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ON GENERALIZED GRILL CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, We introduce a new class of continuous functions namely g-G-continuousfunctions, g-G-irresolute and study some of their properties in topological spaces.

Keywords: g-G-continuous, g-G-irresolute

1. INTRODUCTION

In 1970, Levine first introduced the concept of generalized closed (briefly, g-closed) sets were defined and investigated. The idea of grill on a topological space was first introduced by Choquet in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many topological concept quite effectively. In 2007, Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. The aim of this paper is to introduce g-G-continuous and g-G-irresolute and investigate the relations of g-G-continuous functions between such functions.

2. PRELIMINARIES

Throughout this paper, (X, τ) (or X) represent a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A, respectively. The power set of X will be denoted by $\wp(X)$. A collection G of a nonempty subsets of a space X is called a grill (Andrijevic, 1986) on X if

(1) $A \in G$ and $A \subseteq B \Rightarrow B \in G$,

(2) $A, B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

For any point x of a topological space (X, τ), $\tau(x)$ denote the collection of all open neighbourhoods of x.

We recall the following results which are useful in the sequel.

2.1 Definition (Arya and Nour, 1990). Let (X, τ) be a topological space and G be a grill on X. The mapping $\Phi : \wp(X) \rightarrow \wp(X)$, denoted by $\Phi_G(A, \tau)$ for $A \in \wp(X)$ or simply $\Phi(A)$ called the operator associated with the grill G and the topology τ and is defined by

$\Phi_G(A) = \{x \in X \mid A \cap U \in G, \forall U \in \tau(x)\}$.

Let G be a grill on a space X. Then a map $\Psi : \wp(X) \rightarrow \wp(X)$ is defined by $\Psi(A) = A \cup \Phi(A)$, for all $A \in \wp(X)$. The map Ψ satisfies Kuratowski closure axioms. Corresponding to a grill G on a topological space (X, τ), there exists a unique topology τ_G on X given by

$\tau_G = \{U \subset X \mid \Psi(X-U) = X-U\}$, where for any $A \subset X$, $\Psi(A) = A \cup \Phi(A) = \tau_G - cl(A)$. For any grill G on a topological space by (X, τ, G).

2.2. Definition A subset A of a topological space (X, τ) is called

1) a pre-open set (Mashhour *et al.,* 2009) if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.

2) a semi-open set (Levine, 1963) if $A \subseteq cl(int(A))$ and a semi-closed set if $intl(cl(A)) \subseteq A$.

3) an α-open set (Njastad, 1965) if $A \subseteq int(cl(int(A)))$ and an α - closed set (Maki *et al.,* 1993) if $cl(int(cl(A))) \subseteq A$.

4) a semi-preopen set (Andrijevic, 1986) if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set (Arokiarani *et al.,* 1999) if $(int(cl(A))) \subseteq A$.

2.3. Definition A subset A of a topological space (X, τ) is called

1) a generalized closed set (briefly g-closed) (Levine, 1970) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

2) a semi-generalized closed set (briefly sg-closed) (Bhattacharya and Lahiri, 1987) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).

3) a generalized semi-closed set (briefly gs-closed) (Arya and Nour, 1990) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

4) a generalized α-closed set (briefly gα-closed) (Maki *et al.,* 1993) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

5) an α-generalized closed set (briefly α g-closed) (Maki *et al.,* 1994) if $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

6) a generalized semi-preclosed set (briefly gsp-closed) (Dontchev, 1995) if $spcl(A) \subseteq U$ whenever A $\subseteq$ U and U is open in (X, τ).

7) a generalized preclosed set (briefly gp-closed) (Maki *et al.,* 1996) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

2.4. Definition A function f: (X, τ) $\rightarrow$ (Y, σ) iscalled

1) g-continuous (Balachandran *et al.,* 1991) if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V in (Y, σ),
2) gp-continuous (Arokiarani *et al.,* 1999) if $f^{-1}(V)$ is gp-closed in (X, τ) for every closed set V in (Y, σ),
3) gsp-continuous (Dontchev, 1995) if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V in (Y,σ),
4) gα-continuous (Mashhour *et al.,* 1982) if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V in (Y, σ),
5) gs-continuous (Sundaram *et al.,* 1992) if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V in (Y,σ),
6) αg-continuous (Mashhour *et al.,* 1982) if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V in (Y, σ),

2.4. Theorem. (Arya and Nour, 1990) 1) If G_1 and G_2 are two grills on a space X with $G_1 \subset G_2$, then $\tau_{G1} \subset \tau_{G2}$.

