde{A}^I = (a, b, c; a', b, c') = \{(a, b, c); (a', b, c')\}$



Membership and Non-membership functions of Triangular Intuitionistic Fuzzy number

2.6. Arithmetic Operations of Triangular Intuitionistic Fuzzy Number

Let $\tilde{A}^I = \{(a, b, c); (a', b, c')\}$ and $\tilde{B}^I = \{(d, e, f); (d', e, f')\}$. Then

i. Addition:
$\tilde{A}^I + \tilde{B}^I = \{(a+d, b+e, c+f); (a'+d', b+e, c'+f')\}$

ii. Subtraction:
$\tilde{A}^I - \tilde{B}^I = \{(a-f, b-e, c-d); (a'-f', b-e, c'-d')\}$

2.7. Score function

Let $\tilde{A}^I = \{(a, b, c); (a', b, c')\}$ be a Triangular Intuitionistic Fuzzy Number, then we define a Score function for membership and non-membership values respectively as $S(\widetilde{A}^{I\alpha}) = \frac{a+2b+c}{4}$ and $S(\widetilde{A}^{I\beta}) = \frac{a'+2b+c'}{4}$

2.8. Ranking using Score function

Let $\widetilde{A}^{I} = \{(a,b,c);(a',b,c')\}$ and $\widetilde{B}^{I} = \{(d,e,f);(d',e,f')\}$ be two Triangular Intuitionistic Fuzzy Numbers and $S(\widetilde{A}^{I\alpha})$, $S(\widetilde{A}^{I\beta})$ and $S(\widetilde{B}^{I\alpha})$, $S(\widetilde{B}^{I\beta})$ be the scores of $\widetilde{A}^{I}$ and $\widetilde{B}^{I}$ respectively

i. If $S(\widetilde{A}^{I\alpha}) \le S(\widetilde{B}^{I\alpha})$ and $S(\widetilde{A}^{I\beta}) \le S(\widetilde{B}^{I\beta})$ then $\widetilde{A}^{I} < \widetilde{B}^{I}$.

ii. If $S(\widetilde{A}^{I\alpha}) \ge S(\widetilde{B}^{I\alpha})$ and $S(\widetilde{A}^{I\beta}) \ge S(\widetilde{B}^{I\beta})$ then $\widetilde{A}^{I} > \widetilde{B}^{I}$.

iii. If $S(\widetilde{A}^{I\alpha}) = S(\widetilde{B}^{I\alpha})$ and $S(\widetilde{A}^{I\beta}) = S(\widetilde{B}^{I\beta})$ then $\widetilde{A}^{I} = \widetilde{B}^{I}$.

3. FUZZY SEQUENCING PROBLEM

The Sequencing Problem with uncertain processing time is termed as fuzzy sequencing problem. Algorithms for different types of fuzzy sequencing problems are proposed to sequence the jobs to be processed in various machines, with minimum total processing time. The assumptions for the classical problem are also applicable for fuzzy sequencing problem.

3.1. Algorithm for solving different types of fuzzy sequencing problem

3.1.1. Processing 'n' jobs on two machines

Let A_1', A_2'... A_n' be the processing times of 'n' jobs on Machine 1 and B_1', B_2'...B_n' be the processing times of 'n' jobs on Machine 2. The problem is to find the order in which the 'n' jobs are to be processed through two machines with the minimum total elapsed time.

Procedure:

Step 1: Use the ranking function to identify the minimum processing time from the given list of processing times A_1', A_2'... A_n' and B_1', B_2'... B_n'.

Step 2: If the minimum processing time is A_s' (i.e., job number 's' on machine 1) then do the s^{th} job first in the sequence. If the minimum processing time is B_t' (i.e., job number 't' on machine 2) then do the t^{th} job last in the sequence.

Step 3:

a) If there is a tie in minimum processing of both machines (i.e., $A_s' = B_t'$), process the s^{th} job first and t^{th} job last in the sequence.

b) If the tie for the minimum occurs among the processing time on Machine 1, choose the job corresponding to the minimum of processing time on Machine 2 and process it first.

c) If the tie for the minimum occurs among the processing time on Machine 2, choose the job corresponding to the minimum of processing time on Machine 1 and process it last.

