SUPRA bTµ - CLOSED SETS IN MINIMAL STRUCTURES

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ABSTRACT

We introduce a new set called mbTµ-closed set in a supra topological spaces which are defined on a family of sets satisfying some minimal conditions.

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1. INTRODUCTION

In Mashhour et al. (1983) introduced Supra topological spaces and studied S-continuous maps and S*-continuous maps. Popa and Noiri (2000) introduced concept of minimal structure on a nonempty set. Also they introduced the notation mX-open set and mX-closed set and characterize these sets using mX-cl and mX-int operators respectively.

In this paper, we introduced a new class mX-structures set called minimally bTµ-closed set called as mbTµ-closed set in supra topological spaces. Further, we study the properties of mbTµ-closed sets in supra topological spaces.

2. PRELIMINARIES

Let (X,µ) be a supra topological space and A be a subset of X. The closure of A and interior of A are denoted by clµ(A) and intµ(A) respectively in supra topological spaces.

Definition 2.1 (Mashhour et al., 1983; Sayed and Noiri, 2010)

A subfamily of µ of X is said to be a supra topology on X, if

(i) X, ∅ ∈ µ
(ii) if Ai ∈ µ for all i ∈ I then ∪Ai ∈ µ.

The pair (X,µ) is called supra topological space. The elements of µ are called supra open sets in (X,µ) and complement of a supra open set is called a supra closed set.

Definition 2.2 (Sayed and Noiri, 2010)

(i) The supra closure of a set A is denoted by clµ(A) and is defined as

clµ(A) = ∩{B: B is a supra closed set and A ⊆ B}.

(ii) The supra interior of a set A is denoted by intµ(A) and defined as

intµ(A) = ∪{B: B is a supra open set and A ⊇ B}.

Definition 2.3 (Mashhour et al., 1983)

Let (X,τ) be a topological spaces and µ be a supra topology on X. We call µ a supra topology associated with τ if τ ⊆ µ.

Definition 2.4 (Andrijevic, 1996)

Let (X,µ) be a supra topological space. A set A is called a supra b-open set if A ⊆ clµ(intµ(A))∪intµ(clµ(A)). The complement of a supra b-open set is called a supra b-closed set.

Definition 2.5 (Arockiarani and Pricilla, 2011a)

A subset A of (X,µ) is called supra generalized b–closed set if bclµ(A) ⊆ U whenever A ⊆ U and U is supra open. The complement of supra generalized b-closed set is supra generalized b-open set.

Definition 2.6 (Arockiarani and Pricilla, 2011b)

A subset A of (X,µ) is called Tµ-closed set if bclµ(A) ⊆ U whenever A ⊆ U and U is gµb-open in (X,µ). The complement of Tµ-closed set is called Tµ-open set.

Definition 2.7 (Arockiarani and Pricilla, 2012)

A subset A of a supra topological space (X,µ) is called supra generalized b-regular closed set if
Let $X$ be a nonempty set and $P(X)$ the power set of $X$. A subfamily $m_X$ of $P(X)$ is called a minimal structure on $X$ if it satisfies several basic properties and some characterizations of super T$^\mu$-closed sets and super $T^\mu$-open sets on minimal structures.

**Definition 2.10** (Popa and Noiri, 2000)

Let $X$ be a nonempty set and $m_X$ a minimal structure on $X$ satisfying property B. For a subset $A$ of $X$, the following properties hold:

(i) $A \in m_X$ if and only if $m_X \cap int(A) = A$

(ii) $A$ is $m_X$-closed if and only if $m_X \cap cl(A) = A$

(iii) $m_X \cap int(A)$ is $m_X$-open and $m_X \cap cl(A)$ is $m_X$-closed.

**Definition 2.11** (Popa and Noiri, 2000)

A minimal structure $m_X$ on a nonempty set $X$ is said to have property B if the union of any family of subsets belongs to $m_X$.

