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RESEARCH ARTICLE

SOFT $\alpha\omega\ddot{I}_s$ – NORMAL AND REGULAR SPACES IN SOFT IDEAL TOPOLOGICAL SPACES

N. Chandramathi[1], V. Kiruthika[2*] and P. Nithya[3]

[1]Assistant Professor, Department of Mathematics, Government Arts College, Udumalpet, Tamilnadu, India.
drmathimaths@gmail.com
[2*]Research Scholar, Department of Mathematics, Government Arts College,Udumalpet, Tamilnadu, India.
kiruthi.v3@gmail.com
[3]Research Scholar, Department of Mathematics, Government Arts College, Udumalpet, Tamilnadu, India.
nithyapalanisamy2599@gmail.com
* V. Kiruthika[2] E-mail : . kiruthi.v3@gmail.com

Abstract

In this paper, we introduce a soft $\alpha\omega\ddot{I}_s$ – Normal and Regular spaces in soft ideal topological spaces. Furthermore, we introduce to Enlightment and Edification of some properties, theorems and examples, which is the of the concept of soft mappings in soft ideal topological spaces.

KEYWORDS

Soft set, soft ideal topological space, Soft $\alpha\omega\ddot{I}_s$ -Closed set, Soft $\alpha\omega\ddot{I}_s$ – Hausdorff space, Soft $\alpha\omega\ddot{I}_s$ – Normal space, Soft $\alpha\omega\ddot{I}_s$ – Regular space.

Introduction

1. Introduction

The concept of soft set theory was introduced by Molodstov [5] and the concept of soft ideal theory, soft local function was introduced by A.Kandil,et.al [1]. The concepts of soft αω-closed sets was introduced by S.Jafari,et.al [12]. The concepts of soft $\alpha\omega\ddot{I}_s$- Closed sets was introduced by N.Chandramathi and V.Kiruthika [7]. The concepts of Soft Hausdorff space, Soft Normal space and Soft Regular Space was introduced by M. Shabir and M. Naz [6].

2. Preliminaries

Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U.

2.1 Definition [5]

Let D be a non-empty subset of E and a soft set over U is a parameterized family of subsets of an initial universe U. For a particular e ∈ E, F(e) may be considered the set of e-approximate elements of the soft set (F,E) and if e ∉ E, then F(e) = ∅ ,that is, $(F, \mathrm{E}) = \{f(e): e \in Đ \subseteq \mathrm{E}, f: \mathrm{E} \to P(U)\}$ is called a soft set over U. Then the family of all these soft sets denoted by $SS(U)_E$.

2.2 Definition[5]

Let Շ$_s$ be the collection of soft sets over U. Then Շ$_s$ is said to be a soft topology on U if satisfies the following axioms:

i. (∅, E), (U, E) belongs to Շ$_s$.
ii. The union of any number of soft sets in Շ$_s$ belongs to Շ$_s$.
iii. The intersection of two number of soft sets in Շ$_s$ belongs to Շ$_s$.

The triplet (U,Շ$_s$,E) is said to be soft topological space and we note that the member of Շ$_s$ are said to be Շ$_s$-soft open sets.

2.3 Definition [1]

Let Ī be a non-null collection of soft sets over an initial universe U with the same set of parameter E. Then $\bar{I}$ containing $SS(U)_E$ is called as a soft Ideal on U with same set E if,

i. $(F, E) \in \bar{I}$ and $(G, E) \in \bar{I}$ then $(F, E) \cup (G, E) \in \bar{I}$.
ii. $(F, E) \in \bar{I}$ and $(G, E) \subseteq (F, E)$ then $(G, E) \in \bar{I}$.

2.4 Definition[11]

Let (U,Շ$_s$,E) be a soft topological space and x,y ∈ U such that x≠y. Then (U,Շ$_s$,E) is called a soft Hausdorff space, if there exist soft open soft sets (F, E) and (G, E) such that x ∈ (F, E), y ∈ (G, E) and $(F, E) \cap (G, E)$ = (∅, E).