Step 4: Cancel the jobs already assigned and repeat steps 2 to 4 until all the jobs have been assigned.

The resulting order will minimise the total elapsed time and it is known as optimal sequence.

Step 6: After obtaining an optimal sequence as stated above, the minimum total elapsed time and also the idle time on machines 1 and 2 are calculated as follows:

Minimum Total elapsed time = Time out of the last job on machine 2.

Idle time for machine 1 = Total elapsed time - time when the last job is out of machine 1

Idle time for machine 2 = Time at which the first job on machine 1 finishes in a sequence

$$+\sum_{i=2}^{n}\left\{\begin{matrix}(\text{time when the i}^{\text{th}}\text{ job starts on machine 2})\\ -(\text{time when the (i}-1)^{\text{th}}\text{ job finishes on machine 2})\end{matrix}\right\}$$

3.1.2. Processing 'n' jobs on three machines

Let A_1', A_2'... A_n' be the processing times of 'n' jobs on Machine 1, B_1', B_2'... B_n' be the processing times of 'n' jobs on Machine 2 and C_1', C_2'... C_n' be the processing times of 'n' jobs on Machine 3. There is no standard procedure to obtain an optimal sequence for processing 'n' jobs on 3 Machines. So, we have to convert this type of problems into a two machine problem by satisfying any one or both of the following conditions.

1. Min $(A_i') \geq$ Max (B_i'), for i= 1, 2 .... n
2. Min $(C_i') \geq$ Max (B_i'), for i= 1, 2 .... n

We use ranking function to determine the minimum or maximum processing time. If one of the above conditions is satisfied, we introduce two new machines G and H such that the processing times on G and H are given by

G $=A_i' + B_i'$, for i= 1, 2... n

$H = B_i' + C_i'$, for i= 1, 2… n
Now we can proceed to determine the optimal sequence using 3.1.1.
After obtaining an optimal sequence, the minimum total elapsed time and also the idle time on machines 1, 2 and 3 are calculated as follows:

Minimum Total elapsed time = Time out of the last job on machine 3.
Idle time for machine 1 = Total elapsed time - time when the last job is out of machine 1
Idle time for machine 2 = (Total elapsed time - time when the last job is out of machine 2)
+ Time at which the first job in a sequence finishes on machine 1

$$+\sum_{i=2}^{n}\left\{\begin{array}{c}(\text{time when the } i^{th} \text{ job starts on machine } 2)\\ -(\text{time when the } (i-1)^{th} \text{ job finishes on machine } 2)\end{array}\right\}$$

Idle time for machine 3 = Time at which the first job in a sequence finishes on machine 2

$$+\sum_{i=2}^{n}\left\{\begin{array}{c}(\text{time when the } i^{th} \text{ job starts on machine } 3)\\ -(\text{time when the } (i-1)^{th} \text{ job finishes on machine } 3)\end{array}\right\}$$

3.1.3. Processing 'n' jobs on 'm' machines
Let there be 'n' jobs which are to be processed through 'm' machines M_1, M_2 …M_m in the order M_1,M_2…M_m and T_{ik} be the time taken by the i^{th} job on k^{th} machine.

Procedure
Step 1: Use ranking function to identify Min T_{i1} (Minimum time for the first machine), Min T_{im}(Minimum time on the last machine) and Max (T_{ik}) for k=2, 3… m-1 and i=1, 2 …n (Maximum time on intermediate machines).

Step 2: Check the following conditions:
(i) Min(T_{i1})$\geq$ Max(T_{ik})
(ii) Min(T_{im})$\geq$ Max(T_{ik})

Step 3: If the conditions in step 2 are not satisfied, the problem cannot be solved by this method, hence go to next step.