**Lemma 2.12** (Popa and Noiri, 2000)

Let $X$ be a nonempty set and $m_X$ a minimal structure on $X$ satisfying property B. For a subset $A$ of $X$, the following properties hold:

(i) $A \in m_X$ if and only if $m_X \cap int(A) = A$

(ii) $A$ is $m_X$-closed if and only if $m_X \cap cl(A) = A$

(iii) $m_X \cap int(A)$ is $m_X$-open and $m_X \cap cl(A)$ is $m_X$-closed.

**Definition 2.12**

Let $(X, m_X)$ be an m-space. A set $A$ is called a mb$^\mu$-open set if $A \subseteq m_X \cap int(m_X \cap int(A)) \cap m_X \cap cl(A)$. The complement of a mb$^\mu$-open set is called a mb$^\mu$-closed set.

**Definition 2.13**

Let $(X, m_X)$ be an m-space. A set $A$ is called a mb$^\mu$-open set if $A \subseteq m_X \cap int(m_X \cap int(A)) \cap m_X \cap cl(A)$. The complement of a mb$^\mu$-open set is called a mb$^\mu$-closed set.

**Definition 2.14**

Let $(X, m_X)$ be an m-space. A set $A$ is called a mb$^\mu$-closed set. The complement of a mb$^\mu$-closed set is called a mb$^\mu$-open set.

**Definition 2.15**

Let $(X, m_X)$ be an m-space. A subset $A$ of $X$ is said to be minimal supra g b-closed if $m_X \cap cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is m supra-open.

**Definition 2.16**

Let $(X, m_X)$ be an m-space. A subset $A$ of $X$ is said to be minimal supra gb-closed if $m_X \cap cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is m supra-open.

**Definition 2.17**

Let $(X, m_X)$ be an m-space. A subset $A$ of $X$ is said to be minimal supra T-closed if $m_X \cap cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is m supra-open.

**Definition 2.18**

Let $(X, m_X)$ be an m-space. A set $A$ is called a mb$^\mu$-open set if $A = m_X \cap int(m_X \cap int(A))$. The complement of a mb$^\mu$-open set is called a mb$^\mu$-closed set.

**3. mb$^\mu$-CLOSED SETS IN MINIMAL STRUCTURES**

**Definition 3.1**

Let $(X, m_X)$ be an m-space. A subset $A$ of $X$ is said to be minimal super T$^\mu$-closed if $m_X \cap cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is mT$^\mu$-open.

**Remark 3.2**

Let $(X, m_X)$ be a supra topological space and $m_X$ be minimal structure on $X$. If $m_X = \mu$, then an mb$^\mu$-closed set is mb$^\mu$-closed set in $X$.

In this section, let $(X, m_X)$ be a supra topological space and $m_X$ be a minimal structure on $X$. We obtain several basic properties and some characterizations of mb$^\mu$-closed sets and mb$^\mu$-open sets on m-space.

**Theorem 3.3**

Let $m_X$ have the property B. A subset $A$ of $X$ is mb$^\mu$-closed in $(X, m_X)$ if $m_X \cap bcl(A) = A$ contains no nonempty mb$^\mu$-closed set in $X$.

**Proof** Suppose that $F$ is a nonempty mb$^\mu$-closed subset of $m_X \cap bcl(A) = A$. Now $F \subseteq m_X \cap bcl(A) = A$. Then $F \subseteq m_X \cap bcl(A) \cap A$, since $m_X \cap bcl(A) = m_X \cap bcl(A) \cap A$. Therefore $F \subseteq m_X \cap bcl(A)$ and $F \subseteq A$. Since $F$ is mb$^\mu$-closed set and $A$ is mb$^\mu$-closed, $m_X \cap bcl(A) \subseteq F$. That is $F \subseteq m_X \cap bcl(A)$. Hence $F \subseteq m_X \cap bcl(A) \cap A = F$. That is $F = \emptyset$. Thus $m_X \cap bcl(A) = A$ contains no nonempty mb$^\mu$-closed set.