2.5 Definition [11]

Let (U,Շ$_s$,E, $\bar{I}$) be a soft ideal topological space and x,y ∈ U such that x≠y. Then (U,Շ$_s$,E, $\bar{I}$) is called a soft - $\bar{I}$ - Hausdorff space, if there exist soft $\bar{I}$ - open soft sets (F, E) and (G, E) such that x ∈ (F, E), y ∈ (G, E) and $(F, E) \cap (G, E)$ = (∅, E).

2.6 Definition[2]

Let (U,Ꞇ $_s$,E) be a soft topological space over U; (H,E) and (K,E) be two disjoint soft closed sets over U, if there exist soft open sets (F,E) and (G,E) such that (H,E) ⊆ (F,E), (K,E) ⊆ (G,E) and (F,E) ∩ (G,E) = (∅, E), then (U,Ꞇ $_s$,E) is called a soft normal space.

2.7 Definition[2]

Let (U,Ꞇ $_s$,E, $\bar{I}$) be a soft ideal topological space over U; (H,E) and (K,E) be two disjoint soft closed sets over U, if there exist soft $\bar{I}$- open sets (F,E) and (G,E) such that (H,E) ⊆ (F,E), (K,E) ⊆ (G,E) and (F,E) ∩ (G,E) = (∅, E), then (U,Ꞇ $_s$,E, $\bar{I}$) is called a soft $\bar{I}$ - normal space.

2.8 Definition[2]

Let (U,Ꞇ $_s$,E) be a soft topological space over U and let (H,E) be a soft closed in U and x ∈ X such that x ∉ (H,E), if there exist soft open sets (F_1,E) and (F_2,E) such that x ∈ (F_1,E), (G,E) ⊆ (F_2,E) and (F_1,E) ∩ (F_2,E) = (∅, E), then (U,Ꞇ $_s$,E) is called a soft regular space.

2.9 Definition [2]

Let (U,Ꞇ $_s$,E, $\bar{I}$) be a soft ideal topological space over U and let (H,E) be a soft closed in U and x ∈ X such that x ∉ (H,E), if there exist soft $\bar{I}$ - open sets (F_1,E) and (F_2,E) such that x ∈ (F_1,E), (G,E) ⊆ (F_2,E) and (F_1,E) ∩ (F_2,E) = (∅, E), then (U,Ꞇ $_s$,E, $\bar{I}$) is called a soft $\bar{I}$- regular space.

3. SOFT αωÏ$_s$ – HAUSDORFF SPACE IN SOFT IDEAL TOPOLOGICAL SPACES

In this section we introduce and study about a new Hausdorff space is known as soft αωÏ$_s$ – Hausdorff space in Soft ideal topological spaces.

3.1 Definition

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space and (x,Q),(y,Q) ∈ (Ṿ,Q) such that (x,Q) ≠(y,Q). Then (Ṿ,Ꞇ $_s$,Q,Ï$_s$) is called a soft αωÏ$_s$ - Hausdorff space, if there exist soft αωÏ$_s$ - open sets (F, Q) and (G, Q) such that (x,Q) ∈ (Ẋ,Q), (y,Q) ∈ (Ẏ,Q) and (Ẋ, Q) ∩ ($\dot{Y}$, Q) = ({},Q).

3.2 Example

Let Ṿ = {α, β,γ,δ },

Q = {σ },

Ꞇ $_s$ = {({},Q),(Ṿ,Q),(Ƒ$_1$,Q),(Ƒ$_2$,Q),(Ƒ$_3$,Q)}

Ï$_s$ = {({},Q),(Ƒ$_4$,Q), (Ƒ$_5$,Q), (Ƒ$_6$,Q)} where

(Ƒ$_1$,Q) = {{},{α }}, (Ƒ$_2$,Q) = {{β },{α,β }},

(Ƒ$_3$,Q) = {{Ṿ}, {β }},

(Ƒ$_4$,Q) = {{},{γ }},

(Ƒ$_5$,Q) ={{},{β,δ }},

(Ƒ$_6$,Q) ={{β,δ },{β,γ,δ }}

are soft sets over Ṿ.

Here, {{},{β,δ }} and {{},{γ }}

are soft αωÏ$_s$ – Hausdorff space in (Ṿ,Ꞇ $_s$,Q,Ï$_s$).

