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**RESEARCH ARTICLE**

# ON SOMBOR INDEX OF m-SHADOW GRAPH OF SOME GRAPHS

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## ABSTRACT

Sombor index ($SO$) is a new vertex-degree-based topological index proposed by I. Gutman which is defined as, $\sum_{x_i x_j \in E(G)} \sqrt{\left(d_G(x_i)\right)^2 + \left(d_G(x_j)\right)^2}$ where $d_G(x_i)$ denotes the degree of $x_i{}^{th}$ vertex of G and $i \neq j$. This new topological invariant is applied in the areas of chemical graph theory. In this paper we have obtained the Sombor index of $m$−shadow graphs of path graph $D_m(P_n)$, cycle graph $D_m(C_n)$, ladder graph $D_m(L_n)$ and tadpole graph $D_m(T_{n,p})$. We have also concluded that, the Sombor index of m-shadow graph of any regular or finite connected graph $G$ is $SO(G)m^3$.

**Mathematics subject classification** : 05C09.

**Keywords:** Sombor index, $m$-Shadow graph, Ladder graph, Tadpole graph.

## 1. INTRODUCTION

All the graphs used in this paper are simple connected and undirected. Let $G = (V, E)$ be such a simple graph which is finite and $V(G) = \{x_1, x_2, \cdots, x_n\}$ denotes the vertex set with $n$ vertices and $E(G) = e_{ij}(G)$ denotes the edge set where $e_{ij}$ is an edge with $i$ and $j$ as endpoints. Here $d_G(x_j)$ refers to the degree of the $j^{th}$ vertex in G and the minimum, maximum degrees of any vertex $x_j \in V(G)$ are $\delta_G(x_j)$ and $\Delta_G(x_j)$ respectively for $j = 1, 2, \cdots, n$. The number which is invariant under graph automorphisms is known as graphical invariant and it is often referred to the structural invariant of a graph. The word topological index *(TI)* generally refers the graphical invariant related to molecular graph theory. There are various vertex-degree-based graphical invariants introduced and studied extensively 26 which is usually referred as "topological indices", whose general formula is,

$$TI = \sum_{x_i x_j \in E(G)} \Phi\left(d_G(x_i), d_G(x_j)\right)$$

where $\Phi(a, b)$ is a function with a commutative property, i.e, $\Phi(a, b) = \Phi(b, a)$. The ordered pair $(a, b)$, where $a = d_G(x_i)$ and $b = d_G(x_j)$, is the degree-coordinate (or d-coordinate) of the edge $e_{ij} = x_i x_j \in E(G)$. For example, $\Phi\left(d_G(x_i), d_G(x_j)\right) = d_G(x_i) + d_G(x_j)$ and $d_G(x_i) d_G(x_j)$ as in first and second Zagreb indices respectively or $\sqrt{d_G(x_i)^2 + d_G(x_j)^2}$ as in Sombor index [3], a novel degree-based index introduced by I. Gutman in 2021.



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The Sombor index was inspired from the geometric interpretation of degree-radii of the edges and for a graph G, the Sombor index SO(G) is defined as,

$$SO(G) = \sum_{x_i x_j \in E(G)} \sqrt{d_G(x_i)^2 + d_G(x_j)^2}$$

also I. Gutman has established several mathematical properties related to Sombor index in 34. This new topological index has gained attraction among researchers of chemical graph theory 1579.

## 2. PRELIMINARIES

A graph G is an ordered pair $(V(G), E(G))$, where $V(G)$ denotes the vertex set and $E(G)$ denotes the edge set. If G has the same end vertices, it is called a *loop* and an undirected, loopless graph is said to be a *simple graph*. A graph G is *finite* if its order and size are finite. In a graph G, the *minimum degree* $\delta(G)$ is the minimum number of edges that are incident from any vertex $v \in V$(G) and *maximum degree* $\Delta(G)$ is the maximum number of edges that are incident from any vertex $v \in V(G)$.

**Definition 2.1:** The Shadow graph $D_2(G)$ of a simple connected graph G is obtained by taking two copies of G, i.e., $G'$ and $G''$ and joining each vertex $u' \in G'$ to the neighbors of the corresponding vertex $u'' \in G''$.

