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# ON TOTALLY SUPRA N-CONTINUOUS FUNCTION AND TOTALLY SUPRA N-CLOSED MAP

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**ABSTRACT**

In this paper, we introduce the concept of totally supra N-continuous function and totally supra N-closed map and investigated the relationship of these functions with other functions.
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**Keywords:** Totally supra N-continuous function, totally supra N-closed map.

## 1. INTRODUCTION

In 1983, Mashhour *et al.* (1983) introduced the notion of supra topological spaces and studied, continuous functions and $s^*$-continuous functions. Jamal M.Mustafa (2012) introduced and studies a class of functions called totally supra b-continuous and slightly supra b-continuous functions in supra topological spaces.

In this paper, we introduce the concept of totally supra N-continuous function and totally supra N-closed map and investigated the relationship of these functions with other functions in supra topological spaces.

## 2. PRELIMINARIES

**Definition 2.1**(Mashhour *et al.*,1983)

A subfamily μ of X is said to be supra topology on X if

i) $X, \phi \in \mu$

ii) If $A_i \in \mu \ \forall i \in j$ then $\cup A_i \in \mu$. The pair (X,μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra

open set is called supra closed sets and it is denoted by $\mu^c$.

**Definition 2.2** (Mashhour *et al.*, 1983)

The supra closure of a set A is denoted by clμ (A), and is defined as supra cl(A) = ∩ {B : B is supra closed and A ⊆ B}.

The supra interior of a set A is denoted by intμ(A), and is defined as supraint(A) = ∪ {B : B is supra open and A ⊇ B}.

**Definition 2.3** (Mashhour *et al.*, 1983) Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ , if τ ⊆ μ.

**Definition 2.4** Let (X, μ) be a supra topological space. A set A of X is called

(i) supra semi- open set (Levine, 1991), if A ⊆ clμ(intμ(A)).

(ii) supra α -open set (Devi *et al.,* 2008), if A ⊆ intμ(clμ (intμ (A))).

(iii) supra Ω closed set (Noiri and Sayed, 2005), if sclμ(A) ⊆ intμ (U ),whenever A ⊆ U , U is supra open set.

(iv) supra N-closed set (Vidyarani and Vigneshwaran, 2013), if Ωclμ (A) ⊆ U , whenever A ⊆ U , U is supra α open set.

The complement of above supra closed set is supra open and vice versa.

**Definition 2.5**

A map f:(X, τ ) → (Y, σ) is said to be

(i) supra N-continuous (Vidyarani and Vigneshwaran, 2013a), if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).
(ii) Perfectly supra N-continuous (Vidyarani and Vigneshwaran, 2013a), if $f^{-1}(V)$ is supra clopen in (X, τ) for every supra N-closed set V of (Y, σ).
(iii) strongly supra N-continuous (Vidyarani and Vigneshwaran, 2013a), if $f^{-1}(V)$ is supra closed in (X, τ) for every supra N-closed set V of (Y, σ).
(iv) supra N-closed map (Vidyarani and Vigneshwaran, 2013b), if f(V) is supra N-closed in (Y, σ) for every supra closed set V of (X, τ ).

(v) strongly supra N-closed map (Vidyarani and Vigneshwaran, 2013b), if f(V) is supra N-closed in (Y, σ) for every supra N-closed set V of (X, τ ).

## 3. TOTALLY SUPRA N-CONTINUOUS FUNCTIONS

**Definition 3.1** A map f:(X, τ ) → (Y, σ) is called totally supra continuous function if the inverse image of every supra open set in (Y, σ) is supra clopen in (X, τ ).

**Definition 3.2** A map f:(X, τ ) → (Y, σ) is called totally supra N-continuous function if the inverse image of every supra open set in (Y, σ) is supra N-clopen in (X, τ ).