Step 5: Convert the 'n' job 'm' machine problem into 'n' job two machine problem by considering two machines P and Q such that
$P_{ij} = T_{i1}+T_{i2}+ \dots + T_{i(m-1)}$
$Q_{ij} = T_{i2}+T_{i3}+ \dots +T_{im}$

Step 6: Now we can proceed to determine the optimal sequence using 3.1.1. After obtaining an optimal sequence, the minimum total elapsed time and also the idle time on machines can be determined.

4. NUMERICAL EXAMPLES

4.1. There are five jobs, each of which is to be processed through two machines M_1, M_2 in the order M_1M_2. Processing time (in hours) is given below.

Jobs	Machine (M_1)	Machine (M_2)
A	**(2, 3, 4 ; 1,3,5)**	**(2, 4, 6 ; 1, 4, 7)**
B	**(6, 8, 10 ; 4, 8, 12)**	**(5, 10, 15 ; 3, 10, 17)**
C	**(3, 5, 7 ; 1, 5, 9)**	**(3, 6, 9 ; 2, 6, 10)**
D	**(4, 7, 10 ; 3, 7, 11)**	**(3, 5, 7 ; 1 ,5, 9)**
E	**(2, 4, 6 ; 1, 4, 7)**	**(6, 8, 10 ; 4, 8, 12)**

Obtain the optimal sequence and also determine the minimum total elapsed time and idle time for each of the machine.

Solution:

Jobs	A	B	C	D	E
Order of Cancellation	(1)	(5)	(3)	(4)	(2)

Optimal Sequence: A-E-C-B-D

To find the total elapsed time:

Jobs	Machine(M_1)	
	Time in	**Time out**
A	(0, 0, 0; 0, 0, 0)	(2, 3, 4; 1, 3, 5)
E	(2, 3, 4; 1, 3, 5)	(4, 7, 10; 2, 7, 12)
C	(4, 7, 10; 2, 7, 12)	(7, 12, 17; 3, 12, 21)
B	(7, 12, 17; 3,12, 21)	(13, 20, 27; 7, 20, 33)
D	(13, 20, 27; 7, 20, 33)	(17, 27, 37; 10, 27, 44)

Jobs	Machine(M_2)	
	Time in	**Time in**
A	(2, 3, 4; 1, 3, 5)	(4, 7, 10; 2, 7, 12)
E	(4, 7, 10; 2, 7, 12)	(10, 15, 20; 6, 15, 24)
C	(10, 15, 20; 6, 15, 24)	(13, 21, 29; 8, 21, 34)
B	(13, 21, 29; 8, 21, 34)	(18, 31, 44; 11, 31, 51)
D	(18, 31, 44; 11, 31, 51)	(21, 36, 51; 12, 36, 60)

Jobs	Idle Time (M_1)	Idle Time (M_2)
A	-	(2, 3, 4; 1, 3, 5)
E	-	(-6, 0, 6; -10, 0, 10)
C	-	(-10, 0, 10; -18, 0, 18)
B	--	(-16, 0, 16; -26, 0, 26)
D		(-26, 0, 26; -40, 0, 40)
	(-16, 9, 34; -32, 9, 50)	-
Total	(-16, 9, 34; -32, 9, 50)	(-56, 3, 62; -93, 3, 99)

Minimum total elapsed time = (21, 36, 51; 12, 36, 60) hours
Idle time for Machine M_1 = (-16, 9, 34; -32, 9, 50) hours
Idle time for Machine M_2 = (-56, 3, 62; -93, 3, 99) hours

4.2. There are seven jobs, each of which is to be processed through three machines M_1, M_2 and M_3 in the order $M_1M_2M_3$. Processing time (in hours) is given below.