**Definition 2.2:** A *m*-Shadow graph of G denoted by $D_m(G)$ is a graph obtained by taking *m*-copies of G, i.e., $G', G'', \dots, G^{(m)}$ and then joining each vertex $u^i \in G^i$, I $\in [1, \mathrm{m}-1]$ to all the neighbors of the corresponding vertex $v^j \in G^{i+1}, G^{i+2}, \dots, G^{(m)}, i < j \leq m.$ 8

**Definition 2.3:** The path between any two vertices *u* and *v* is a sequence of ordered adjacent for $q_0 = u, q_1, q_2, q_3, \dots, q_{k-1} = v$ and $q_k = w$, where $u, v, w \in V(G)$ and are distinct.

**Definition 2.4:** The Ladder graph is a planar undirected graph which is the Cartesian product of two path graphs with only one edge and is denoted by $L_n = P_n \times P_2$.

**Definition 2.5:** The Tadpole graph is a special type of graph consisting of a cycle $C_n$ of atleast $n \geq 3$ vertices and a path $P_p$ with *p* vertices connected at a common vertex. It is denoted by $T_{n,p}$.

Some of the important implications are given below:

**Preposition 2.1:** Let $K_n$ be the complete graph of order *n*, and $\overline{K_n}$, a null graph, be its complement. Then for any graph G of order *n*, $SO(\overline{K_n}) \leq SO(G_n) \leq SO(K_n)$. Equality holds if and only if $G \cong \overline{K_n}$ or $G \cong K_n$. Recall that, SO($\overline{K_n}$) = 0 and $SO(K_n) = \frac{n(n-1)^2}{\sqrt{2}}$.

**Preposition 2.2:** For a path graph $P_n$, the Sombor index is given by,

$$SO(P_n) = \begin{cases} \sqrt{2} & for\ n = 2 \\ 2\sqrt{5} + 2(n-3)\sqrt{2} & for\ n \geq 3 \end{cases}$$

and $SO(P_n) \leq SO(G) \leq SO(K_n)$.

## 3. SOMBOR INDEX (SO) OF SOME GRAPHS

**Theorem 3.1.** Let $P_n$ be a path graph of order *n*, then the Sombor index of *m*-shadow graph of path graph $P_n$ is,

$$SO(D_m(P_n)) = \begin{cases} SO(P_{n=2})m^3 & for\ n = 2 \\ SO(P_n)m^3 & for\ n \geq 3 \end{cases}$$

*Proof.* Let $V[D_m(P_n)] = \{v_j', v_j'', \dots, v_j^m : j \in [1,n]\}$ be the vertex set, where $v_j', v_j'', \dots, v_j^m$ be the vertices of m-copies of path graph $P_n$. The vertex $v_j'$ are adjacent to $v_l'', v_l''', \dots, v_l^m$ only where $v_j'', v_j''', \dots, v_j^m$ are adjacent.

Since $D_m(P_n)$ are constructed from *m*-copies of $P_n$, the total number of vertices and edges are given by, $V|D_m(P_n)| = mn$ and $E|D_m(P_n)| = m^2(n-1)$ respectively. The minimum degree $\delta(D_m(P_n)) = m, \forall n$ and the maximum degree is $\Delta(D_m(P_n)) = m$ for $n = 2$ and $\Delta(D_m(P_n)) = 2m$ for $n \geq 3$.