3.3 Theorem

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space. Then every soft Hausdorff space is soft αωÏ$_s$ – Hausdorff.

Proof: Let (Ṿ,Ꞇ $_s$,Q) be a soft Hausdorff space and we let two distinct points

(x,Q) ≠ (y,Q) in (Ṿ,Q).

Then there exist two disjoint soft open sets (Ẋ,Q),(Ẏ,Q) ∈ Ꞇ s with (x,Q) ∈ (Ẋ,Q) and (y,Q) ∈ (Ẏ,Q) and (Ẋ,Q) ∩ (Ẏ,Q) = (∅, Q).

Since every soft open set is soft αωÏ$_s$ – open, then (Ẋ,Q) and (Ẏ,Q) are soft αωÏ$_s$ – open. Thus (x,Q) and (y, Q) are separated by soft αωÏ$_s$ – open neighborhoods. Therefore the space is soft αωÏ$_s$ – Hausdorff.

3.4 Theorem

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space. Then every soft ω – Hausdorff is soft αωÏ$_s$ – Hausdorff.

Proof: Suppose that (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be the soft ω – Hausdorff. Then for any two distinct points (x,Q) ≠ (y,Q) there exist soft ω– open sets (Ẋ,Q) and (Ẏ,Q) with (x,Q) ∈ (Ẋ,Q), (y,Q) ∈ (Ẏ,Q) and (Ẋ,Q) ∩ (Ẏ,Q) = ({}, Q). But we know that every soft ω– open set is soft αωÏ$_s$ – open, so (Ẋ, Q) and (Ẏ, Q) are soft soft αωÏ$_s$– open and disjoint. Hence the space is soft αωÏ$_s$– Hausdorff.

The converse of the above theorem need not be true as seen from the following Example.

3.5 Example

Let Ṿ = {α, β,γ,δ },

Q = {σ },

Ꞇ $_s$ = {({},Q),(Ṿ,Q),(Ƒ$_1$,Q),(Ƒ$_2$,Q),(Ƒ$_3$,Q)}

Ï$_s$ = {({},Q),(Ƒ$_4$,Q), (Ƒ$_5$,Q), (Ƒ$_6$,Q)}

Where

(Ƒ$_1$,Q) = {{},{α }},

(Ƒ$_2$,Q) = {{β },{α,β }},

(Ƒ$_3$,Q) = {{Ṿ}, {β }},

(Ƒ$_4$,Q) = {{},{γ }},

(Ƒ$_5$,Q) ={{},{β,δ }},

(Ƒ$_6$,Q) ={{β,δ },{β,γ,δ }}

are soft sets over Ṿ.

Here, {{β },{α,β }} and {{},{β,δ }}

are not soft ω– Hausdorff space in (Ṿ,Ꞇ s,Q,Ï$_s$).

3.6 Theorem

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space.

i. Let Ï$_s$ = ({},Q). Then (Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏ$_s$ – Hausdorff if and only if it is soft ω – Hausdorff.
ii. Let Ï$_s$ = P(Ṿ, Q). Then (Ṿ, Ꞇ $_s$, Q ,Ï$_s$) is soft Hausdorff if and only if it is soft αωÏ$_s$ – Hausdorff.

Proof:

i. Let Ï$_s$ = ({}, Q). Then (Ƒ, Q)ω^* = Cl(Ƒ, Q) and ωCl^{*s}(Ƒ,Q) = (Ƒ,Q) ∪ (Ƒ,Q)ω^* = Cl(Ƒ,Q) for every subset (Ƒ,Q) of (Ṿ,Q). Therefore, we have soft α-

Ïs – open set is equal to the soft open set and hence, (Ṿ,Ϲ s, Q ,Ïs) is soft αωÏs – Hausdorff if and only if it is soft ω – Hausdorff.

ii. Let Ïs = P({Ṿ, Q }. Then (Ƒ, Q)ω* = ({}, Q) and ωCl*s(Ƒ,Q) = (Ƒ,Q) for every soft subset (Ƒ,Q) of Ṿ. Let (Ƒ, Q) belongs to soft α-Ïs – open. Then (Ƒ,Q) ⊆ ωCl*s(int(Ƒ,Q)) = int (Ƒ,Q) and hence, (Ṿ, Ϲ s,Q,Ïs) is soft Hausdorff if and only if it is soft αωÏs – Hausdorff.