**Theorem 3.3** Every strongly supra N-continuous function is totally supra N-continuous function.

**Proof** Let f:(X, τ ) → (Y, σ) be a strongly supra N-continuous function. Let V be supra open set in (Y, σ). Then V is supra N-open set in (Y, σ), since every supra open set is supra N-open set. Since f is strongly supra N-continuous function $f^{-1}(V)$ is both supra open and supra closed in (X, τ ). Implies $f^{-1}(V)$ is supra N-clopen in (X, τ ). Therefore f is totally supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.4** Let X=Y={a, b, c} and τ = {X, φ, {a},{b},{a,b},{b, c}} , σ = {Y, φ, {a} }. N-closed set in (X, τ ) are {X, φ, {a},{b},{a,b},{b, c}}. N-closed set in (Y, σ) are {Y, φ, {a},{b},{c},{a,b},{b, c},{a,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is totally supra N-continuous but not strongly supra N-continuous, since V={a,c} is supra N-closed in (Y, σ) but $f^{-1}$({a,c}) = {a,b} is supra open but not supra closed set in (X, τ ).

**Theorem 3.5** Every totally supra N-continuous function is supra N-continuous function.

**Proof** Let f:(X, τ ) → (Y, σ) be a totally supra N-continuous function. Let V be supra open set in (Y, σ). Since f is totally supra N-continuous function, then $f^{-1}(V)$ is supra N-clopen in (X, τ ). Implies $f^{-1}(V)$ is supra N-open in (X, τ ). Therefore f is supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.6** Let X=Y={a, b, c} and τ = {X, φ, {a},{b},{a,b},{b, c}} , σ = {Y, φ, {a},{b,c} }. N-closed set in (X, τ ) are {X, φ, {a},{b},{a,b},{b, c}}. N-closed set in (Y, σ) are {Y, φ, {a},{b},{c},{a,b},{b, c},{a,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=c, f(b)=b, f(c)=a. Here f is supra N-continuous but not totally supra N-continuous, since V={a} is supra open in (Y, σ) but $f^{-1}$({a}) = {c} is supra N-closed but not supra N-open set in (X, τ ).

**Theorem 3.7** Every totally supra continuous function is supra N-continuous function.

**Proof** Let f:(X, τ ) → (Y, σ) be a totally supra continuous function. Let V be supra open set in (Y, σ). Since f is totally supra continuous function, then $f^{-1}(V)$ is supra clopen in (X, τ ). Implies $f^{-1}(V)$ is supra N-clopen in (X, τ ). Hence $f^{-1}(V)$ is supra N-open in (X, τ ). Therefore f is supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.8** Let X=Y={a, b, c} and τ = {X, φ, {a},{b},{a,b},{b, c}} , σ = {Y, φ, {a},{b,c} }. N-closed set in (X, τ ) are {X, φ, {a},{b},{a,b},{b, c}}. N-closed set in (Y, σ) are {Y, φ, {a},{b},{c},{a,b},{b, c},{a,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=c, f(b)=b, f(c)=a. Here f is supra N-continuous but not totally supra continuous, since V={b,c} is supra open in (Y, σ) but $f^{-1}$({b,c}) = {a,b} is supra open but not supra closed set in (X, τ ).

**Theorem 3.9** Every totally supra continuous function is totally supra N-continuous function.

**Proof** Let f:(X, τ ) → (Y, σ) be a totally supra continuous function. Let V be supra open set in (Y, σ). Since f is totally supra continuous function, then $f^{-1}(V)$ is supra clopen in (X, τ ). Implies $f^{-1}(V)$ is supra N-clopen in (X, τ ). Therefore f is totally supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.10** Let X=Y={a, b, c} and τ = {X, φ, {a}} , σ = {Y, φ, {a},{b},{a,b},{b,c} }. N-closed set in (X, τ) are {X, φ, {a},{b},{c},{a,b},{b, c},{a,c}}. N-closed set in (Y, σ) are {Y, φ, {a},{b},{b, c},{a,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is totally supra N-continuous but not totally supra continuous, since V={a,b} is supra open in (Y, σ) but $f^{-1}$({a,b}) = {a,b} is not supra clopen set in (X, τ ).