By I. Gutman, $SO(G) = \sum_{v_j v_l \in E(G)} \sqrt{(d_G(v_j))^2 + (d_G(v_l))^2}$

Therefore for $n = 2$,

$$SO(D_m(P_{n=2})) = E|D_m(P_2)|\sqrt{(\delta_{D_m(P_2)}(v_j))^2 + (\Delta_{D_m(P_2)}(v_l))^2}$$

$$= m^2(2-1)\sqrt{m^2 + m^2}$$

$$= m^3\sqrt{2}$$

$$= SO(P_{n=2})m^3$$

For $n \geq 3$, there are $2m^2$ pair of vertices with both minimum and maximum degree and $(n-3)m^2$ pair of vertices with maximum degree. Therefore,

$$SO(D_m(P_n))$$

$$= 2m^2\sqrt{(\delta_{D_m(P_n)}(v_j))^2 + (\Delta_{D_m(P_n)}(v_l))^2} + (n\text{-}3)m^3\sqrt{(\Delta_{D_m(P_n)}(v_j))^2 + (\Delta_{D_m(P_n)}(v_l))^2}$$

$$= 2m^2\sqrt{m^2 + 4m^2} + (n-3)m^3\sqrt{m^2 + 4m^2}$$

$$= 2m^2\sqrt{5} + 2(n-3)m^3\sqrt{2}$$

$$= SO(P_n)m^3$$

**Theorem 3.2.** For a complete graph $K_n$ with *n*-vertices, the Sombor index of *m*-shadow graph of $K_n$ is $\mathrm{SO}(\mathrm{D_m}(\mathrm{K_n})) = \mathrm{SO}(\mathrm{K_n})m^3$ where $\mathrm{SO}(\mathrm{K_n}) = \frac{n(n-1)^3}{\sqrt{2}}$

Using Preposition2.2, the above theorems yields the following property, for any graph G with $n$-vertices, the Sombor index of $m$-shadow graph of G is,

$$SO(D_m(P_n)) \leq SO(D_m(G_n)) \leq SO(D_m(K_n))$$

**Lemma 3.3.** For a cycle $C_n$ the Sombor index is $SO(C_n) = 2n\sqrt{2}$

*Proof.* The order and size of $C_n$ are given by, $V|(C_n)| = E|(D_m(C_n)| = n$. Since cycle graph is a *2-regular* graph, the minimum and maximum degree are $\delta_{C_n} = \Delta_{C_n} = 2$. The Sombor index

of $C_n$ is,

$$SO(C_n) = E|(C_n)|\sqrt{(\delta_{C_n}(v_j))^2 + (\Delta_{C_n}(v_l))^2}$$
$$= n\sqrt{4+4}$$
$$= 2n\sqrt{2}$$

From Lemma 3.3. we can say that, for a $k$-regular graph of order n, the Sombor index is

$$SO(G_n^k) = \frac{k^2 n}{\sqrt{2}}.$$

**Theorem 3.4.** The $C_n$ be a cycle graph of order $n$, then the Sombor index of m-shadow graph of cycle $C_n$ is $SO(D_m(C_n)) = SO(C_n)m^3$

*Proof.* Let $V[D_m(C_n)] = \{v_j', v_j'', \ldots, v_j^m : j \in [1, n]$ be the vertex set, where $v_j', v_j'', \ldots, v_j^m$ be the vertices of m-copies of cycle graph $C_n$. The vertex $v_j'$ are adjacent to $v_l'', v_l''', \ldots, v_l^m$ only where $v_j'', v_j''', \ldots, v_j^m$ are adjacent.

Since $D_m(C_n)$ are constructed from $m$-copies of $C_n$, the order and size are given by, $V|D_m(C_n)| = mn$ and $E|D_m(C_n)| = m^2 n$ respectively. All cycle graph is a 2-*regular* graph, hence the minimum and maximum degree are $\delta_{D_m(C_n)} = \Delta_{D_m(C_n)} = 2m, \forall n.$

The Sombor index of $D_m(C_n) is,$

$$SO(D_m(C_n)) = E|D_m(C_n)|\sqrt{(\delta_{D_m(C_n)}(v_j))^2 + (\Delta_{D_m(C_n)}(v_l))^2}$$
$$= m^2 n\sqrt{4m^2 + 4m^2}$$
$$= 2nm^3\sqrt{2}$$
$$= SO(C_n)m^3$$

**Lemma 3.5.** For a ladder graph $L_n$, the Sombor index is

$$SO(L_n) = \begin{cases} n(3n-2)\sqrt{2} & for\ n = 1,2 \\ (9n-20)\sqrt{2} + 4\sqrt{13} & for\ n \geq 3 \end{cases}$$

