ON GENERALIZED GRILL CONTINUOUS FUNCTIONS

Indirani, K.¹, 2P. Sathishmohan and V. Rajendran
¹Department of Mathematics, Nirmala College for Women, Coimbatore, TN, India
²Department of Mathematics, KSG College, Coimbatore, TN, India.
*E-mail: iiscsathish@yahoo.co.in

ABSTRACT

In this paper, We introduce a new class of continuous functions namely g-G-continuous functions, g-G-irresolute and study some of their properties in topological spaces.

Keywords: g-G-continuous, g-G-irresolute

1. INTRODUCTION

In 1970, Levine first introduced the concept of generalized closed (briefly, g-closed) sets were defined and investigated. The idea of grill on a topological space was first introduced by Choquet in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many topological concepts quite effectively. In 2007, Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. The aim of this paper is to introduce g-G-continuous and g-G-irresolute and investigate the relations of g-G-continuous functions between such functions.

2. PRELIMINARIES

Throughout this paper, (X, τ) (or X) represent a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A, respectively. The power set of X will be denoted by (X).

Let G be a grill on a space X. Then a map Ψ : ϕ(X) → ϕ(X) is defined by Ψ(A) = A ∪ Φ(A), for all A ∈ ϕ(X). The map Ψ satisfies Kuratowski closure axioms. Corresponding to a grill G on a topological space (X, τ), there exists a unique topology τG on X given by τG = {U ⊆ X | Ψ(X-U) = X-U}, where for any A ⊆ X,

Ψ(A) = A ∪ Φ(A) = τG - cl(A). For any grill G on a topological space by (X, τ, G).

2.2. Definition A subset A of a topological space (X, τ) is called

1) a pre-open set (Mashhour et al., 2009) if A ⊆ cl(int(A)) and a semi-open set if cl(int(A)) ⊆ A.
2) a semi-open set (Levine, 1963) if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
3) an α-open set (Njastad, 1965) if A ⊆ cl(int(A)) and an α-closed set (Maki et al., 1993) if cl(int(cl(A))) ⊆ A.
4) a semi-preopen set (Andrzejic, 1986) if A ⊆ cl(int(cl(A))) and a semi-preclosed set (Arockiarani et al., 1999) if (int(cl(A))) ⊆ A.

2.3. Definition A subset A of a topological space (X, τ) is called

1) a generalized closed set (briefly g-closed) (Levine, 1970) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2) a semi-generalized closed set (briefly sg-closed) (Bhattacharya and Lahiri, 1987) if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
3) a generalized semi-closed set (briefly gs-closed) (Arya and Nour, 1990) if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
4) a generalized α-closed set (briefly ga-closed) (Maki et al., 1993) if acl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).

5) an α-generalized closed set (briefly α g-closed) (Maki et al., 1994) if acl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

6) a generalized semi-preclosed set (briefly gsp-closed) (Dontchev, 1995) if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

7) a generalized preclosed set (briefly gp-closed) (Maki et al., 1996) if pccl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

2.4. Definition A function f: (X, τ) → (Y, σ) is called

1) g-continuous (Balachandran et al., 1991) if f⁻¹(V) is g-closed in (X, τ) for every closed set V in (Y, σ),

2) gp-continuous (Arokiarani et al., 1999) if f⁻¹(V) is gp-closed in (X, τ) for every closed set V in (Y, σ),

3) gsp-continuous (Dontchev, 1995) if f⁻¹(V) is gsp-closed in (X, τ) for every closed set V in (Y, σ),

4) gα-continuous (Masihpour et al., 1982) if f⁻¹(V) is gα-closed in (X, τ) for every closed set V in (Y, σ),

5) gs-continuous (Sundaram et al., 1992) if f⁻¹(V) is gs-closed in (X, τ) for every closed set V in (Y, σ),

6) gαg-continuous (Masihpour et al., 1982) if f⁻¹(V) is gαg-closed in (X, τ) for every closed set V in (Y, σ).

2.4. Theorem (Arya and Nour, 1990) 1) If G₁ and G₂ are two grills on a space X with G₁ ⊆ G₂, then τ₀G₁ ⊆ τ₀G₂.

2) If G is a grill on a space X and B ⊈ G, then B is closed in (X, τ, G).