Jobs	**Machine (M_1)**	**Machine (M_2)**	**Machine (M_3)**
A	**(2, 3, 4; 1, 3, 5)**	**(2, 4, 6; 1, 4, 7)**	**(3, 6, 9; 1, 6, 11)**
B	**(7, 8, 9; 6, 8, 10)**	**(2, 3, 4; 1, 3, 5)**	**(4, 7, 10; 2, 7, 12)**
C	**(4, 7, 10; 2, 7, 12)**	**(1, 2, 3; -1, 2, 5)**	**(3, 5, 7; 1, 5, 9)**
D	**(2, 4, 6; 1, 4, 7)**	**(3, 5, 7; 1, 5, 9)**	**(7, 11, 15; 5, 11, 17)**
E	**(8, 9, 10; 7, 9, 11)**	**(-1, 1, 3; -2, 1, 4)**	**(3, 5, 7; 1, 5, 9)**
F	**(7,8,9;6,8,10)**	**(2,4,6;1,4,7)**	**(3,6,9;1,6,11)**
G	**(4,7,10;2,7,12)**	**(2,3,4;1,3,5)**	**(10,12,14;9,12,15)**

Obtain the optimal sequence and also determine the minimum total elapsed time and idle time for each of the machine.

Solution:

Since the problem is a three machine problem, we convert this into a two machine problem.
For that, it has to satisfy any one or both of the following conditions

i. Min(M_1) ≥ Max(M_2)

ii. Min(M_3) ≥ Max(M_2)

Here Min (M_1) = (2, 3, 4; 1, 3, 5) and Max (M_2) = (3, 5, 7; 1, 5, 9) = Min (M_3).

(i.e.,) i. Min (M_1) $\not\geq$ Max (M_2)

ii. Min (M_3) = Max (M_2)

Therefore, the second condition is satisfied. We convert the problem into a two machine problem as H and K. The processing time of the twomachines H and K for 7 jobs are as follows:

H= M_1+ M_2 and K= M_2+ M_3

Job	**H**	**K**	**Order of Cancellation**
A	(4, 7, 10; 2, 7, 12)	(5, 10, 15; 2, 10, 18)	(2)
B	(9, 11, 13; 7, 11, 15)	(6, 10, 14; 3, 10, 17)	(6)
C	(5, 9, 13; 1, 9, 17)	(4, 7, 10; 0, 7, 14)	(3)
D	(5, 9, 13; 2, 9, 16)	(10, 16 ,22; 6, 16, 26)	(4)
E	(7, 10, 13; 5, 10, 15)	(2, 6, 10; 1, 6, 13)	(1)
F	(9, 12, 15; 7, 12, 17)	(5, 10, 15; 2, 10, 18)	(7)
G	(6, 10, 14; 3, 10, 17)	(12, 15, 18; 10, 15, 20)	(5)

Optimal sequence: A-D-G-F-B-C-E

Job	Machine (M_1)		
	Time in	Time out	Idle time
A	(0, 0, 0; 0, 0, 0)	(2, 3, 4; 1, 3, 5)	-
D	(2, 3, 4; 1, 3, 5)	(4, 7, 10; 2, 7, 12)	-
G	(4, 7, 10; 2, 7, 12)	(8, 14, 20; 4, 14, 24)	-
F	(8, 14, 20; 4, 14, 24)	(15, 22, 29; 10, 22, 34)	-
B	(15, 22, 29; 10, 22, 34)	(22, 30, 38; 16, 30, 44)	-
C	(22, 30, 38; 16, 30, 44)	(26, 37, 48; 18, 37, 56)	-
E	(26, 37, 48; 18, 37, 56)	(34, 46, 58; 25, 46, 67)	-
			(-21, 13, 47; -45, 13, 71)
Total			(-21, 13, 47; -45, 13, 71)

Job	Machine (M_2)		
	Time in	Time out	Idle Time
A	(2, 3, 4; 1, 3, 5)	(4, 7, 10; 2, 7, 12)	(2, 3, 4; 1, 3, 5)
D	(4, 7, 10; 2, 7, 12)	(7, 12, 17 ; 3, 12, 21)	(-6, 0, 6; -10, 0, 10)
G	(8, 14, 20; 4, 14, 24)	(10, 17, 24; 5, 17, 29)	(-9, 2, 13; -17, 2, 21)
F	(15, 22, 29; 10, 22, 34)	(17, 26, 35; 11, 26, 41)	(-9, 5, 19; -19, 5, 29)
B	(22, 30, 38; 16, 30, 44)	(24, 33, 42; 17, 33, 49)	(-13, 4, 21; -25, 4, 33)
C	(26, 37, 48; 18, 37, 56)	(27, 39, 51; 17, 39, 61)	(-16, 4, 24;- 31, 4, 39)
E	(34, 46, 58; 25, 46, 67)	(33, 47, 61; 23, 47, 71)	(-17, 7, 31;- 36, 7, 50)
			(-24, 12, 48; -49, 12, 73)
Total			(-92, 37, 166; -186, 37, 260)