2) If G is a grill on a space X and $B \notin G$, then B is closed in (X, τ, G).

3) For any sunset A of a space X and any grill G on X, $\Phi(A)$ is τ_G-closed.

2.5. Theorem (Arya and Nour, 1990) Let (X, τ) be a topological space and G be any grill on X. Then

1) $A \subseteq B$ ($\subseteq X$) $\Rightarrow \Phi(A) \subseteq \Phi(B)$;

2) $A \subseteq X$ and $A \notin G \Rightarrow \Phi(A) = \phi$;

3) $\Phi(\Phi(A)) \subseteq \Phi(A) = cl(\Phi(A)) \subseteq cl(A)$, for any $A \subseteq X$;

4) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$ for any A, $B \subseteq X$;

5) $A \subseteq \Phi(A) \Rightarrow cl(A) = \tau_G - cl(A) = cl(\Phi(A)) = \Phi(A)$;

6) $U \in \tau$ and $\tau \setminus \{\phi\} \subseteq G \Rightarrow U \subseteq \Phi$ (U);

7) If $U \in \tau$ then $U \cap \Phi(A) = U \cap \Phi(U\cap A)$, for any $A \subseteq X$.

2.6. Theorem Let (X,τ) be a topological space and G be any grill on X. Then, for any A, $B \subseteq X$.

1) $A \subseteq \Psi(A)$ (Arya and Nour, 1990);

2) $\Psi(\phi) = \phi$ (Arya and Nour, 1990);

3) $\Psi(A\cup B) = \Psi(A) \cup \Psi(B)$ (Arya and Nour, 1990);

4) $\Psi(\Psi(A)) = \Psi(A)$ (Arya and Nour, 1990);

5) Int (A) $\subset$ int(Ψ(A));

6) Int($\Psi(A \cap B)$) $\subset$ Int(Ψ(A));

7) Int($\Psi(A \cap B)$) $\subset$ Int(Ψ(B));

8) Int (Ψ(A)) $\subset \Psi$(A);

9) $A \subseteq B \Rightarrow \Psi(A) \subseteq \Psi(B)$.

3. g - G – CONTINUOUS FUNCTIONS

3.1. Definition A subset A of a topological space (X, τ, G) is called a generalized grill closed

(briefly g - G - closed) set if Ψ (A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X.

3.2. Definition A function f: (X, τ, G) $\rightarrow$ (Y, σ) is said to be g-G-continuous, if the inverse

image of every open set in (Y, σ) is g-G-open in (X, τ, G).

3.3. Definition A function f: (X, τ, G) $\rightarrow$ (Y, σ, H) is said to be g-G-irresolute, if $f^{-1}(A)$ is

g-G-open in (X, τ, G) for every g-H-open set in (Y, σ, H).

3.4. Theorem Every g-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) $\rightarrow$ (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is g-open in (X, τ, G). Since every g closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.5. Example Let X =Y= {a,b,c} , τ = {ϕ, {a}, {b}, {a.b}, X}, σ ={ϕ, {a}, X} and G = {{a},

{a,b}, X}. Define the function f: (X, τ, G) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = a, f(c) = c. Then f is g-G-continuous but not g-continuous. Since for the g-G-open set V = {a} in (Y, σ), $f^{-1}(V)$ is g-G-closed but not g-closed in (X, τ, G).

3.6. Theorem Every gs-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is gs-open in (X, τ, G). Since every gs closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.7. Example Let X = {a,b,c}, τ = {ϕ, {a}, {a,b}, {a.c}, X} , σ ={ϕ, {a}, {a,c} X} and G =

{{b}, {a,b},X} Define the function f: (X, τ, G) → (Y, σ) by f(a) = a, f(b) = c, f(c) = b. Then f is g-G-continuous but not gs-continuous. Since for the g-G-open set V = {a} in (Y, σ), $f^{-1}(V)$ is g-G closed but not gs-closed in (X, τ, G).