3) For any subset A of a space X and any grill G on X, Φ(A) is τ₀G-closed.

2.5. Theorem (Arya and Nour, 1990) Let (X, τ) be a topological space and G be any grill on X. Then

1) A ⊆ B (⊆ X) ⇒ Φ(A) ⊆ Φ(B);

2) A ⊆ X and A ⊈ G ⇒ Φ(A) = φ;

3) Φ(Φ(A)) ⊆ Φ(A) = cl(Φ(A)) ⊆ cl(A), for any A ⊆ X;

4) Φ(A ∪ B) = Φ(A) ∪ Φ(B) for any A, B ⊆ X;

5) A ⊆ Φ(A) ⇒ cl(A) = τ₀G - cl(A) = cl(Φ(A)) = Φ(A);

6) U ∈ τ and τ \ {φ} ⊆ G ⇒ U ⊆ Φ(U);

7) If U ∈ τ then U ∩ Φ(A) = U ∩ Φ(U ∩ A), for any A ⊆ X.

2.6. Theorem Let (X, τ) be a topological space and G be any grill on X. Then, for any A, B ⊆ X.

1) A ⊆ Ψ(A) (Arya and Nour, 1990);

2) Ψ(φ) = φ (Arya and Nour, 1990);

3) Ψ(A ∪ B) = Ψ(A) ∪ Ψ(B) (Arya and Nour, 1990);

4) Ψ(Ψ(A)) = Ψ(A) (Arya and Nour, 1990);

5) Int(A) ⊆ int(Ψ(A));

6) Int(Ψ(A ∩ B)) ⊆ int(Ψ(A));

7) Int(Ψ(A ∩ B)) ⊆ Int(Ψ(B));

8) Int(Ψ(A)) ⊆ Ψ(A);

9) A ⊆ B ⇒ Ψ(A) ⊆ Ψ(B).

3. g - G - CONTINUOUS FUNCTIONS

3.1. Definition A subset A of a topological space (X, τ, G) is called a generalized grill closed (briefly g - G - closed) set if Ψ(A) ⊆ U whenever A ⊆ U and U is open in X.

3.2. Definition A function f: (X, τ, G) → (Y, σ) is said to be g-G-continuous, if the inverse image of every open set in (Y, σ) is g-G-open in (X, τ, G).

3.3. Definition A function f: (X, τ, G) → (Y, σ, H) is said to be g-G-irresolute, if f⁻¹(A) is g-G-open in (X, τ, G) for every g-H-open set in (Y, σ, H).

3.4. Theorem Every g-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous function. Let V be any open set in (Y, σ). Then f⁻¹(V) is g-open in (X, τ, G). Since every g closed set is g - G - closed set, f⁻¹(V) is g-G-open in (X, τ, G). Therefore, f is g-G-continuous.

3.5. Example Let X = Y = {a, b, c}, τ = {φ, {a}, {b}, {a, b}, X}, σ = {φ, {a}, X}, and G = {{a}, {a, b}, X}. Define the function f: (X, τ, G) → (Y, σ, H) by f(a) = b, f(b) = a, f(c) = c. Then f is g-G-continuous but not g-continuous. Since for the g-G-open set V = {{a}} in (Y, σ), f⁻¹(V) is g-G-open but not g-closed in (X, τ, G).

3.6. Theorem Every gs-continuous function is g-G-continuous but not conversely.
Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is g-open in $(X, \tau, G)$. Since every gs closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$. Therefore $f$ is g-G-continuous.

3.7. Example Let $X = \{a,b,c\}, \tau = \{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a,c\}, X\}$ and $G = \{\{b\}, \{a,b\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then $f$ is g-G-continuous but not gs-G-continuous. Since for the g-G-open set $V = \{a\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not gs-closed in $(X, \tau, G)$.