Job	Machine (M_3)		
	Time in	Time out	Idle Time
A	(4, 7, 10; 2, 7, 12)	(7, 13, 19; 3, 13, 23)	(4, 7, 10; 2, 7, 12)
D	(7, 13, 19; 3, 13, 23)	(14, 24, 34; 8, 24, 40)	(-12, 0, 12; -20, 0, 20)
G	(14, 24, 34; 8, 24, 40)	(24, 36, 48; 17, 36, 55)	(-20, 0, 20; -32, 0, 32)
F	(24, 36, 48; 17, 36, 55)	(27, 42, 57; 18, 42, 66)	(-24, 0, 24; -38, 0, 38)
B	(27, 42, 57; 18, 42, 66)	(31, 49, 67; 20, 49, 78)	(-30, 0, 30; -48, 0, 48)
C	(31, 49, 67; 20, 49, 78)	(34, 54, 74; 21, 54, 87)	(-36, 0, 36; -58, 0, 58)
E	(34, 54, 74; 21, 54, 87)	(37, 59, 81; 22, 59, 96)	(-40, 0, 40; -66, 0, 66)
Total			(-158, 7, 172; -260, 7, 274)

Minimum total elapsed time = (37, 59, 81; 22, 59, 96) hours
Idle time for Machine M_1 = (-21, 13, 47; -45, 13, 71) hours
Idle time for Machine M_2 = (-92, 37, 166; -186, 37, 260) hours
Idle time for Machine M_3 = (-158, 7, 172; -260, 7, 274) hours

4.3. Obtain an optimal sequence for the following sequencing problem of four jobs and four machines when passing is not allowed, of which processing time (in hours) are given below. Also calculate the minimum total elapsed time and idle time for each of the machines.

Job	Machine (M_1)	Machine (M_2)	Machine (M_3)	Machine (M_4)
A	(10,13,16;9,13,17)	(3,8,13;2,8,14)	(6,7,8;5,7,9)	(12,14,16;10,14,18)
B	(11,12,13;10,12,14)	(3,6,9;1,6,11)	(3,8,13;2,8,14)	(17,19,21;15,19,23)
C	(5,9,13;4,9,14)	(6,7,8;5,7,9)	(3,8,13;2,8,14)	(14,15,16;12,15,18)
D	(3,8,13;2,8,14)	(4,5,6;3,5,7)	(3,6,9;1,6,11)	(14,15,16;12,15,18)

Solution:
To find an Optimal Sequence, we convert the 4 Machine problem into a 2 Machine problem. For that, it has to satisfy any one of the following conditions.

i) Min (M_1) ≥ Max (M_2 , M_3)
ii) Min (M_4) ≥ Max (M_2 , M_3)

Here Min (M_1) =(3, 8, 13; 2, 8, 14) and Min (M_4) =(12, 14, 16; 10, 14, 18)
Max (M_2) = (3, 8, 13; 2, 8, 14) = Max (M_3)
Max (M_2, M_3) = (3, 8, 13; 2, 8, 14)
i.e.,

i) Min (M_1) = Max (M_2, M_3)
ii) Min (M_4) ≥ Max (M_2, M_3)

Here both the conditions are satisfied. Now, we convert this problem into a 2 machine problem as G and H. The processing time of machines G and H are obtained as follows:
G = M_1+ M_2+ M_3
H= M_2+ M_3+ M_4