4. Soft $\alpha\omega$Ïs – Normal Space In Soft Ideal Topological Spaces

In this section we introduce and study about a new normal space is known as soft αωÏs – Normal space in Soft ideal topological spaces.

4.1 Definition

A soft ideal topological space (Ṿ,Ϲ s,Q,Ïs) is said to be soft αωÏs – Normal space, if for every pair of disjoint soft closed sets (F,Q) and (G,Q), there exist soft αωÏs – open sets (Ẋ,Q) and Ẏ,Q such that (F,Q) ⊆ (Ẋ,Q) and (G,Q) ⊆ (Ẏ,Q).

4.2 Example

Let Ṿ = {α, β, γ },

Q = {σ },

Ϲ s = {({},Q),(Ṿ,Q),(Ƒ1,Q),(Ƒ2,Q)},

Ïs = {({},Q), (Ƒ3,Q)}

where

(Ƒ1,Q) = {{},{α }},

(Ƒ2,Q) = {{β },{α, β }},

(Ƒ3,Q) = {{β }},{γ }} are soft sets over Ṿ.

Here, {{β },{α, β }} and {{β }},{γ }}

are soft αωÏs – Normal space in (Ṿ,Ϲ s,Q,Ïs).

4.3 Theorem Let (Ṿ, Ϲ s,Q,Ïs) be a soft ideal topological space then the following statements are equivalent:

i. (Ṿ,Ϲ s,Q,Ïs) is soft αωÏs – Normal space.
ii. For all soft αωÏs – closed set (F,Q) and for each neighborhood (Ẋ,Q) of (F,Q) there exist soft αωÏs – open neighborhood (Ẏ,Q) of (F,Q) such that soft αωÏsCl(Ẏ,Q) ⊆ (Ẋ,Q)
iii. For each pair of the soft αωÏs – closed set (F,Q) and (G,Q) there exists soft αωÏs – open neighborhood (Ẋ,Q) of (F,Q) such that soft αωÏsCl(Ẋ,Q) ∩ (G,Q) = ({},Q).
iv. For each pair of the soft αωÏs – closed set (F,Q) and (G,Q) there exists soft αωÏs – open neighborhood (Ẋ,Q) of (F,Q) and (Ẏ,Q) of (G,Q) such that (Ẋ,Q) ∩ (Ẏ,Q) = ({},Q).

Proof:

(i) ⇒ (ii) Let (Ṿ,Ϲ s,Q,Ïs) is soft αωÏs – Normal space. Let (F, Q) soft αωÏs – closed set and $(\dot{X}, Q)$ be any open neighborhood (F, Q). Let the soft αωÏs – closed set (F, Q) and

$(V, Q) - (\dot{X}, Q)$ and $(F, Q) \subseteq (\dot{X}, Q)$

implies that

$(F, Q) \cap ((V, Q) - (\dot{X}, Q)) = (\{\}, Q)$.

Since (Ṿ, Q) is soft αωÏs – Normal space there exist soft αωÏs – open neighborhood $(\dot{Y}, Q)$ of (F, Q) and (Z, Q) of $((V, Q) - (\dot{X}, Q))$

such that

$(\dot{Y}, Q) \cap (Z, Q) = (\{\}, Q)$.

Now

$(\dot{Y}, Q) \cap (Z, Q) = (\{\}, Q)$

which implies that

$(\dot{Y}, Q) \subseteq ((V, Q) - (Z, Q))$.

Since

$(V, Q) - (Z, Q)$ is soft αωÏs – closed, soft αωÏsCl$(\dot{Y}, Q)$ ⊆ soft αωÏsCl$((V, Q) - (Z, Q)) = (V, Q) - (Z, Q)$.

Therefore soft αωÏsCl$(\dot{Y}, Q) \cap (Z, Q) = (\{\}, Q)$ and soft αωÏsCl$(\dot{Y}, Q) \cap ((V, Q) - (\dot{X}, Q)) \subseteq$ soft αωÏsCl$(\dot{Y}, Q) \cap (Z, Q) = (\{\}, Q)$, thus soft αωÏsCl$(\dot{Y}, Q) \subseteq (\dot{X}, Q)$.

