Kong. Res. J. 2(1) : 36-39, 2015 ISSN 2349-2694
Kongunadu Arts and Science College, Coimbatore.

DOI:10.26524/krj63

SUPRA bT^{μ} - CLOSED SETS IN MINIMAL STRUCTURES

Krishnaveni, K. and M. Vigneshwaran*
Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore.
*E.mail: vignesh.mat@gmail.com

ABSTRACT

We introduce a new set called mbT^{μ} - closed set in a supra topological spaces which are defined on a family of sets satisfying some minimal conditions.

Mathematics subject classification: 54A05

Keyword: mbT^{μ}-closed set

1. INTRODUCTION

In Mashhour *et al.* (1983) introduced Supra topological spaces and studied S-continuous maps and S^*-continuous maps . Popa and Noiri (2000) introduced concept of minimal structure on a nonempty set. Also they introduced the notation m_X-open set and m_X- closed set and characterize these sets using m_X-cl and m_X- int operators respectively.

In this paper, we introduced a new class m_X-structures set called minimally bT^{μ} - closed set called as mbT^{μ} - closed set in supra topological spaces. Further, we study the properties of mbT^{μ} - closed sets in supra topological spaces.

2. PRELIMINARIES

Let (X,μ) be a supra topological space and A be a subset of X. The closure of A and interior of A are denoted by $cl^{\mu}(A)$ and $int^{\mu}(A)$ respectively in supra topological spaces. Let (X,m_X) be an m-space where X is a nonempty set and m_X is the minimal structure defined on X. The m_X-cl^{μ} and m_X-int^{μ} denotes the m_X-closure and m_X- interior on (X,m_X) respectively on supra topological space.

Definition 2.1 (Mashhour *et al.,* 1983; Sayed and Noiri, 2010)

A subfamily of μ of X is said to be a supra topology on X, if

(i) $X,\phi\in\mu$

(ii) if $A_i\in\mu$ for all $i\in J$ then $\cup A_i\in\mu$.

The pair (X,μ) is called supra topological space. The elements of μ are called supra open sets in (X,μ) and complement of a supra open set is called a supra closed set.

Definition2.2 (Sayed and Noiri, 2010)

(i) The supra closure of a set A is denoted by $cl\mu(A)$ and is defined as

$cl\mu(A)=\cap\{B:B$ is a supra closed set and $A\subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int\mu(A)$ and defined as

$int\mu(A)=\cup\{B:$ B is a supra open set and $A\supseteq B\}$.

Definition 2.3 (Mashhour *et al.,* 1983)

Let (X,τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau\subset\mu$.

Definition 2.4 (Andrijevic, 1996)

Let (X,μ) be a supra topological space. A set A is called a supra b-open set if $A\subseteq cl\mu(int\mu(A))\cup int\mu(cl\mu(A))$. The complement of a supra b-open set is called a supra b-closed set.

Definition 2.5 (Arockiarani and Pricilla, 2011a)

Let (X,μ) be a supra topological space . A set A of X is called supra generalized b – closed set (simply gμb – closed) if $bcl\mu(A)\subseteq U$ whenever $A\subseteq U$ and U is supra open. The complement of supra generalized b-closed set is supra generalized b-open set.

Definition 2.6 (Arockiarani and Pricilla, 2011b)

A subset A of (X,μ) is called Tμ-closed set if $bcl\mu(A)\subseteq U$ whenever $A\subseteq U$ and U is gμb- open in (X,μ).The complement of Tμ-closed set is called Tμ-open set.

Definition 2.7 (Arockiarani and Pricilla, 2012)

A subset A of a supra topological space (X,μ) is called supra generalized b-regular closed set if

$bcl\mu(A) \subseteq U$ and whenever $A \subseteq U$ and U is supra regular open of (X,μ). The complement of supra generalized b-regular closed set is called supra generalized b-regular open set.

Definition 2.8 (Krishnaveni and Vigneshwaran, 2013)

A subset A of a supra topological space (X,μ) is called $bT\mu$-closed set if $bcl\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $T\mu$ -open in (X,μ).The complement of supra $bT\mu$ -closed set is called supra $bT\mu$ -open set.

