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RESEARCH ARTICLE

SOFT $\mathcal{G}$ COMPACTNESS AND SOFT $\mathcal{G}$ CONNECETEDNESS IN SOFT GRILL TOPOLOGICAL SPACES

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Abstract

This paper aims to investigate the concepts of connectedness and compactness to soft grill topological space (X, $\tau_{\mathcal{S}}$, $\zeta_{\mathcal{S}}$, A). The concepts of $\zeta_{\mathcal{S}} - \mathcal{G}$ connected sets, $\zeta_{\mathcal{S}} - \mathcal{G}$ separated sets, and $\zeta_{\mathcal{S}} - \mathcal{G}$ compactness are introduced in soft topological spaces with soft grill. Furthermore, we use $\zeta_{\mathcal{S}} - \mathcal{G}$ closed sets to refine existing theorems, and we illustrate the remarks with a variety of cases.

keywords

Soft sets, soft connected, soft $\zeta_{\mathcal{S}}$ connected , soft $\zeta_{\mathcal{S}} - \mathcal{G}$ connected set, soft $\zeta_{\mathcal{S}} - \mathcal{G}$ separated, soft $\zeta_{\mathcal{S}} - \mathcal{G}$ compactness.

1. Introduction

Soft grill topological spaces were first formulated by Rodyna. A et-al[7,8]. Subsequently, the idea of soft generalized closed sets in the setting of soft grill topological spaces was explored in [1]. In this paper, we aim to introduce the concept of soft $\zeta_{\mathcal{S}} - \mathcal{G}$ connectedness and soft $\zeta_{\mathcal{S}} - \mathcal{G}$ compactness has been investigated its essential characteristics in depth.

2. Preliminaries

2.1 Definition[8]

A non empty collection $\mathcal{G} \subseteq SS$(X, A) of soft sets over X is called a soft grill, if the following conditions hold:

(i) If $\mathcal{F}_{\mathrm{A}} \in \mathcal{G}$ and $\mathcal{F}_{\mathrm{A}} \subseteq \mathcal{H}_{\mathrm{A}}$, which implies $\mathcal{H}_{ß} \in \mathcal{G}$.

(ii) If $\mathcal{F}_{\mathrm{A}} \subseteq \mathcal{H}_{\mathrm{A}} \in \mathcal{G}$, which implies $\mathcal{F}_{\mathrm{A}} \in \mathcal{G}$ or $\mathcal{H}_{\mathrm{A}} \in \mathcal{G}$.

The quadruplet (X,, τ, A, $\mathcal{G}$) is said to be soft grill topological space.

2.2 Definition[1]

Let $\zeta_{\mathcal{S}}$ be a soft grill over a soft topological space ($\mathcal{X}$, $\tau_{\mathcal{S}}$, $\mathcal{A}$). A soft set $\mathcal{F}_{ß}$ is called $\zeta_{\mathcal{S}}$ generalized closed set (briefly $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set), if $\chi_{\zeta}(\mathcal{F}_{ß}) \subseteq \mathcal{U}_{\mathcal{A}}$, whenever $\mathcal{F}_{ß} \subseteq \mathcal{U}_{\mathcal{A}}$ and $\mathcal{U}_{\mathcal{A}}$ is soft open in ($\mathcal{X}$, $\tau_{\mathcal{S}}$, $\mathcal{A}$). The complement of such set will be called $\zeta_{\mathcal{S}} - \mathcal{G}$ open set (resp. $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open set).

2.3 Definition[10]

A bijection f : (X,τ,E) → (Y,σ,K) is called Soft homeomorphism if f is both Soft continuous and Soft open map.

2.4 Definition[9]

A soft topological space (X, $\tau_{\mathcal{S}}$,A) is soft compact if each cover of X by a soft open sets has a finite subcover.

2.5 Definition[9]

Let $\mathcal{G}$ be a soft grill topological space (X, τ, E). A soft set F is called soft $\mathcal{G}$ compact if for every cover $\{U_i \in \tau / i \in \setminus \Gamma\}$ of F by a soft open sets, there exists a finite subset Γ_0 of Γ such that $(\widetilde{X} - \coprod_{i \in \Gamma_0} U_i) \notin \zeta_{\mathcal{S}}$.