(ii) ⇒ (iii) Let (F, Q) and (G, Q) be disjoint soft αωÏs – closed sets.

Since

$(F, Q) \cap (G, Q) = (\{\}, Q)$ and $(V, Q) - (G, Q)$ be soft αωÏs – open, we have (H,Q) ⊆ $(V, Q) - (G, Q)$.

Hence

$(V, Q) - (G, Q)$ is soft αωÏs – open neighborhood $(\dot{X}, Q)$ of (F, Q) such that soft αωÏsCl$(\dot{X}, Q) \subseteq (V, Q) - (G, Q)$. Therefore, soft αωÏsCl$(\dot{X}, Q) \cap (G, Q) = (\{\}, Q)$.

(iii) ⇒ (iv) Let (F, Q) and (G, Q) be disjoint soft αωÏs – closed sets. By (iii) there exists soft αωÏs – open neighborhood $(\dot{X}, Q)$ of (F, Q) such that soft αωÏsCl$(\dot{X}, Q) \cap (G, Q) = (\{\}, Q)$. Now (G, Q) and soft αωÏsCl$(\dot{X}, Q)$ are disjoint soft αωÏs – closed sets. Hence by (iii), there exists soft αωÏs – open neighborhood $(\dot{Y}, Q)$ of (G, Q) such that soft αωÏsCl$(\dot{Y}, Q) \cap (G, Q) = (\{\}, Q)$.

(iv) ⇒ (i) Let (F, Q) and (G, Q) be disjoint soft αωÏs – closed sets. By (iv) there exists soft αωÏs – open neighborhood $(\dot{X}, Q)$ of (F, Q) and $(\dot{Y}, Q)$ of (G, Q) such that $(\dot{X}, Q) \cap (\dot{Y}, Q) = (\{\}, Q)$. Since $(\{\}, Q) \subseteq (\dot{X}, Q) \cap (\dot{Y}, Q) = (\{\}, Q)$. Hence, (Ṿ,Ϲ s,Q,Ïs) is soft αωÏs – Normal.

4.4 Theorem

Let (Ṿ,Ϲ s,Q,Ïs) be a soft ideal topological space. Then every soft ω– Normal is soft αωÏs – Normal.

Proof:

Suppose ((Ṿ,Ϲ s,Q,Ïs) is soft ω-Normal. Let (F, Q) and (G, Q) be disjoint soft ω – closed sets. Then $(F, Q) \cap (G, Q) = (\{\}, Q)$, if (F, Q) and (G, Q) are soft ω – closed sets. By inclusion of soft ω –closed classes, each of (F, Q) and (G, Q) are also soft αωÏs – closed. Since the space is αωÏs – Normal space. Then there exist soft αωÏs-open sets $(\dot{X}, Q)$ and $(\dot{Y}, Q)$ such that $(F, Q) \subseteq (\dot{X}, Q)$ and $(G, Q) \subseteq (\dot{Y}, Q)$, $(\dot{X}, Q) \cap (\dot{Y}, Q) = (\{\}, Q)$.

The converse of the above theorem need not be true as seen from the following Example.

4.5 Example

Let Ṿ = {$\alpha, \beta, \gamma, \delta$},
Q = {σ},
Ꞇ $_s$ = {({},Q),(Ṿ,Q),(Ƒ$_1$,Q),(Ƒ$_2$,Q),(Ƒ$_3$,Q)}
Ï$_s$ = {({},Q),(Ƒ$_4$,Q), (Ƒ$_5$,Q)} where
(Ƒ$_1$,Q) = {{},{α}},
(Ƒ$_2$,Q) = {{β},{α,β}},
(Ƒ$_3$,Q) = {{Ṿ}, {β}},
(Ƒ$_4$,Q) = {{},{γ}},
(Ƒ$_5$,Q) ={{β,δ},{β,γ,δ}}
are soft sets over Ṿ.
Here, (Ƒ$_3$,Q) and (Ƒ$_4$,Q) are soft ω – Normal but not soft αωÏ$_s$ – Normal space in (Ṿ,Ꞇ s,Q,Ïs).