Remark 2.9 (Krishnaveni and Vigneshwaran, 2013) The following relations are well known

Supra closed
↓
Supra bT-closed → supra gb-closed
↓
Supra gbr-closed

Definition 2.10 (Popa and Noiri, 2000)

Let X be a nonempty set and P(X) the power set of X. A subfamily m_X of P(X) is called a minimal structure (m- structure) on X if $\varphi \epsilon m_X$ and $X \epsilon m_X$. The pairs (X, m_X) is called a minimal space (or m-space).

Definition 2.11 (Popa and Noiri, 2000)

A minimal structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belongs to m_X.

Lemma 2.12 (Popa and Noiri, 2000)

Let X be a nonempty set and m_X a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:

(i) $A \epsilon m_X$ if and only if m_X- int(A) = A
(ii) A is m_X-closed if and only if m_X-cl(A) = A
(iii) m_X-int(A) ϵ m_X-open and m_X-cl(A) is m_X-closed.

Definition 2.13

Let (X,m_X) be an m-space. A set A is called a mb^μ-open set if $A \subseteq m_X\text{-}cl\mu(m_X\text{-}int\mu(A)) \cup m_X\text{-}int\mu(m_X\text{-}cl\mu(A))$. The complement of a mb^μ-open set is called a mb^μ-closed set.

Definition 2.14

Let (X,m_X) be an m-space. A set A is called a m supra regular-open set if A= m_X- clμ(m_X- intμ(A)). The complement of a m supra regular-open set is called a m supra regular-closed set.

Definition 2.15

Let (X,m_X) be an m-space. A subset A of X is said to be minimal supra g b-closed($mg^\mu b$- closed) if m_X-$bcl^\mu(A) \subseteq G$ whenever $A \subseteq G$ and G is m supra-open.

Definition 2.16

Let (X,m_X) be an m-space. A subset A of X is said to be minimal supra gbr-closed ($mg^\mu br$- closed) if m_X- $bcl^\mu(A) \subseteq G$ whenever $A \subseteq G$ and G is m supra regular-open.

Definition 2.17

Let (X,m_X) be an m-space. A subset A of X is said to be minimal supra T-closed(mT^μ- closed) if m_X-$bcl^\mu(A) \subseteq G$ whenever $A \subseteq G$ and G is m supra gb-open.

Definition 2.18

Let (X,m_X) be an m-space. A set A is called a m supra regular open set if A = m_X-intμ(m_X-clμ(A)). The complement of a m supra regular open set is called a m supra regular closed set.

3. mbT^μ-CLOSED SETS IN MINIMAL STRUCTURES

Definition 3.1

Let (X,m_X) be an m-space. A subset A of X is said to be minimal supra bT closed (mbT^μ- closed) if m_X-$bcl^\mu(A) \subseteq G$ whenever $A \subseteq G$ and G is mT^μ-open.

Remark 3.2

Let(X,μ) be a supra topological space and m_X be minimal structure on X. If $m_X = \mu$, then an mbT^μ-closed set is bT^μ-closed set in X.

In this section, let (X,μ) be a supra topological space and m_X be an m-structure on X. We obtain several basic properties and some characterizations of mbT^μ - closed sets and mbT^μ - open sets on m-space.

Theorem 3.3

Let m_X have the property B. A subset A of X is mbT^μ - closed in (X,m_X) iff m_X-$bcl^\mu(A)$ –A contains no non empty mT^μ - closed set in X.

Proof Suppose that F is a nonempty mT^μ-closed subset of m_X-$bcl^\mu(A)$ – A. Now $F \subseteq m_X\text{-}bcl^\mu(A) - A$. Then $F \subseteq m_X\text{-}bcl^\mu(A) \cap A^c$, since $m_X\text{-}bcl^\mu(A) - A = m_X\text{-}bcl^\mu(A) \cap A^c$. Therefore $F \subseteq m_X\text{-}bcl^\mu(A)$ and $F \subseteq A^c$. Since F^c is mT^μ- open set and A is mbT^μ - closed, $m_X\text{-}bcl^\mu(A) \subseteq F^c$. That is $F \subseteq m_X\text{-}bcl^\mu(A)^c$. Hence $F \subseteq m_X\text{-}bcl^\mu(A) \cap m_X\text{-}bcl^\mu(A)^c = \varphi$. That is $F = \varphi$. Thus m_X-$bcl^\mu(A)$ – A contains no nonempty mT^μ - closed set.

