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SEMI #GENERALIZED α–CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper we introduce and discuss some basic properties and preservation theorems of slightly s#gα-continuous functions.

Keywords: clopen, s#gα-open, slightly s#gα-continuity.

1. INTRODUCTION

The notion of s#gα-closed sets in a topological space was introduced by V. Kokilavani and M. Vivek Prabu, 2013. The concept of slightly continuous functions were introduced and investigated by R.C. Jain, 1980.

In this paper we introduce the notion of slightly s#gα-continuous functions and discuss their basic properties. Throughout this paper X, Y and Z denote the topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by int(A) and cl(A) respectively.

2. PRELIMINARIES

2.1. Definition A subset A of a topological space X is said to be

(i) g-closed (Levine, 1970) if cl(A) ⊆ U, whenever A ⊆ U and U is open. The complement of a g-closed set is said to be g-open.

(ii) g#α-closed (Nano *et al.,* 2004) if αcl(A) ⊆ U, whenever A ⊆ U and U is g-open. The complement of a g#α-closed set is said to be g#α-open.

(iii) #gα-closed (Devi *et al.,* 2009) if αcl(A) ⊆ U, whenever A ⊆ U and U is g#α-open. The complement of a #gα-closed set is said to be #gα-open.

(iv) s#gα-closed (Vivek Prabu and Kokilavani, 2013) if scl(A) ⊆ U, whenever A ⊆ U and U is #gα-open. The complement of a s#gα-closed set is said to be s#gα-open.

2.2. Definition A function f : X ⟶ Y is s#gα-continuous (Vivek Prabu and Kokilavani, 2013) if f^{-1}(V) is s#gα-closed in X for every closed set V in Y.

2.3. Definition A function f : X ⟶ Y is slightly continuous (Jain, 1980) if f^{-1}(V) is open in X for every clopen set V in Y.

3. SLIGHTLY s#gα-CONTINUOUS FUNCTIONS

3.1. Definition A function f : X ⟶ Y is said to be slightly s#gα-continuous if f^{-1}(V) is s#gα-open in X for every clopen set V in Y.

3.2. Theorem For a function f : X ⟶ Y, the following are equivalent:

(a) f is slightly s#gα-continuous.

(b) f^{-1}(V) ∈ s#gαO (X) for each V ∈ CO (Y).

(c) f^{-1}(V) is s#gα-clopen for each V ∈ CO (Y).

Proof: (a) ⟹ (b): Let V ∈ CO (Y) and let x ∈ f^{-1}(V). Then f(x) ∈ V. Since f is slightly s#gα-continuous, there is a U ∈ s#gαO (X, x) such that f(U) ⊂ V. Thus f^{-1}(U) = $\cup_x${U: x ∈ f^{-1}(V)}, that is f^{-1}(U) is a union of s#gα-open sets. Hence f^{-1}(U) ∈ s#gαO (X).

(b) ⟹ (c): Let V ∈ CO (Y). Then (Y – V) ∈ CO (X). By hypothesis f^{-1}(Y – V) = X – f^{-1}(V) ∈ s#gαO (X). Thus f^{-1}(V) is s#gα-closed.

(c) ⟹ (a): The proof is obvious.

3.3. Theorem If f : X ⟶ Y is slightly s#gα-continuous and g : Y ⟶ Z is slightly s#gα-continuous, then their composition g∘f is slightly s#gα-continuous.

Proof: Let V ∈ CO (Z), then g^{-1}(V) ∈ CO (Y). Since f is slightly s#gα-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ ∈ s#gαO (X). Thus g∘f is slightly s#gα-continuous.

3.4. Theorem The following are equivalent for a function f : X ⟶ Y:

(a) f is slightly s#gα-continuous.

(b) for each x ∈ X and for each V ∈ CO (Y, f(x)), there exists a s#gα-clopen set U such that f(U) ⊂ U.

(c) for each closed set F of Y, f^{-1}(F) is s#gα-closed.

(d) f(cl(A)) ⊂ s#gαcl(f(A)) for each A ⊂ X.

(e) cl(f^{-1}(B)) for each B ⊂ Y.

Proof: (a) $\Longrightarrow$ (b): Let x ∈ X and V ∈ CO (Y, f(x)), by theorem 3.2 $f^{-1}(V)$ is clopen. Put U = $f^{-1}(V)$, then x ∈ U and f(U) ⊂ V.

(b) $\Longrightarrow$ (c): It is obvious.

(c) $\Longrightarrow$ (d): Since $s^{\#}g\alpha cl(f(A))$ is the smallest $s^{\#}g\alpha$-closed set containing f(A), hence by (c), we have (d).