4.6 Theorem

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space which is soft αωÏs – Normal. Then the following hold:

i. For every soft closed set (F,Q) and a soft ω-open set (G,Q) containing (F,Q), there exist soft αωÏ$_s$ – open set $(\dot{X},Q)$ such that $(F,Q) \subseteq$ int$_s$*$(\dot{X},Q) \subseteq (\dot{X},Q) \subseteq (G,Q)$.
ii. For every soft ω-closed set (F,Q) and every open set (G,Q) containing (F,Q), there exists soft αωÏ$_s$ – open set $(\dot{X},Q)$ such that $(F,Q) \subseteq (\dot{X},Q) \subseteq$ cl$_s$*$(\dot{X},Q) \subseteq (G,Q)$.

Proof:

(i) Let (F,Q) be a soft closed set and (G,Q) be a soft ω-open set containing (F,Q). Then $(F,Q) \cap ((\dot{V},Q) - (G,Q))$ = $(\{\},Q)$ where (F,Q) be a soft closed and $(\dot{V},Q) - (G,Q)$ is soft ω-closed set. Then there exist disjoint soft αωÏ$_s$ – open set $(\dot{X},Q)$ and $(\dot{Y},Q)$ such that $(F,Q) \subseteq (\dot{X},Q)$ and $((\dot{V},Q) - (G,Q)) \subseteq (\dot{Y},Q)$. Since $(\dot{X},Q) \cap (\dot{Y},Q) = (\{\},Q)$, we have $(\dot{X},Q) - (G,Q) \subseteq (\dot{Y},Q)$. Therefore, $(F,Q) \subseteq$ int$_s$*$(\dot{X},Q) \subseteq (\dot{X},Q) \subseteq ((\dot{V},Q) - (G,Q)) \subseteq (G,Q)$.

(ii) Let (F,Q) be a soft ω - closed set and (G,Q) be a soft α-Ï$_s$-open set containing (F,Q). Then $((\dot{V},Q) - (G,Q))$ be a soft α-Ï$_s$-closed set contained in the soft ω – open set $((\dot{V},Q) - (G,Q))$. By (i), there exist soft αωÏ$_s$ – open set $(\dot{Y},Q)$ such that $(\dot{X},Q) - (G,Q) \subseteq$ int$_s$*$(\dot{Y},Q) \subseteq (\dot{Y},Q) \subseteq ((\dot{V},Q) - (F,Q))$. Therefore, $(F,Q) \subseteq ((\dot{V},Q) - (\dot{Y},Q)) \subseteq$ cl$_s$*$((\dot{V},Q) - (\dot{Y},Q)) \subseteq (G,Q)$. If $(\dot{X},Q) = ((\dot{V},Q) - (\dot{Y},Q))$, then $(F,Q) \subseteq (\dot{X},Q) \subseteq$ cl$_s$*$(\dot{X},Q) \subseteq (G,Q)$ and so $(\dot{X},Q)$ is the required soft αωÏ$_s$ – closed set in (Ṿ,Ꞇ $_s$,Q,Ï$_s$).

5. SOFT αωÏs – REGULAR SPACE IN SOFT IDEAL TOPOLOGICAL SPACES

In this section we introduce and study about a new regular space is known as soft αωÏs –Regular space in Soft ideal topological spaces.

5.1 Definition

A soft ideal topological space ((Ṿ,Ꞇ $_s$,Q,Ï$_s$) is said to be soft αωÏs – Regular space, if for each pair of point x and a soft closed set (G,Q) not containing (x,Q), there exist disjoint soft αωÏs – open sets (Ẋ,Q) and Ẏ,Q such that x ⊆ (Ẋ,Q) and (G,Q) ⊆ (Ẏ,Q).