*Proof.* The order and size of ($L_n$) are given by, $V|L_n| = 2n$ and $E|L_n| = 3n-2$ respectively. The minimum and maximum degree for $n = 1,2$ are, $\delta_{L_n} = \Delta_{L_n} = n$ and for $n \geq 3$, $\delta_{L_n} = 2$ and $\Delta_{L_n} = 3$ respectively. Therefore the Sombor index of $L_n$, for $n = 1,2$ is,

$$SO(L_n) = E|L_n|\sqrt{(\delta_{L_n}(v_j))^2 + (\Delta_{L_n}(v_l))^2}$$
$$= (3n-2)\sqrt{n^2 + n^2}$$
$$= n(3n-2)\sqrt{2}$$

For $n \geq 3$, there are 2 pair of vertices with minimum degree, 4 pair of vertices with both minimum and maximum degree and $3n-8$ pair of vertices with maximum degree.

$$SO(L_n) = 2\sqrt{(\delta_{L_n}(v_j))^2 + (\Delta_{L_n}(v_l))^2} + 4\sqrt{(\delta_{L_n}(v_j))^2 + (\Delta_{L_n}(v_l))^2}$$
$$+ (3n-8)\sqrt{(\Delta_{L_n}(v_j))^2 + (\Delta_{L_n}(v_l))^2}$$

$$= 2\sqrt{4+4} + 4 + 4\sqrt{4+9} + (3n-8)\sqrt{9+9}$$
$$= 4\sqrt{2} + 4\sqrt{13} + (9n-24)\sqrt{9+9}$$
$$= (9n-20)\sqrt{2} + 4\sqrt{13}$$

**Theorem 3.6.** Let $L_n$ be a ladder graph of order *2n*, then the Sombor index of m-shadow graph of ladder graph $L_n$ is,

$$SO(D_m(L_n)) = \begin{cases} SO(L_{\{n=1,2\}})m^3 & for\ n = 1,2 \\ SO(L_{\{n\}})m^3 & for\ n \geq 3 \end{cases}$$

*Proof.* Let $V[D_m(L_n)] = \{v_j', v_j'', \ldots, v_j^m : j \in [1,2n]$ be the vertex set, where $v_j', v_j'', \ldots, v_j^m$ be the vertices of m-copies of ladder graph $L_n$. The vertex $v_j'$ are adjacent to $v_l'', v_l''', \ldots, v_l^m$ only where $v_j'', v_j''', \ldots, v_j^m$ are adjacent.

Since $D_m(L_n)$ are constructed from *m*-copies of $L_n$, the order and size are given by, $V|D_m(L_n)| = 2mn$ and $E|D_m(L_n)| = m^2(3n-2)$ respectively. The minimum and maximum degree for $n = 1,2$ are, $\delta_{D_m(L_n)} = \Delta_{D_m(L_n)} = mn$ and for $n \geq 3$, $\delta_{D_m(L_n)} = 2m$ and $\Delta_{D_m(L_n)} = 3m$ respectively. Therefore the Sombor index for $n = 1,2$ is,

$$SO(D_m(L_n)) = E|D_m(L_n)|\sqrt{(\delta_{D_m(L_n)}(v_j))^2 + (\Delta_{D_m(L_n)}(v_l))^2}$$
$$= m^2(3n-2)\sqrt{(mn)^2 + (mn)^2}$$
$$= n(3n-2)m^3\sqrt{2} = SO(L_n)m^3$$

For $n \geq 3$, there are $2m^2$ pair of vertices with minimum degree, $4m^2$ pair of vertices with both minimum and maximum degree and $(3n-8)m^2$ pair of vertices with maximum degree.