3.8. Theorem Every sg-continuous function is g-G-continuous but not conversely.

Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is sg-open in $(X, \tau, G)$. Since every sg closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.9. Example Let $X = \{a,b,c\}, \tau = \sigma = \{\phi, \{a\}, \{a,b\}, X\}$ and $G = \{\{b\}, \{a,b\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Then $f$ is g-G-continuous but not g-G-continuous. Since for the g-G-open set $V = \{a\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not sg-closed in $(X, \tau, G)$.

3.10. Theorem Every gq-continuous function is g-G-continuous but not conversely.

Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is gq-open in $(X, \tau, G)$. Since every gq closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.11. Example Let $X = \{a,b,c\}, \tau = \{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a,b\}, X\}$ and $G = \{\{b\}, \{a,b\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then $f$ is g-G-continuous but not gq-continuous. Since for the g-G-open set $V = \{a\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not gq-closed in $(X, \tau, G)$.

3.12. Theorem Every gq-continuous function is g-G-continuous but not conversely.

Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is gq-open in $(X, \tau, G)$. Since every gq closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.13. Example Let $X = \{a,b,c\}, \tau = \sigma = \{\{b\}, X\}$ and $G = \{\{a\}, \{a,b\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = a$. Then $f$ is g-G-continuous but not gq-continuous. Since for the g-G-open set $V = \{b\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not gq-closed in $(X, \tau, G)$.

3.14. Theorem Every gp-continuous function is g-G-continuous but not conversely.

Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is gp-open in $(X, \tau, G)$. Since every gp-closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.15. Example Let $X = \{a,b,c\}, \tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$, $\sigma = \{\phi, \{a\}, \{a,b\}, X\}$ and $G = \{\{a\}, \{a,c\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then $f$ is g-G-continuous but not gp-continuous. Since for the g-G-open set $V = \{a\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not gp-closed in $(X, \tau, G)$.

3.16. Theorem Every gsp-continuous function is g-G-continuous but not conversely.

Proof. Let $f: (X, \tau, G) \to (Y, \sigma)$ be an g-continuous. Let $V$ be any open set in $(Y, \sigma)$. Then $f^{-1}(V)$ is gsp-open in $(X, \tau, G)$. Since every gsp-closed set is $g - G$ closed set, $f^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.17. Example Let $X = \{a,b,c\}, \tau = \{\phi, \{a\}, \{b\}, \{a,c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a,b\}, X\}$ and $G = \{\{a\}, \{a,b\}, X\}$. Define the function $f: (X, \tau, G) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then $f$ is g-G-continuous but not gsp-continuous. Since for the g-G-open set $V = \{a\}$ in $(Y, \sigma)$, $f^{-1}(V)$ is g-G closed but not gsp-closed in $(X, \tau, G)$.

3.18. Theorem Let $f: (X, \tau, G) \to (Y, \sigma)$ be g-G-continuous and $g: (Y, \tau) \to (Z, \eta)$ is continuous then $g \circ f: (X, \tau, G) \to (Z, \eta)$ is g-G-continuous.

Proof. Let $g$ be a continuous function and $V$ be any open in $(Z, \eta)$, then $f^{-1}(V)$ is open in $(Y, \sigma)$. Since $f$ is g-G-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is g-G-open in $(X, \tau, G)$.

3.19. Theorem Let $f: (X, \tau, G) \to (Y, \sigma, H)$ and $g: (Y, \tau, H) \to (Z, \eta, L)$ are g-G-irresolute then $g \circ f: (X, \tau, G) \to (Z, \eta, L)$ is g-G-irresolute.

Proof. Let $g$ be a g-G-irresolute and $V$ be any g-G-open in $(Z, \eta, L)$, then $f^{-1}(V)$ is g-G-open in $(Y, \sigma, H)$. Since $f$ is g-G-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is g-G-irresolute in $(X, \tau, G)$. Hence $g \circ f$ is g-G-irresolute.
Remarks

\( g \alpha \cdot \text{continuous} \rightarrow ag \cdot \text{continuous} \rightarrow gp \cdot \text{continuous} \)

\( \text{continuous} \rightarrow g \cdot G \cdot \text{continuous} \)

\( gs \cdot \text{continuous} \rightarrow sg \cdot \text{continuous} \rightarrow gsp \cdot \text{continuous} \)

REFERENCES