Conversely, assume that $m_X \cap bcl(A) = A$ contains no nonempty mb$^\mu$-closed set. Let $A \subseteq G$, $G$ is mb$^\mu$-open. Suppose that $m_X \cap bcl(A) = A$ contains no nonempty mb$^\mu$-closed set. Let $A \subseteq G$, $G$ is mb$^\mu$-open. Suppose that $m_X \cap bcl(A)$ is not contained in $G$. Then $m_X \cap bcl(A) \cap G = \emptyset$. That is $A = A$ is a nonempty mb$^\mu$-closed set.
of \( m_X \cdot bcl(A) - A \), which is a contradiction. Therefore \( m_X \cdot bcl(A) \subseteq G \) and hence \( A \) is \( mbT^\mu \)-closed.

**Theorem 3.4**

For subsets \( A \) and \( B \) of \( X \), the following properties hold:

(i) If \( A \) is \( m_X \)-supra closed, then \( A \) is \( mbT^\mu \)-closed.

(ii) If \( m_X \) has the property \( B \) and \( A \) is \( mbT^\mu \)-closed and \( mT^\mu \)-open then \( A \) is \( m_X \)-supra closed.

(iii) If \( A \) is \( mbT^\mu \)-closed and \( A \subseteq B \subseteq m_X \cdot bcl(A) \), then \( B \) is \( mbT^\mu \)-closed.

**Proof** (i) Let \( A \) be an \( m_X \)-supra closed set in \((X,m_X)\). Let \( A \subseteq G \), where \( G \) is \( mT^\mu \)-open in \((X,m_X)\). Since \( A \) is \( m_X \)-supra closed, \( m_X \cdot cl(A) = A \), we know that \( m_X \cdot bcl(A) \subseteq m_X \cdot cl(A) = A \). Therefore \( A \) is \( mbT^\mu \)-closed.

(ii) Since \( A \) is \( mT^\mu \)-open and \( mbT^\mu \)-closed, we have \( m_X \cdot bcl(A) \subseteq A \). Therefore \( A \) is \( m_X \)-supra closed.

(iii) Let \( A \) be \( mbT^\mu \)-closed, \( m_X \cdot bcl(B) - B \subseteq m_X \cdot bcl(A) - A \), and since \( m_X \cdot bcl(A) - A \) contains no empty \( mT^\mu \)-closed set, neither does \( m_X \cdot bcl(B) - B \). By theorem 3.3, the result follows.

**Theorem 3.5**

Union of two \( mbT^\mu \)-closed sets is \( mbT^\mu \)-closed.

**Proof** Assume that \( A \) and \( B \) are \( mbT^\mu \)-closed sets in \( X \). Let \( G \) be an \( mT^\mu \)-open set in \( X \) such that \( A \cup B \subseteq G \). Then \( A \subseteq G \) and \( B \subseteq G \). Since \( A \) and \( B \) are \( mbT^\mu \)-closed, \( m_X \cdot bcl(A) \subseteq G \) and \( m_X \cdot bcl(B) \subseteq G \). Hence, \( m_X \cdot bcl(A \cup B) \subseteq m_X \cdot bcl(A) \cup m_X \cdot bcl(B) \subseteq G \). Therefore \( A \cup B \) is \( mbT^\mu \)-closed.

**Theorem 3.6**

Every \( m_X \)-supra closed set in \( X \) is \( mbT^\mu \)-closed in \( X \).

**Proof** Let \( G \) be an \( mT^\mu \)-open set such that \( A \subseteq G \). Since \( A \) is \( m_X \)-supra closed, \( m_X \cdot cl(A) = A \), then \( A \subseteq G \). We know that \( m_X \cdot bcl(A) \subseteq m_X \cdot cl(A) \) and \( m_X \cdot bcl(A) \subseteq G \). Therefore \( A \) is \( mbT^\mu \)-closed.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.7**

Consider the \( m \)-space \( X = \{a,b,c\} \) with minimal structure \( m_X = \{X,\emptyset,\{a\},\{a,b\}\} \) and \( mbT^\mu \)-closed are \( \{X,\emptyset,\{a\},\{a,b\}\} \). The set \( \{b\} \) is \( mbT^\mu \)-closed but not \( m_X \)-supra closed set.

**Theorem 3.8**

Every \( mbT^\mu \)-closed in \( X \) is \( mg^b \)-closed in \( X \) but not conversely.