$$SO(D_m(L_n))$$
$$= 2m^2\sqrt{(\delta_{D_m(L_n)}(v_j))^2 + (\Delta_{D_m(L_n)}(v_l))^2}$$
$$+ 4m^2\sqrt{(\delta_{D_m(L_n)}(v_j))^2 + (\Delta_{D_m(L_n)}(v_l))^2}$$
$$+ (3n-8)m^2\sqrt{(\Delta_{D_m(L_n)}(v_j))^2 + (\Delta_{D_m(L_n)}(v_l))^2}$$

$$= 2m^2\sqrt{4m^2 + 4m^2} + 4m^2\sqrt{4m^2 + 9m^2}$$
$$+ (3n-8)m^2\sqrt{9m^2 + 9m^2}$$

$$= (9n-20)m^3\sqrt{2} + 4m^3\sqrt{13} = SO(L_n)m^3$$

**Lemma 3.7.** For a tadpole graph $T_{\{n,p\}}$, the Sombor index is

$$SO(T_{\{n,p\}}) = \begin{cases} 1\sqrt{10} + 2\sqrt{13} + 2(n+p-3)\sqrt{2} & for\ p = 1 \\ 1\sqrt{5} + 3\sqrt{13} + 2(n+p-4)\sqrt{2} & for\ p \geq 2 \end{cases}$$

*Proof.* The order and size of $T_{\{n,p\}}$ are given by, $V|(T_{\{n,p\}})| = E|(T_{\{n,p\}})| = n + p$ . The graph has three types of degree: minimum degree $\delta_{T_{\{n,p\}}}$, maximum degree $\Delta_{T_{\{n,p\}}}$ and $(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}})$ . The minimum and maximum degree are, $\delta_{T_{\{n,p\}}} = 1$ and $\Delta_{T_{\{n,p\}}} = 3$ respectively. Therefore, the Sombor index of $T_{\{n,p\}}$ for $p = 1$ has 1 pair of vertices with both minimum and maximum degree, 2 pair of vertices with maximum and $(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}})$ degrees and $(n + p - 3)$ pair of vertices with $(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}})$ degrees,

$$\begin{aligned} SO(T_{\{n,p=1\}}) &= 1\sqrt{\left\{\left(\delta_{T_{\{n,p\}}}(v_j)\right)^2 + \left(\Delta_{T_{\{n,p\}}}(v_l)\right)^2\right\}} \\ &+ 2\sqrt{\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_j)^2 + \left(\Delta_{T_{\{n,p\}}}(v_l)\right)^2} \\ &+ (n+p \\ &- 3)\sqrt{\left\{\left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_j)\right)^2 + \left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_l)\right)^2\right\}} \\ &= 1\sqrt{10} + 2\sqrt{13} + 2(n+p-3)\sqrt{2} \end{aligned}$$

For $p \geq 2$, there 1 pair of vertices with minimum and $\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)$ degree, $3$ pair of vertices with maximum and $\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)$ degrees and $(n + p - 4)$ pair of vertices with $\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)$ degrees. Therefore,

$$\begin{aligned} SO(T_{\{n,p\}}) = {} & 1\sqrt{\left(\delta_{T_{\{n,p\}}}(v_j)\right)^2 + \left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_l)\right)^2} \\ &+ 3\sqrt{\left\{\left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_j)\right)^2 + \left(\Delta_{T_{\{n,p\}}}(v_l)\right)^2\right\}} \\ &+ (n+p \\ &- 4)\sqrt{\left\{\left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_j)\right)^2 + \left(\left(\Delta_{T_{\{n,p\}}} - \delta_{T_{\{n,p\}}}\right)(v_l)\right)^2\right\}} \end{aligned}$$

$$= 1\sqrt{\{5\}} + 3\sqrt{\{13\}} + 2(n+p-4)\sqrt{\{2\}}$$

**Theorem 3.8.** Let $T_{\{n,p\}}$ be a tadpole graph of order *n+p*, then the Sombor index of *m*-shadow graph of tadpole graph $T_{\{n,p\}}$ is,

$$SO(D_m(T_{\{n,p\}})) = \begin{cases} SO\left(T_{\{n,p=1\}}\right)m^3 & for\ p = 1 \\ SO\left(T_{\{n,p\}}\right)m^3 & for\ p \geq 2 \end{cases}$$