5.2 Example 2

Let Ṿ = {$\alpha, \beta, \gamma, \delta$},
Q = {σ},
Ꞇ $_s$ = {({},Q),(Ṿ,Q),
(Ƒ$_1$,Q),(Ƒ$_2$,Q),(Ƒ$_3$,Q)}
Ï$_s$ = SS (Ṿ)$_Q$ where
(Ƒ$_1$,Q) = {{},{α}},
(Ƒ$_2$,Q) = {{},{α,β}},
(Ƒ$_3$,Q) = {{Ṿ}, {β}}
are soft sets over Ṿ.
Then (Ṿ,Ꞇ s,Q,Ïs) is soft αωÏ$_s$ – Regular space..

5.3 Theorem

Let ((Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space and x ∈ (Ṿ,Q). Then the following are equivalent :

i. (Ṿ,Ꞇ s,Q,Ïs) is soft αωÏ$_s$ – Regular space.
ii. For every soft ω – closed set (F,Q) such that (x,Q) ∩ (F,Q) = ({},Q), there exist disjoint soft ω – open set (G$_1$,Q) and (G$_2$,Q) such that (x,Q) – (G$_1$,Q) ∈ Ï$_s$ and (F,Q) – (G$_2$,Q) ∈ Ï$_s$.

Proof:

(i) ⇒ (ii) For every soft ω – closed set (F, Q) such that (x,Q) ∩ (F,Q) = ({},Q). Then x ∉ (F,Q). By hypothesis, there exist disjoint soft αωÏ$_s$ – open sets (G$_1$,Q) and (G$_2$,Q) such that (x,Q) – (G$_1$,Q) ∈ Ï$_s$ and (F,Q) – (G$_2$,Q) ∈ Ï$_s$.

(ii) (ii) ⇒ (i) Let (F,Q) be a soft ω – closed set such that x ∉ (F,Q). Then (x,Q) ∩ (F,Q) = ({},Q). there exist disjoint soft αωÏ$_s$ – open sets (G$_1$,Q) and (G$_2$,Q) such that (x, Q) – (G$_1$, Q) ∈ Ï$_s$ and (F,Q) – (G$_2$,Q) ∈ Ï$_s$. Therefore, (Ṿ,Ꞇ $_s$,Q,Ï$_s$ s) is soft αωÏ$_s$ – Regular space.

5.4 Theorem

A soft subspace (Ṿ$_1$,Ꞇ $_{1s}$, Q ,Ï$_{1s}$) of a soft αωÏ$_s$ – Regular space ((Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏ$_1$s – Regular space.

Proof:

Let y ∈ Ṿ$_1$ and (G,Q) be a soft ω – closed set in Ṿ$_1$. Such that y ∉ (G$_1$,Q).

Then (G,Q) = (Ṿ$_1$,Q) ∩ (F,Q) for some soft ω – closed set (F,Q) in Ṿ and y ∉ (F,Q).

Since ((Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏ$_s$ – Regular space, there exist disjoint soft ω – open set (F$_1$, Q) and (F$_2$,Q) in Ṿ such that y ∈ (F$_1$,Q) and ((F,Q) −(F$_1$,Q)) ∈ Ïs. Then, we have ((F,Q) −(F$_2$,Q)) ∩ (Ṿ$_1$,Q) ∈ Ï$_1$s. Since (Ṿ$_1$,Q) ∩ (F$_1$,Q) and (Ṿ$_1$,Q) ∩ (F$_2$,Q) are disjoint soft ω – open sets, (Ṿ$_1$, Ꞇ $_{1s}$, Q ,Ï$_{1s}$) is soft αωÏ$_1$s – Regular space.

5.5 Theorem

Let $m_{\rho\mu}$: $SS(Ṿ)_Q \rightarrow SS(\hat{W})_N$ be a soft function which is bijective, soft irresolute and soft open irresolute. If (Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏs – Regular space, then ($\hat{W}$,Ọ$_s$,N,Ï$_s$) is a soft αω-$m_{\rho\mu}$(Ï$_s$) – Regular space.