2.6 Definition[6]

A non empty soft subsets (F,E), (G.E) of a soft topological spaces (X, $\tau_{\mathcal{S}}$, A) are said to be soft separated sets if cl(F, E) $\widetilde{\cap}$ (G,E)= (F,E) $\widetilde{\cap}$ cl(G,E)=$\widetilde{\emptyset}$.

2.7 Definition[6]

A soft topological space (X, $\tau_{\mathcal{S}}$, A) is said to be soft connected if $\widetilde{X}$ cannot be expressed as the soft union of two soft separated sets (F,A), (G,A) in (X, $\tau_{\mathcal{S}}$, A). Otherwise , (X, $\tau_{\mathcal{S}}$, A) is said to be soft disconnected

3. Soft $\mathcal{G}$ COMPACTNESS SOFT GRILL TOPOLOGICAL SPACES

In this section, we define a novel category of soft generalized compactness within the framework of soft grills as follows:

3.1 Definition

An soft grill topological space (X, τ_S, ζ_S, A) is called ζ_S- $\mathcal{G}$ compact if every ζ_S- $\mathcal{G}$ open cover $\{U_i \in \tau_S / i \in \Delta\}$ of (X, τ_S, ζ_S, A), there exists a finite subset Δ_0 of Δ such that $(\widetilde{X} - \coprod_{i \in \Delta_0} U_i) \notin \zeta_S$.

3.2 Example

Let X ={μ1, μ2, μ3} and $\mathcal{A}$ ={α1, α2},

$\tau_S = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}\}$, and

$\zeta_s = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \widetilde{X}\}$, where $K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over $X_{\mathcal{A}}$, we get the following

$K_1 = \{\{\mu_1\}, \{\mu_1\}\}$,
$K_2 = \{\{\mu_2\}, \{\mu_2\}\}$,
$K_3 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,
$K_4 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$,
$K_5 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$,
$K_6 = \{\{\mu_3\}, \{\mu_3\}\}$,
$K_7 = \{\{\mu_1\}, \{\mu_2\}\}$,
$K_8 = \{\{\mu_1, \mu_2\}, \{\mu_2\}\}$,
$K_9 = \{\{\mu_1\}, \{\mu_3\}\}$,
$K_{10} = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_3\}\}$,
$K_{11} = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_2\}\}$,
$K_{12} = \{\{\mu_2\}, \{\mu_1, \mu_3\}\}$,
$\rho_1 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_3\}\}$,
$\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$,
$\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$,
$\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,
$\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$,
$\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$,
$\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$,
$\rho_8 = \{\mathcal{X}, \{\mu_1, \mu_3\}\}$,
ζ_S- $\mathcal{G}C(X) = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, \rho_1, \rho_2, \rho_3, \rho_4\}$,
ζ_S- $\mathcal{G}O(X) = \zeta_S - \mathcal{G}C(X)^C$.
So (X, τ_S, ζ_S, A) is a soft ζ_S- $\mathcal{G}$ compact.

3.3 Theorem

Every soft compact topological space (X, τ_S, A) is soft ζ_S- $\mathcal{G}$ compact.

Proof

Let $\{U_i \in \tau_S / i \in \Delta\}$ be a cover of $\widetilde{X}$ by soft τ open sets. Then $\widetilde{X} = \coprod_{i \in \Delta_0} U_i$. Since (X, τ_S, A) is a soft compact, there exist a finite subset Δ_0 of Δ such that $\widetilde{X} = \coprod_{i \in \Delta_0} U_i$. Thus $(\widetilde{X} - \coprod_{i \in \Delta_0} U_i) = \widetilde{\emptyset} \notin \zeta_S$. So $\widetilde{X}$ is a ζ_S- $\mathcal{G}$ compact.

3.4 Theorem

Every soft ζ_S compact topological space (X, τ_S, A) is soft ζ_S- $\mathcal{G}$ compact.

Proof

Let $\{U_i \in \tau_S / i \in \Delta\}$ be a cover of $\widetilde{X}$ by soft ζ_S open sets. Then $\widetilde{X} = \coprod_{i \in \Delta_0} U_i$. Since (X, τ_S, ζ_S, A) is a soft ζ_S compact, there exist a finite subset Δ_0 of Δ such that $\widetilde{X} = \coprod_{i \in \Delta_0} U_i$. Thus $(\widetilde{X} - \coprod_{i \in \Delta_0} U_i) = \widetilde{\emptyset} \notin \zeta_S$. So $\widetilde{X}$ is a ζ_S- $\mathcal{G}$ compact.