Jobs	G	H	Order of cancellation
A	(19, 28, 37; 16, 28, 40)	(21, 29, 37; 17, 29, 41)	(4)
B	(17, 26, 35; 13, 26, 39)	(23, 33, 43; 18, 33, 48)	(3)
C	(14, 24, 34; 11, 24, 37)	(23, 30, 37; 19, 30, 41)	(2)

D	(10, 19, 28; 6, 19, 32)	(21, 26, 31; 16, 26, 36)	(1)

Optimal Sequence: D-C-B-A

To find total elapsed time:

Job	Machine (M_1)		
	Time in	Time out	Idle time
D	(0, 0, 0; 0, 0, 0)	(3, 8, 13; 2, 8, 14)	-
C	(3, 8, 13; 2, 8, 14)	(8, 17, 26; 6, 17, 28)	-
B	(8, 17, 26; 6, 17, 28)	(19, 29, 39; 16, 29, 42)	-
A	(19, 29, 39; 16, 29, 42)	(29, 42, 55; 25, 42, 59)	-
			-
Total			(12, 40, 68; -4, 40, 84)

Job	Machine (M_2)		
	Time in	Time out	Idle time
D	(3, 8, 13; 2, 8, 14)	(7, 13, 19; 5, 13, 21)	(3, 8, 13; 2, 8, 14)
C	(8, 17, 26; 6, 17, 28)	(14, 24, 34; 11, 24, 37)	(-11, 4, 19; -15, 4, 23)
B	(19, 29, 39; 16, 29, 42)	(22, 35, 48; 17, 35, 53)	(-15, 5, 25; -21, 5, 31)
A	(29, 42, 55; 25, 42, 59)	(32, 50, 68; 27, 50, 73)	(-19, 7, 33; -28, 7, 42)
			(-1, 32, 65; -18, 32, 82)
Total			(-43, 56, 155; -80, 56, 192)

Job	Machine (M_3)		
	Time in	Time out	Idle time
D	(7, 13, 19; 5, 13, 21)	(10, 19, 28; 6, 19, 32)	(7, 13, 19; 5, 13, 21)
C	(14, 24, 34; 11, 24, 37)	(17, 32, 47; 13, 32, 51)	(-14, 5, 24; -21, 5, 31)
B	(22, 35, 48; 17, 35, 53)	(25, 43, 61; 19, 43, 67)	(-25, 3, 31; -34, 3, 40)
A	(32, 50, 68; 27, 50, 73)	(38, 57, 76; 32, 57, 82)	(-29, 7, 43; -40, 7, 54)
			(-9, 25, 59; -27, 25, 77)
Total			(-70, 53, 176; -117, 53, 223)

Job	Machine (M_4)		
	Time in	Time out	Idle time
D	(10, 19, 28; 6, 19, 32)	(24, 34, 44; 18, 34, 50)	(10, 19, 28; 6, 19, 32)
C	(24, 34, 44; 18, 34, 50)	(38, 49, 60; 30, 49, 68)	(-20, 0, 20; -32, 0, 32)
B	(38, 49, 60; 30, 49, 68)	(55, 68, 81; 45, 68, 91)	(-22, 0, 22; -38 ,0, 38)
A	(55, 68, 81; 45, 68, 91)	(67, 82, 97; 55, 82, 109)	(-26 ,0, 26; -46, 0, 46)
Total			(-58, 19, 96; -110, 19, 148)

Minimum Total elapsed time = (67, 82, 97; 55, 82, 109) hours

Idle time for machine M_1 = (12, 40, 68; -4, 40, 84) hours

Idle time for machine M_2 = (-43, 56, 155; -80, 56, 192) hours

Idle time for machine M_3 = (-70, 53, 176; -117, 53, 223) hours

Idle time for machine M_4 = (-58, 19, 96; -110, 19, 148) hours

5. CONCLUSION

In this paper, we have solved different types of fuzzy sequencing problem using triangular intuitionistic fuzzy numbers. With the help of the proposed algorithm we obtained an optimal sequence and total elapsed time to process all jobs through machines, without converting the problem into a classical sequencing problem. The concept of fuzzy sequencing problem provides an efficient framework in solving the real-life problems.

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