*Proof.* Let $V[D_m(T_{\{n,p\}})] = \{v'_{n_j}, v''_{n_j}, \cdots, v^m_{n_j}, v'_{p_k}, v''_{p_k}, \cdots, v^m_{p_k} : j \in [1,n]$ and $k \in [1,p]$ be the vertex set, where $v'_{n_j}, v''_{n_j}, \cdots, v^m_{n_j}, v'_{p_k}, v''_{p_k}, \cdots, v^m_{p_k}$ denotes the of m-copies of tadpole graph $T_{\{n,p\}}$ with *n+p* number of vertices. Every $v^i_{n_j}, v^i_{p_k}$ $(for\ 1 \leq j \leq n\ and\ 1 \leq k \leq p)$ vertex in the $i^{th}$ copy of $T_{\{n,p\}}$ is adjacent to $v^{i+1}_{n_l}, v^{i+2}_{n_l}, \cdots, v^m_{n_l}, v^{i+1}_{p_q}, v^{i+2}_{p_q}, \cdots, v^m_{p_q}$ of all $i+1,\ i+2, \cdots, m^{th}\ copies\ of\ T_{\{n,p\}}$, wherever $v^i_{n_j}, v^i_{p_k}$ are adjacent.

Since $D_m(T_{\{n,p\}})$ is constructed from m-copies of $T_{\{n,p\}}$, the order and size are given by, $V|D_m(T_{\{n,p\}})| = m(n+p)$ and $E|D_m(T_{\{n,p\}})| = m^2(n+p)$ respectively. The minimum degree is $\delta_{D_m(T_{\{n,p\}})} = m$ and the maximum degree is $\Delta_{D_m(T_{\{n,p\}})} = 3m$. Therefore the Sombor index for $p = 1$ is,

$$SO\left(D_m\left(T_{\{n,p=1\}}\right)\right) = m^2\sqrt{\left\{\left(\delta_{D_m(T_{\{n,p\}})}(v_j)\right)^2 + \left(\Delta_{D_m(T_{\{n,p\}})}(v_l)\right)^2\right\}} + 2m^2\sqrt{\left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_j)\right)^2 + \left(\Delta_{D_m(T_{\{n,p\}})}(v_l)\right)^2} + (n+p-3)m^2\sqrt{\left\{\left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_j)\right)^2 + \left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_l)\right)^2\right\}}$$

$$= m^2\sqrt{\{m^2 + 9m^2\}} + 2m^2\sqrt{\{4m^2 + 9m^2\}} + (n+p-3)m^2\sqrt{\{4m^2 + 4m^2\}}$$

$$= m^3\sqrt{10} + 2m^3\sqrt{13} + 2(n+p-3)m^3\sqrt{2}$$
$$= SO(T_{\{n,p=1\}})m^3$$

For $p \geq 2$, there are $m^2$ pair of vertices with minimum and $\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)$ degree, $3m^2$ pair of vertices with maximum and $\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)$ degrees and $(n+p-4)m^2$ pair of vertices with $\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)$ degrees. Therefore,

$$SO\left(D_m(T_{\{n,p\}})\right) =$$
$$m^2\sqrt{\left(\delta_{D_m(T_{\{n,p\}})}(v_j)\right)^2 + \left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_l)\right)^2} +$$
$$3m^2\sqrt{\left\{\left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_j)\right)^2 + \left(\Delta_{D_m(T_{\{n,p\}})}(v_l)\right)^2\right\}} +$$
$$(n+p-4)m^2\sqrt{\left\{\left(\left(\Delta_{D_m(T_{\{n,p\}})} - \delta_{D_m(T_{\{n,p\}})}\right)(v_j)\right)^2 + \left(\left(\Delta_{D_m}(T_{\{n,p\}}) - \delta_{D_m(T_{\{n,p\}})}\right)(v_l)\right)^2}$$
$$= m^2\sqrt{\{4m^2+m^2\}} + 3m^2\sqrt{\{4m^2+9m^2\}} + (n+p-4)m^2\sqrt{\{4m^2+4m^2\}}$$
$$= m^3\sqrt{\{5\}} + 3m^3\sqrt{\{13\}} + 2(n+p-4)m^3\sqrt{\{2\}}$$
$$= SO(T_{\{n,p\}})m^3$$