The converse of the above theorem is not true as seen from the following example.

3.5 Example

Let X={μ1, μ2, μ3}, $\mathcal{A}$={α1, α2},

$\tau_S = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8\}$

and $\zeta_s = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \widetilde{\mathcal{X}}\}$, where $K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over X, the following

$K_1 = \{\{\mu_1\}, \{\mu_1\}\}$,
$K_2 = \{\{\mu_1, \mu_2\}, X\}$,
$K_3 = \{X, \{\mu_1\}\}$,
$K_4 = \{\{\mu_2\}, \{\mu_2\}\}$,
$K_5 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,
$K_6 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$,
$K_7 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$,
$K_8 = \{\{\mu_3\}, \{\mu_3\}\}$,
$\rho_1 = \{\{\mu_1\}, \{\mu_1, \mu_2\}\}$,
$\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$,
$\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$,
$\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,
$\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$,
$\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$,
$\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$,
$\rho_8 = \{\mathcal{X}, \{\mu_1, \mu_3\}\}$,
$\rho_9 = \{\mu_1, \mu_2\}, \mathcal{X}\}$.
Therefore X is soft ζ_S - $\mathcal{G}$ compact, but not soft ζ_S compact.

3.6 Theorem

Soft $\zeta_S - \mathcal{G}$ closed subset of a soft ζ_S - $\mathcal{G}$ compact space in (X, τ_S, ζ_S, A) is soft ζ_S- $\mathcal{G}$ compact.

Proof

Let F_A be a soft $\zeta_S - \mathcal{G}$ closed set and let $\{U_i \in \tau_S / i \in \Delta\}$ be a cover of F_A by soft ζ_S open sets. Then $F_A \subseteq \coprod_{i \in \Delta_0} U_i$. Since F_A is a soft $\zeta_S - \mathcal{G}$ closed, $\chi_\zeta(\mathcal{F}_A) \subseteq \coprod_{i \in \Delta_0} U_i$. Now $\{U_i \in \tau_S / i \in \Delta\} \cup \{\widetilde{X} - \chi_\zeta(\mathcal{F}_A)\}$ is a cover of $\widetilde{X}$ by soft ζ_S open sets in (X, τ_S, A). Since (X, τ_S, A) is a soft ζ_S- $\mathcal{G}$ compact space, there exist a finite subset Δ_0 of Δ such that $\widetilde{X} - [\coprod_{i \in \Delta_0} U_i \cup (\widetilde{X} - \chi_\zeta(\mathcal{F}_A))] \notin \zeta_S$. Then $F_A - \coprod_{i \in \Delta_0} U_i = \widetilde{X} - [\coprod_{i \in \Delta_0} U_i \cup (\widetilde{X} - \chi_\zeta(\mathcal{F}_A))] \notin \zeta_S$. So, F_A is soft ζ_S- $\mathcal{G}$ compact.

3.7 Corollary

Soft ζ_S closed subset of soft ζ_S- $\mathcal{G}$ compact space is a soft ζ_S- $\mathcal{G}$ compact space.

Proof

Following directly from the fact that, every ζ_S closed set is soft ζ_S- $\mathcal{G}$ closed set in (X, τ_S, ζ_S, A).

3.8 Theorem

If $\mathcal{F}_A$ is soft ζ_S- $\mathcal{G}$ compact subset and G_A is a soft ζ_S- $\mathcal{G}$ open set contained in $\mathcal{F}_A$, then $(\mathcal{F}_A - G_A)$ is a soft ζ_S- $\mathcal{G}$ compact.

Proof

Let $\{U_i \in \tau_{\mathcal{S}}/i \in \Delta\}$ be a cover of $(\mathcal{F}_A - G_A)$ is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open sets. Then $(\mathcal{F}_A - G_A) \subseteq \coprod_{i\in\Delta_0} U_i$, since $G_A \subseteq \mathcal{F}_A$ and G_A is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open set, $\mathcal{F}_A \subseteq \coprod_{i\in\Delta_0} U_i \cup G_A$. Since $\mathcal{F}_A$ is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact, there exist a finite subset Δ_0 of Δ such that $\mathcal{F}_A - [\coprod_{i\in\Delta_0} U_i \cup G_A] \notin \zeta_{\mathcal{S}}$. Thus $[(\mathcal{F}_A - G_A) - \coprod_{i\in\Delta_0} U_i] \notin \zeta_{\mathcal{S}}$. Therefore $(\mathcal{F}_A - G_A)$ is a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact.