Conversely, assume that m_X-$bcl^\mu(A)$ – A contains no nonempty mT^μ - closed set. Let $A \subseteq G$, G is mT^μ-open. Suppose that m_X-$bcl^\mu(A)$ is not contained in G. Then $m_X\text{-}bcl^\mu(A) \cap G^c$ is a nonempty mT^μ - closed set of m_X-$bcl^{\mu}(A)$ – A, which is a contradiction. Therefore m_X-$bcl^{\mu}(A) \subseteq G$ and hence A is mbT^{μ} - closed.

Theorem 3.4

For subsets A and B of X, the following properties hold:

(i) If A is m_X-supra closed, then A is mbT^{μ}-closed.
(ii) If m_X has the property B and A is mbT^{μ}-closed and mT^{μ}-open then A is m_X-supra closed.
(iii) If A is mbT^{μ}-closed and $A \subseteq B \subseteq m_X$-$bcl^{\mu}(A)$, then B is mbT^{μ}-closed.

Proof (i) Let A be an m_X-supra closed set in (X,m_X). Let $A \subseteq G$, where G is mT^{μ}-open in (X,m_X). Since A is m_X-supra closed, m_X-$cl^{\mu}(A)$ = A,we know that m_X-$bcl^{\mu}(A) \subseteq m_X$-$cl^{\mu}(A)$ = A, m_X-$bcl^{\mu}(A) \subseteq G$. Therefore A is mbT^{μ}-closed.

(ii) Since A is mT^{μ}-open and mbT^{μ}-closed, we have m_X-$bcl^{\mu}(A) \subseteq A$. Therefore A is m_X-supra closed.

(iii) Let A is mbT^{μ}-closed, m_X-$bcl^{\mu}(B)$ – B $\subseteq$ m_X-$bcl^{\mu}(A)$ – A, and since m_X-$bcl^{\mu}(A)$ – A contains no nonempty mT^{μ}- closed set, neither does m_X-$bcl^{\mu}(B)$-B. By theorem 3.3, the result follows.

Theorem 3.5

Union of two mbT^{μ}-closed sets is mbT^{μ}-closed.

Proof Assume that A and B are mbT^{μ}- closed sets in X. Let G be an mT^{μ}-open set in X such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are mbT^{μ}-closed, m_X-$bcl^{\mu}(A) \subseteq G$ and m_X-$bcl^{\mu}(B) \subseteq G$. Hence, m_X-$bcl^{\mu}(A \cup B) \subseteq m_X$-$bcl^{\mu}(A) \cup m_X$-$bcl^{\mu}(B) \subseteq G$. Therefore $A \cup B$ is mbT^{μ}-closed.

Theorem 3.6

Every m_X-supra closed set in X is mbT^{μ}-closed in X .

Proof Let G be an mT^{μ}- open set such that $A \subseteq G$. Since A is m_X- supra closed, m_x-$cl^{\mu}(A)$ = A, then -$cl^{\mu}(A) \subseteq G$. We know that m_X-$bcl^{\mu}(A) \subseteq m_X$-$cl^{\mu}(A)$, then m_X-$bcl^{\mu}(A) \subseteq G$. Therefore A is mbT^{μ}-closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7

Consider the m-space X = {a,b,c} with minimal structure m_X = {X,φ,{a},{a,b}}. mbT^{μ}-closed are { X,ϕ, {b}, {c},{b,c}}.The set {b} is mbT^{μ}-closed but not m_X-supra closed set.

Theorem 3.8

Every mbT^{μ}-closed in X is $mg^{\mu}b$-closed in X but not conversely.

Proof Let $A \subseteq G$ and G is m- supra open set in X. We know that m-supra open set is mT^{μ}- open set. since A is mbT^{μ}- closed,we have m_X-$bcl^{\mu}(A) \subseteq G$. Therefore A is $mg^{\mu}b$-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9

Consider the m-space X = {a,b,c} with minimal structure m_X = {X,φ,{a}}. $mg^{\mu}b$- closed are { X,ϕ,{b}, {c}, {a,b},{a,c},{b,c}} and mbT^{μ}- closed are { X,ϕ,{b},{c},{b,c}}.The set {a,b} is $mg^{\mu}b$- closed but not mbT^{μ}- closed set.

Theorem 3.10

Every mbT^{μ}-closed in X is $mg^{\mu}br$-closed in X but not conversely.