**Lemma 3.9.** If $SO(G_n)$ is the Sombor index of a finite connected simple graph G, then the Sombor index of a regular graph $G_n^k$ is $SO(G_n^k) = e(G_n^k)k\sqrt{2}$

*Proof.* For a graph $G = \{V, E\}$ with vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$, and edge set $E(G) = e(G_n)$, the Sombor index can be written as,

$$SO(G_n) = e(G_n)\sqrt{\left\{\left(d_{G_n}(v_j)\right)^2 + \left(d_{G_n}(v_l)\right)^2\right\}}$$

If $G_n^k$ is a regular graph with regularity *k*, then,

$$SO(G_n^k) = e(G_n^k)\sqrt{\{k^2 + k^2\}}$$
$$= e(G_n^k)k\sqrt{2}$$

**Theorem 3.10.** The Sombor index of m−shadow graph of G is $SO\big(D_m(G_n)\big) = SO(G_n)m^3$ and if $G_n^k$ is a *k*-regular graph, then $SO\left(D_m(G_n^k)\right) = SO(G_n^k)m^3$.

*Proof.* The *m*−shadow graph of G has *m* times the order of G and $m^2$ times the size of G, hence the degree of any $v_j \in V(G)$ is also multiplied by *m*. Therefore,

$$SO\big(D_m(G_n)\big) = m^2 e(G_n)\sqrt{\left\{\left(md_{G_n}(v_j)\right)^2 + \left(md_{G_n}(v_l)\right)^2\right\}}$$
$$= m^3\, e(G_n)\sqrt{\left\{\left(d_{G_n}(v_j)\right)^2 + \left(d_{G_n}(v_l)\right)^2\right\}}$$
$$= SO(G_n)m^3$$

Similarly, The *m*-shadow graph of a regular graph $G_n^k$ has *m* times the order of $G_n^k$ and $m^2$ times the size of $G_n^k$ and it is *mk*-regular. Therefore,

$$SO\left(D_m(G_n^k)\right) = m^2\, e(G_n^k)\sqrt{\{(mk)^2 + (mk)^2\}}$$
$$= m^3 e(G_n^k)k\sqrt{2}$$
$$= SO(G_n^k)m^3$$

**4.CONCLUSION**

From the results obtained we arrive at a conclusion that, for any regular or finite connected simple graph G, the Sombor index of m-shadow graph of G is the product of Sombor index of G and cubic times of m, i.e, $SO\big(D_m(G)\big) = SO(G)m^3$.

**REFERENCES**


1. Das, K. C., evik, A. S., Cangul, I. N., & Shang, Y. On Sombor index. Symmetry,(2021). 13(1), 140.
2. De, N., Cancan, M., Alaeiyan, M., & Farahani, M. R. On some degree based topological indices of mk-graph. Journal of Discrete Mathematical Sciences and Cryptography,(2020). 23(6), 1183-1194
3. Gutman, I. Geometric approach to degree-based topological indices: Sombor indices. MATCH Commun. Math. Comput. Chem, (2021). 86(1), 11-16.
4. Liu, H., Gutman, I., You, L., & Huang, Y. (2022). Sombor index: Review of extremal results and bounds. Journal of Mathematical Chemistry, 1-28.

5. Milovanovic, I., Milovanovic, E., Ali, A., & Matejic, M. Some results on the Sombor indices of graphs. Contrib. Math,(2021). 3, 59-67.
6. Ning, W., Song, Y., & Wang, K. More on Sombor index of graphs. Mathematics, (2022). 10(3), 301.
7. Reti, T., Doslic, T., & Ali, A. On the Sombor index of graphs. Contrib. Math,(2021). 3, 11-18.
8. Singh, R., & Patekar, S. C. On the Sombor index and Sombor energy of m-splitting graph and m-shadow graph of regular graphs.(2022). arXiv preprint arXiv:2205.09480.
9. Ulker, A., Grsoy, A., & Grsoy, K. N. The energy and Sombor index of graphs. MATCH Commun. Math. Comput. Chem,(2022). 87, 51-58.

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