3.9 Corollary

If $\mathcal{F}_A$ is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact subset and G_A is a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ closed set of $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$, $(\mathcal{F}_A \cap G_A)$ is a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact.

Proof

Let $\mathcal{F}_A$ is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact subset and G_A is a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ closed set of $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$. Then $(\widetilde{X} - G_A)$ is a $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open set of $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$. By using Theorem 3.7, $(\mathcal{F}_A \cap G_A) = (\mathcal{F}_A - (\widetilde{X} - G_A))$ is a soft $\zeta_{\mathcal{S}}$ - $\mathcal{G}$ compact.

3.10 Theorem

If soft union of two soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact set in $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$ is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact.

Proof

Let $\mathcal{F}_A$ and G_A be a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact sets and let $\{U_i \in \tau_{\mathcal{S}}/i \in \Delta\}$ be a cover of $\mathcal{F}_A \cup G_A$ by soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open sets. Then $\{U_i \in \tau_{\mathcal{S}}/i \in \Delta\}$ is a cover of $\mathcal{F}_A$ and G_A by soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open sets. Since $\mathcal{F}_A$ and G_A are soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact, there exist a finite subset Δ_0 and Δ_1 of Δ such that

$\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij} \notin \zeta_{\mathcal{S}}$ and $G_A - \coprod_{ik\in\Delta_1} U_{ik} \notin \zeta_{\mathcal{S}}$.

Thus $(\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij}) \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik}) \notin \zeta_{\mathcal{S}}$.

So $\mathcal{F}_A = \coprod_{ij\in\Delta_0} U_{ij} \cup (\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij})$

and

$G_A = \coprod_{ik\in\Delta_1} U_{ik} \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik})$.

So $\mathcal{F}_A \cup G_A = \coprod_{ij\in\Delta_0} U_{ij} \cup (\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij}) \cup \coprod_{ik\in\Delta_1} U_{ik} \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik})$.

Hence, $\mathcal{F}_A \cup G_A = \cup [U_{ij} \cup U_{ik}/ ij \in \Delta_0$ and $ik \in \Delta_1] \cup (\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij}) \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik})$.

Which implies that, $\mathcal{F}_A \cup G_A = \cup [U_{ij} \cup U_{ik}/ ij \in \Delta_0$ and $ik \in \Delta_1] \cup (\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij}) \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik})$.

Consequently, $(\mathcal{F}_A \cup G_A) - \{\cup [U_{ij} \cup U_{ik} / ij \in \Delta_0$ and $ik \in \Delta_1]\} = (\mathcal{F}_A - \coprod_{ij\in\Delta_0} U_{ij}) \cup (G_A - \coprod_{ik\in\Delta_1} U_{ik}) \notin \zeta_{\mathcal{S}}$. Therefore $\mathcal{F}_A \cup G_A$ is a $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact.

3.11 Corollary

If finite soft union of soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact sets over X is a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact.

Proof

Let $\delta_1, \delta_2, \ldots, \delta_n$ be a finite soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ compact sets over X. Assume $\{U_{\beta i} \in \frac{\tau}{\beta i} \in \Delta, \beta = 1,2,3, \ldots n\}$ be a cover of $\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \delta_\beta$ by a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open sets. Then $\{U_{\beta i} \in \frac{\tau}{\beta i} \in \Delta, \beta = 1,2,3, \ldots n\}$ is a cover of δ_β for each $\beta = 1,2,3, \ldots n$ by a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open sets. For each $\beta = 1,2,3, \ldots n$, there exist a finite subset Δ_β of Δ such that $\delta_\beta - \coprod_{\beta i\in\Delta_\beta} U_{\beta i} \notin \zeta_{\mathcal{S}}$ .So $\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \delta_\beta - \coprod_{\beta i\in\Delta_\beta} U_{\beta i} \notin \zeta_{\mathcal{S}}$. Hence $\delta_\beta = \coprod_{\beta i\in\Delta_\beta} U_{\beta i} \cup (\delta_\beta - \coprod_{\beta i\in\Delta_\beta} U_{\beta i})$, for each $\beta = 1,2,3, \ldots n$ and thus $\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \delta_\beta = (\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \coprod_{\beta i\in\Delta_\beta} U_{\beta i}) \cup (\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \delta_\beta - \coprod_{\beta i\in\Delta_\beta} U_{\beta i})$. Hence $(\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \coprod_{\beta i\in\Delta_\beta} U_{\beta i}) = \cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} (\delta_\beta - \coprod_{\beta i\in\Delta_\beta} U_{\beta i}) - \left(\cup_{\boldsymbol{\beta=1}}^{\boldsymbol{n}} \coprod_{\beta i\in\Delta_\beta} U_{\beta i}\right) \notin \zeta_{\mathcal{S}}$.