3.8. Theorem Every sg-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is sg-open in (X, τ, G). Since every sg closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.9. Example Let X = {a,b,c}, τ = σ = {ϕ, {a}, {a,b}, X} and G = { {b},{b,c}, X}. Define the

function f: (X, τ, G) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c. Then f is g-G-continuous but not g-continuous. Since for the g-G-open set V = {a} in (Y, σ), $f^{-1}(V)$ is g-G closed but not sg-closed in (X, τ, G).

3.10. Theorem Every αg-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is αg -open in (X, τ, G). Since every αg-closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.11. Example Let X = {a,b,c}, τ = {ϕ, {a}, {a,b}, {a,c}, X}, σ ={ϕ, {b}, {b,c} X} and G =

{{b}, {a,b}, X}. Define the function f: (X, τ, G) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a. Then f is g-G-continuous but not αg -continuous. Since for the g-G-open set V = {b} in (Y, σ), $f^{-1}(V)$ is g- G closed but not αg -closed in (X, τ, G).

3.12. Theorem Every gα-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is gα -open in (X, τ, G). Since every gα-closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.13. Example Let X = {a,b,c}, τ = σ = {{b}, X} and G = {{a}, {a,b}, X}. Define the function f: (X, τ, G) → (Y, σ) by f(a) = c, f(b) = a, f(c) = a. Then f is g-G-continuous but not gα-continuous. Since for the g-G-open set V = {b} in (Y, σ), $f^{-1}(V)$ is g- G closed but not gα -closed in (X, τ, G).

3.14. Theorem Every gp-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is gp -open in (X, τ, G). Since every gp-closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.15. Example Let X = {a,b,c}, τ = {ϕ, {a}, {b}, {a,b}, X}, σ = {ϕ, {a}, {a,b}, X} and G =

{{a}, {a,c}, X}. Define the function f: (X, τ, G) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c. Then f is g-G-continuous but not gp-continuous. Since for the g-G-open set V = {a} in (Y, σ), $f^{-1}(V)$ is g- G closed but not gp -closed in (X, τ, G).

3.16. Theorem Every gsp-continuous function is g-G-continuous but not conversely.

Proof. Let f: (X, τ, G) → (Y, σ) be an g-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is gsp -open in (X, τ, G). Since every gsp-closed set is g – G – closed set, $f^{-1}(V)$ is g-G-open in (X, τ, G).Therefore is g-G-continuous.

3.17. Example Let X = {a,b,c}, τ = {ϕ, {a}, {a,b}, {a,c}, X}, σ = {ϕ, {a}, {a,b}, X} and G =

{{a}, {a,b}, X}. Define the function f: (X, τ, G) → (Y, σ) by f(a) = a, f(b) = c, f(c) = b. Then f is g-G-continuous but not gsp-continuous. Since for the g-G-open set V = {a} in (Y, σ), $f^{-1}(V)$ is g- G closed but not gsp -closed in (X, τ, G).

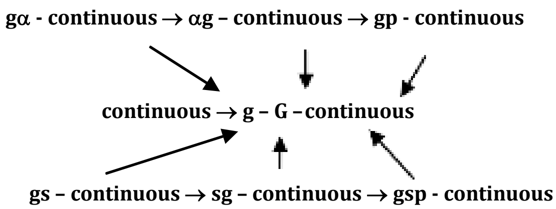
3.18. Theorem Let f: (X, τ, G) → (Y, σ) is g-G-continuous and g: (Y, τ) → (Z, η) is continuous then g o f : (X, τ, G) → (Z, η) isg-G-continuous.

Proof. Let g be a continuous function and V be any open in (Z, η), then $f^{-1}(V)$ is open in (Y, σ). Since f is g-G-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is g-G-open in (X, τ, G).Hence g o f is g-G-continuous.

3.19. Theorem Let f: (X, τ, G) → (Y, σ, H) and g: (Y, τ, H) → (Z, η, L) are g-G-irresolute then g o f : (X, τ, G) → (Z, η, L) is g-G-irresolute.

Proof. Let g be a g-G-irresolute and V be any g-L-open in (Z, η, L), then $f^{-1}(V)$ is g-G-open in (Y, σ, H). Since f is g-G-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is g-G-irresolute in (X, τ, G).Hence g o f is g-G-irresolute.

320. Remarks



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