**Theorem 3.11** If f:(X, τ ) → (Y, σ) is totally supra N-continuous and g: (Y, σ) →(Z,η) is supra continuous then gof: (X, τ ) →(Z,η) is totally supra N-continuous.

**Proof** Let V be supra open set in Z. Since g is supra continuous, then $g^{-1}(V)$ is supra open set in Y. Since f is totally supra N-continuous, then $f^{-1}(g^{-1}(V))$ is supra N-clopen in X. Hence gof is totally supra N-continuous.

**Theorem 3.12** If f:(X, τ ) → (Y, σ) is perfectly supra N-continuous and g: (Y, σ) →(Z,η) is totally supra N-continuous then gof: (X, τ ) →(Z,η) is totally supra N-continuous.

**Proof** Let V be supra open set in Z. Since g is totally supra N-continuous, then $g^{-1}(V)$ is supra N-closed and supra N-open set in Y. Since f is perfectly supra N-continuous, then $f^{-1}g^{-1}(V)$ is supra clopen in X. Implies $f^{-1} g^{-1}(V)$ is supra N-clopen in X. Hence gof is totally supra N-continuous.

## 4. TOTALLY SUPRA N-CLOSED MAP

**Definition 4.1** A map f:(X, τ) → (Y, σ) is said to be totally supra N-closed map, if f(V) is supra clopen in (Y, σ) for every supra N-closed set V of (X, τ ).

**Theorem 4.2** Every totally supra N-closed map is supra N-closed map.

**Proof** Let f:(X, τ ) → (Y, σ) be a totally supra N-closed map. Let V be supra closed set in (X, τ ), then V is supra N-closed set in (X, τ ), since every supra closed set is supra N-closed set. Since f is totally supra N-closed map, then f(V) is supra clopen in (Y, σ). Implies f(V) is supra closed in (Y, σ). Therefore f(V) is supra N-closed in (Y, σ). Therefore f is supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.3** Let X=Y={a, b, c} and τ = {X, φ, {a,b}} , σ = {Y, φ, {a,b},{b,c} }. N-closed set in (X, τ ) are {X, φ, {c},{a,c},{b, c}}. N-closed set in (Y, σ) are {Y, φ, {a},{c},{a,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=c, f(b)=b, f(c)=a. Here f is supra N-closed map but not totally supra N-closed map, since V={b,c} is supra N-closed in (X, τ ) but f ({b,c}) = {a,b} is supra open but not supra closed set in (Y, σ).

**Theorem 4.4** Every totally supra N-closed map is strongly supra N-closed map.

**Proof** Let f:(X, τ ) → (Y, σ) be a totally supra N-closed map. Let V be supra N-closed set in (X, τ ). Since f is totally supra N-closed map, then f(V) is supra clopen in (Y, σ). Implies f(V) is supra closed in (Y, σ). Therefore f(V) is supra N-closed in (Y, σ). Therefore f is strongly supra N-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.5** Let X=Y={a, b, c} and τ = {X, φ,{a},{a,b}} , σ = {Y, φ, {a},{c},{a,c} }. N-closed set in (X, τ ) are {X, φ, {b},{c},{a,c},{b, c}}. N-closed set in (Y, σ) are {Y, φ, {a},{b},{c},{a,b},{b,c}}. f:(X, τ ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is strongly supra N-closed map but not totally supra N-closed map, since V={b} is supra N-closed in (X, τ ) but f ({b}) = {c} is supra open but not supra closed set in (Y, σ).

**Theorem 4.6** If f:(X, τ) → (Y, σ) is totally supra N-closed map and g: (Y, σ) →(Z,η) is totally supra N-closed map then gof: (X, τ ) →(Z,η) is totally supra N-closed map.

**Proof** Let V be supra N-closed set in X, then f(V) is supra clopen in Y, since f is totally supra N-closed map. Implies f(V) is supra closed in Y. Then f(V) is supra N-closed in Y, since every supra closed set is supra N-closed set. Since g is totally supra N-closed map g(f(V)) is supra clopen in Z. Hence gof is totally supra N-closed map.

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