4. SOFT $\mathcal{G}$ CONNECETEDNESS IN SOFT GRILL TOPOLOGICAL SPACES

In this section, we introduce soft $\zeta_{\mathcal{S}}$ - $\mathcal{G}$ separated and soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ connectedness in soft grill topological spaces and we workout some basic theorem.

4.1 Definition

Any two soft non empty subsets S_A and T_A of a soft grill tological spaces $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$ are said to be soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$separated, if $S_A \cap \zeta_{\mathcal{S}}$- $\mathcal{G}$ cl$(T_A) = \emptyset = \zeta_{\mathcal{S}}$- $\mathcal{G}$ cl$(S_A) \cap T_A$. If $\widetilde{X} = S_A \cup T_A$ such that S_A and T_A are $\zeta_{\mathcal{S}}$- $\mathcal{G}$ separated, it is said to be S_A and T_A from a soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ separated of $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$.

4.2 Definition

An soft grill topological space $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$ is called $\zeta_{\mathcal{S}}$- $\mathcal{G}$ connected if $\widetilde{X}$ cannot be written as the disjoint union of two non empty $\zeta_{\mathcal{S}}$- $\mathcal{G}$ separated sets. A soft subset of $\widetilde{X}$ is $\zeta_{\mathcal{S}}$- $\mathcal{G}$ connected if it is $\zeta_{\mathcal{S}}$- $\mathcal{G}$ connected as a subspace.

4.3 Example

Let X $=\{\mu_1, \mu_2, \mu_3\}$, $\mathcal{A}=\{\alpha_1, \alpha_2\}$, $\tau_{\mathcal{S}} = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8\}$ and $\zeta_s = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \widetilde{X}\}$, where K_1, $K_2, K_3, K_4, K_5, K_6, K_7, K_8, \rho_1, \rho_2,$ $\rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over X, the following

$K_1 = \{\{\mu_1\}, \{\mu_2\}\}$,

$K_2=\{\{\mu_2, \mu_3\}, \{\mu_1, \mu_3\}\}$,

$K_3 = \{\{\mu_2\}, \{\mu_3\}\}$,

$K_4 = \{\{\mu_2\}, \{\mu_2\}\}$,

$K_5 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,

$K_6 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$,

$K_7 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$,

$K_8 = \{\{\mu_3\}, \{\mu_3\}\}$,

$\rho_1 = \{\{\mu_1\}, \{\mu_1, \mu_2\}\}$,

$\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$,

$\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$,

$\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$,

$\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$,

$\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$,

$\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$,

$\rho_8 = \{ X, \{\mu_1, \mu_3\}\}$,

$\rho_9 = \{\mu_1, \mu_2\}, X\}$.

Therefore X is soft $\zeta_{\mathcal{S}}$- $\mathcal{G}$ connected.

4.4 Theorem

A soft grill topological space (X, τ_S, ζ_S, A) is soft ζ_S- $\mathcal{G}$ connected if and only if $\widetilde{X}$ cannot be express as the disjoint union of two non empty soft ζ_S- $\mathcal{G}$ open sets.