Proof: Let (G,N) be a soft ω – closed set in ($\hat{W}$,N) and (w,N) ∈ ($\hat{W}$,N) such that (w,N) ∉ (G,N). Since $m_{\rho\mu}$ is a homeomorphism soft function, then there exist (ṿ,Q) ∈ (Ṿ,Q) such that $m_{\rho\mu}$(ṿ,Q) = (w,N) and (F,Q) = $m^{-1}_{\rho\mu}$(G,N) is a soft ω – closed set in (Ṿ,Q) such that (Ṿ,Q) ∉ (F,Q). By hypothesis, there exist disjoint soft ω – open set (F_1,Q) and (F_2,Q) such that (Ṿ,Q) ∈ (F_1,Q) and (F_2,Q) – (F,Q) ∈ Ï$_s$. It follows that, (F,Q)∈ F_Q(e) for all e ∈ Q and $m_{\rho\mu}$[(F_2,Q) – (F,Q)] ∈ $m_{\rho\mu}$(Ï$_s$). Hence $m_{\rho\mu}$(ṿ,Q) = (w,N) ∈ $m_{\rho\mu}$[F_Q(e)] for all e ∈ Q. Therefore, ($\hat{W}$,Ọ$_s$,N,Ï$_s$) is a soft αω-$m_{\rho\mu}$(Ï$_s$) – Regular space.

5.6 Theorem

Let (Ṿ,Ꞇ $_s$,Q,Ï$_s$) be a soft ideal topological space and (ṿ,Q) ∈ ($Ṿ$,Q)then the following statements are equivalent:

i. (Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏ$_s$ – Regular space.
ii. For all soft αωÏ$_s$ – open set (F, Q), there exist soft αωÏ$_s$ – open neighborhood $(ṿ, Q) \in (F, Q)$ and soft αωÏ$_s$Cl$(F, Q) - (\dot{X}, Q)$ ∈ Ïs.
iii. For all soft αωÏ$_s$ – closed set (G, Q), there exist soft αωÏ$_s$ – open neighborhood $(ṿ, Q) \notin (G, Q)$ and soft αωÏ$_s$Cl$(\dot{X}, Q) \cap (G, Q)$ ∈ Ïs.

Proof:

(i) ⇒ (ii) Let (F, Q) be a soft αωÏ$_s$– open set such that $(ṿ, Q) \in (F, Q)$. Then $(F, Q)'$ is a soft αωÏ$_s$– closed set such that $(ṿ, Q) \notin (F, Q)'$. It follows by (i), there exist disjoint soft αωÏ$_s$ – open sets (H, Q) and (K,Q) such that $(ṿ, Q)$ ∈ (H, Q) and $(F, Q)'$ – (K, Q) = (K, Q)' – (F, Q) ∈ Ï$_s$. Since (H, Q) ∩ (K,Q) = ({}, Q). Then (H, Q) ⊆ (K,Q)'. So soft αωÏ$_s$Cl(H, Q) ⊆ (K,Q)'. Hence, soft αωÏ$_s$Cl(H, Q) − (F, Q)⊆ (K,Q)' − (F, Q) ∈ Ï$_s$ which implies that soft αωÏ$_s$Cl$(F, Q) - (\dot{X}, Q)$ ∈ Ï$_s$.

(ii) ⇒ (iii) Let (G, Q) soft αωÏ$_s$– closed set such that $(ṿ, Q) \notin (G, Q)$. Then $(G, Q)'$ soft αωÏ$_s$– open set such that $(ṿ, Q) \in (G, Q)'$ from (ii), there exist soft αωÏ$_s$– open set (J, Q)

such that $(ṿ, Q) \in (J, Q)$ and soft αωÏ$_s$Cl(J, Q) − (G, Q)'= soft αωÏ$_s$Cl$(J, Q) \cap (G, Q)$ ∈ Ï$_s$.

(iii) ⇒ (i) Let (K, Q) be a soft αωÏ$_s$– closed set such that $(ṿ, Q) \notin (K, Q)$. From (iii) there exist soft αωÏ$_s$– open set (J, Q) such that $(ṿ, Q) \in (J, Q)$ and soft αωÏ$_s$Cl$(J, Q) \cap (G, Q)$ = (J, Q) – [soft αωÏ$_s$Cl(J, Q)]' ∈ Ïs, where, (J, Q) and [soft αωÏ$_s$Cl(J, Q)]' are disjoint soft αωÏ$_s$– o pen sets. Therefore, (Ṿ,Ꞇ $_s$,Q,Ï$_s$) is soft αωÏ$_s$ – Regular space.

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