Proof

Let $A \subseteq G$ and G is m- supra regular open set in X. We know that m-supra regular open set is mT^{μ}- open set. Since A is mbT^{μ}- closed, we have m_X-$bcl^{\mu}(A) \subseteq G$. Therefore A is $mg^{\mu}br$-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11 Consider the m-space X = {a,b,c} with minimal structure m_X = {X,φ,{a}}.The set {a,b} is $mg^{\mu}br$- closed but not mbT^{μ}- closed set.

Theorem 3.12

For each x∈X, {x} is mT^{μ}- closed in X or {x} is mbT^{μ}- closed set in X.

Proof If {x} is not mT^{μ}- closed. Then the only mT^{μ}-open set containing $\{x\}^c$ in X. Also, the m_X-$bcl^{\mu}(\{x\})^c$ is contained in X and hence {x} is mbT^{μ}- closed set in X.

Theorem 3.13

Let m_X have property B. Let A be a subset of X ,then A is mbT^{μ}- closed iff m_X-$bcl^{\mu}(A)$-A does not contain any nonempty m_X-supra closed set.

Proof Suppose that F is nonempty mbT^{μ}- closed subset of m_X-$bcl^{\mu}(A)$-A. Now $F \subseteq m_X$-$bcl^{\mu}(A)$ – A. Then $F \subseteq m_X$-$bcl^{\mu}(A) \cap A^c$, since m_X-$bcl^{\mu}(A)$ – A = m_X-$bcl^{\mu}(A) \cap A^c$. Therefore $F \subseteq m_X$-$bcl^{\mu}(A)$ and $F \subseteq A^c$. Since F^c is mbT^{μ}- open set and A is mbT^{μ} - closed, m_X-$bcl^{\mu}(A) \subseteq F^c$. That is $F \subseteq [m_X$-$bcl^{\mu}(A)]^c$. Hence $F \subseteq m_X$-$bcl^{\mu}(A) \cap [m_X$-$bcl^{\mu}(A)]^c$ = φ. That is F = φ. Thus m_X-$bcl^{\mu}(A)$ – A contains no nonempty mbT^{μ} - closed set.

Conversely, assume that m_X-$bcl^{\mu}(A)$ – A contains no nonempty m_X- supra closed set. Let $A \subseteq G$, G is mbT^{μ}- open. Suppose that m_X-$bcl^{\mu}(A)$ is not contained in G. Then m_X-$bcl^{\mu}(A) \cap G^c$ is a nonempty

mbT^{μ} - closed set of m_X-$bcl^{\mu}(A) - A$, which is a contradiction. Therefore m_X-$bcl^{\mu}(A) \subseteq G$ and hence A is mbT^{μ} - closed.

Remark 3.14 From the above observation we get the following implications

m_X- Supra closed

↓

m_X- Supra bT-closed → m_X – Supra gb-closed

↓

m_X-Supra gbr-closed

REFERENCES

Andrijevic D., (1996).On b-open sets, mat.Vesnik **48**(1-2): 59-64.

Arockiarani I. and M.T. Pricilla, (2012). On generalized b-regular closed sets, In supra topological spaces, *Asian J Curr Eng Math* **1**(1): 1-4.

Arockiarani, I. And M.T. Pricilla, (2011b). On supra T-closed sets, *Int J Mathematics Arch* **2**(8): 1376-1380 .

Arockiarani, I. and M.T. Pricilla, (2011a). On supra generalized b-closed sets. *Antartica J Math* **8**.

Krishnaveni, K. and M. Vigneshwaran, (2013). On bT-Closed sets in supra topological Spaces, *Int J Math Arch* **4**(2): 1-6.

Mashhour, A.S., A.A. Allam, F.S. Mohamoud and F.H. Khedr, (1983). On supra Topological spaces, *Indian J Pure Appl Math*No.4, 14(1983),502-510.

Noiri, T., (2006). 11th meetings on topological spaces Theory and its Applications, Fukuoka University Seminar House. 1-9.

Parimelazhagan, R., K. Balachandran and N. Nagaveni, (2009). On Weakly Generalized Closed Sets in Minimal Structure *Int J Contemp Math Sci* **4**(27): 1335-1343.

Popa V. And T. Noiri, (2000). On M-continuous functions, Anal.Univ. Dunarea De Jos Galati.Ser.Mat.Fiz.Mec.Teor.(2). **18**(23): 31-41.

Sayed, O.R. and T. Noiri, (2010). On b-open sets and supra b-Continuity on Topological spaces, *Eur J Pure Appl Math* **3**(2): 295-302.