**Proof** Let \( A \subseteq G \) and \( G \) is \( m \)-supra open set in \( X \). We know that \( m \)-supra open set is \( mT^\mu \)-open set. Since \( A \) is \( mbT^\mu \)-closed, we have \( m_X \cdot bcl(A) \subseteq G \). Therefore \( A \) is \( mg^b \)-closed set in \( X \).

The converse of the above theorem need not be true as seen from the following example.

**Example 3.9**

Consider the \( m \)-space \( X = \{a,b,c\} \) with minimal structure \( m_X = \{X,\emptyset,\{a\}\} \). Suppose that \( m_X \cdot bcl(A) \) is \( mg^b \)-closed but not \( mbT^\mu \)-closed set.

**Theorem 3.10**

Every \( mbT^\mu \)-closed in \( X \) is \( mg^b \)-closed in \( X \) but not conversely.

**Proof** Let \( A \subseteq G \) and \( G \) is \( m \)-supra regular open set in \( X \). We know that \( m \)-supra regular open set is \( mT^\mu \)-open set. Since \( A \) is \( mbT^\mu \)-closed, we have \( m_X \cdot bcl(A) \subseteq G \). Therefore \( A \) is \( mg^b \)-closed set in \( X \).

The converse of the above theorem need not be true as seen from the following example.

**Example 3.11** Consider the \( m \)-space \( X = \{a,b,c\} \) with minimal structure \( m_X = \{X,\emptyset,\{a\}\} \). Suppose that \( mg^b \)-closed but not \( mbT^\mu \)-closed set.

**Theorem 3.12**

For each \( x \in X \), \( \{x\} \) is \( mbT^\mu \)-closed in \( X \) or \( \{x\} \) is \( mbT^\mu \)-closed set in \( X \).

**Proof** If \( \{x\} \) is not \( mbT^\mu \)-closed. Then the only \( mT^\mu \)-open set containing \( \{x\} \) is \( X \). Also, the \( m_X \cdot bcl(\{x\}) \) is contained in \( X \) and hence \( \{x\} \) is \( mbT^\mu \)-closed set in \( X \).

**Theorem 3.13**

Let \( m_X \) have property B. Let \( A \) be a subset of \( X \), then \( A \) is \( mbT^\mu \)-closed iff \( m_X \cdot bcl(A) \) is not any empty \( m_X \)-supra closed set.

**Proof** Suppose that \( F \) is nonempty \( mbT^\mu \)-closed subset of \( m_X \cdot bcl(A) \). Now \( F \subseteq m_X \cdot bcl(A) \setminus A \). Then \( F \subseteq m_X \cdot cl(A) \) since \( m_X \cdot bcl(A) \). Therefore \( F \subseteq m_X \cdot cl(A) \) and \( F \subseteq \emptyset \). Since \( F \) is \( mbT^\mu \)-open set and \( A \) is \( mbT^\mu \)-closed, \( m_X \cdot cl(A) \subseteq \emptyset \). That is \( F \subseteq m_X \cdot cl(A) \). Hence \( F \subseteq m_X \cdot cl(A) \) and \( F \subseteq \emptyset \). That is \( F = \emptyset \). Thus \( m_X \cdot cl(A) \) is not any empty \( mbT^\mu \)-closed set.

Conversely, assume that \( m_X \cdot bcl(A) \) is not any empty \( m_X \)-supra closed set. Let \( A \subseteq G \), \( G \) is \( mbT^\mu \)-open. Suppose that \( m_X \cdot bcl(A) \) is not contained in \( G \). Then \( m_X \cdot bcl(A) \cap G \) is a nonempty
mbT$^u$ - closed set of $m_X$-bcl$(A) - A$, which is a contradiction. Therefore $m_X$-bcl$(A) \subseteq G$ and hence $A$ is mbT$^u$- closed.

**Remark 3.14** From the above observation we get the following implications

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m_X - \text{Supra closed} \downarrow
\]
\[
m_X - \text{Supra bT-closed} \rightarrow m_X - \text{Supra gb-closed} \downarrow
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\[
m_X - \text{Supra gbr-closed}
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**REFERENCES**