(d) $\Longrightarrow$ (e): For each B ⊂ Y, $f(cl(f^{-1}(B))) \subset s^{\#}g\alpha cl(f(f^{-1}(B))) \subset s^{\#}g\alpha cl(f(B))$. Hence $f(cl(f^{-1}(B))) \subset f^{-1}(s^{\#}g\alpha cl(f(B))) \Longrightarrow cl(f^{-1}(B)) \subset s^{\#}g\alpha cl(f(B))$.

(e) $\Longrightarrow$ (a): Let V ∈ CO (Y). Then (Y – V) ∈ CO (X), by (e), we have $cl(f^{-1}(Y - V)) \subset f^{-1}(s^{\#}g\alpha cl(Y - V)) = f^{-1}(Y - V)$, since every closed set is $s^{\#}g\alpha$-closed, thus $f^{-1}(Y - V) = X - f^{-1}(V)$ is closed and thus $s^{\#}g\alpha$-closed. Hence $f^{-1}(V) \in s^{\#}g\alpha O$ (X) and f is slightly $s^{\#}g\alpha$-continuous.

3.5. Definition A function f : X ⟶ Y is called almost contra $s^{\#}g\alpha$-continuous if $f^{-1}(V)$ is $s^{\#}g\alpha$-closed in X for every regular open set V in Y.

3.6. Theorem The following are equivalent for a function f : X ⟶ Y:

(a) f is almost contra $s^{\#}g\alpha$-continuous.

(b) $f^{-1}(F) \in s^{\#}g\alpha C$ (X) for every F ∈ RO (Y).

(c) for each x ∈ X and for each regular closed subset F in Y containing f(x), there exists a $s^{\#}g\alpha$-closed set U in X containing x such that f(U) ⊆ F.

(d) for each x ∈ X and for each regular closed subset V in Y not containing f(x), there exists a $s^{\#}g\alpha$-open set K in X not containing x such that $f^{-1}(V) \subseteq K$.

(e) $f^{-1}(int(cl(G))) \in s^{\#}g\alpha C$ (X) for every open subset G of Y.

(f) $f^{-1}(cl(int(F))) \in s^{\#}g\alpha O$ (X) for every closed subset F of Y.

Proof: (a) $\Longrightarrow$ (b): Let F ∈ RO (Y). Then Y – F ∈ RC (Y). By (a), $f^{-1}(Y - F) = X - f^{-1}(F) \in s^{\#}g\alpha O$ (X). Hence $f^{-1}(F) \in s^{\#}g\alpha C$ (X).

(b) $\Longrightarrow$ (a): The Proof is similar.

(b) $\Longrightarrow$ (c): Let F be any regular open set in Y containing f(x). By (b), $f^{-1}(F) \in s^{\#}g\alpha C$ (X) and x ∈ $f^{-1}(F)$. Take U = $f^{-1}(F)$. Then f(U) ⊆ F.

(c) $\Longrightarrow$ (b): Let F be any regular open set in Y and x ∈ $f^{-1}(F)$. From (c), there exists a $s^{\#}g\alpha$-closed set U in X containing x such that f(U) ⊆ F. We have $f^{-1}(F)$ = U. Thus $f^{-1}(F)$ is $s^{\#}g\alpha$-closed.

(c) $\Longrightarrow$ (d): Let V be any regular closed set in Y not containing f(x). Then Y – V is a regular open set containing f(x). By (c), there exists a $s^{\#}g\alpha$-closed set U in X containing x such that f(U) ⊆ Y – V. Hence $U \subseteq f^{-1}(Y - V) \subseteq X - f^{-1}(V)$ and then $f^{-1}(V) \subseteq X - U$.Take H = X – U, we obtain that H is a $s^{\#}g\alpha$-open set in X not containing x.

(d) $\Longrightarrow$ (c): The Proof is similar.

(b) $\Longrightarrow$ (e): Let G be an open subset of Y. Since int(cl(G)) is regular open, then by (b), it follows that $f^{-1}(int(cl(G))) \in s^{\#}g\alpha C$ (X).

(e) $\Longrightarrow$ (b): The Proof is similar.

(a) $\Longrightarrow$ (f): Let F be a closed subset of Y. Since cl(int(G)) is regular closed, then by $f^{-1}(F) \in s^{\#}g\alpha C$ (X), it follows that $f^{-1}(cl(int(F))) \in s^{\#}g\alpha O$ (X).

(f) $\Longrightarrow$ (a): The Proof is similar.

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