Proof

Let $\widetilde{X}$ be a soft ζ_S- $\mathcal{G}$ connected, and S_A and T_A be a two disjoint non empty soft ζ_S- $\mathcal{G}$ open subsets of $\widetilde{X}$ such that $\widetilde{X} = S_A \cup T_A$. Then S_A and T_A are ζ_S- $\mathcal{G}$ closed in (X, τ_S, ζ_S, A). Hence, $S_A \cap \zeta_S$- $\mathcal{G}$ cl(T_A)=$\emptyset$ = ζ_S- $\mathcal{G}$ cl(S_A) $\cap T_A$. Then $\widetilde{X}$ is not soft ζ_S- $\mathcal{G}$ connected. Which is contradiction our hypothesis. This proves that $\widetilde{X}$ cannot be express as the union of two disjoint non empty soft ζ_S- $\mathcal{G}$ open subsets of $\widetilde{X}$. Conversely, suppose that $\widetilde{X} = S_A \cup T_A$, $S_A \neq \emptyset \neq T_A$ and $S_A \cap \zeta_S$- $\mathcal{G}$ cl(T_A)=$\emptyset$ = ζ_S- $\mathcal{G}$ cl(S_A) $\cap T_A$. Thus S_A and T_A be a two disjoint non empty soft ζ_S - $\mathcal{G}$ open subsets of $\widetilde{X}$,which is contradiction . Hence $\widetilde{X}$ is soft ζ_S - $\mathcal{G}$ connected.

4.5 Theorem

If F_A is soft ζ_S- $\mathcal{G}$ connected set in soft grill topological space (X, τ_S, ζ_S, A), then it is contained in $S_A \cup T_A$, where S_A and T_A are soft ζ_S- $\mathcal{G}$separated, then either $F_A \subset S_A$or $F_A \subset T_A$.

Proof

Now $F_A = (F_A \cap S_A) \cup (F_A \cap T_A)$, where $F_A \cap S_A$ and $F_A \cap T_A$ are soft ζ_S - $\mathcal{G}$ separated sets. So either $F_A \cap S_A \neq \emptyset$ or $F_A \cap T_A \neq \emptyset$ and hence $F_A \subset S_A$or $F_A \subset T_A$.

4.6 Theorem

If a soft subset S_A be a soft grill topological spaces (X, τ_S, ζ_S, A) is soft ζ_S - $\mathcal{G}$ connected, then there exist a soft ζ_S- $\mathcal{G}$ connected set Q_A satisfying $Q_A \subset S_A \subset \zeta_S$- $\mathcal{G}$cl(Q_A).

Proof

Assume $S_A = E_A \cup F_A$, E_A and F_A are soft ζ_S- $\mathcal{G}$ connected sets. Then $Q_A \subset E_A$ and $Q_A \subset F_A$ and hence either $S_A \subset \zeta_S$- $\mathcal{G}$cl(Q_A) $\subset \zeta_S -$ $\mathcal{G}$cl(E_A) $\subset (\widetilde{X}/F_A)$ or $S_A \subset (\widetilde{X}/F_A)$. Hence either $E_A = \emptyset$ or $F_A = \emptyset$.

4.7 Theorem

If soft grill topological spaces (X, τ_S, ζ_S, A), the following statement are equivalent:

1. $\widetilde{X}$ is soft ζ_S- $\mathcal{G}$ connected.
2. $\widetilde{X}$ cannot be written as the union of two disjoint non empty ζ_S- $\mathcal{G}$ open sets.
3. $\widetilde{X}$ contains no nonempty subset which is both ζ_S- $\mathcal{G}$ open and ζ_S- $\mathcal{G}$ closed.

Proof

(1)→(2): Assume $\widetilde{X}$ be a soft ζ_S- $\mathcal{G}$ connected and if $\widetilde{X}$ can be expressed as the union of two disjoint nonempty sets S_A and T_A are ζ_S- $\mathcal{G}$ open sets. Consequently $S_A \subset \widetilde{X}/T_A$. Now ζ_S - $\mathcal{G}$ cl(S_A)$\subset$ ζ_S - $\mathcal{G}$ cl($\widetilde{X}/T_A$)= $\widetilde{X}/T_A$. Hence ζ_S - $\mathcal{G}$ cl(S_A) $\cap$ T_A = $\emptyset$. Similarly we can prove $S_A \cap \zeta_S$- $\mathcal{G}$cl(T_A)=$\emptyset$. Which is contradiction to that X is soft ζ_S - $\mathcal{G}$ connected. Therefore, (X, τ_S, ζ_S, A) , cannot be written as the union of two disjoint non empty ζ_S- $\mathcal{G}$ open sets.

(2)→(3): Assume $\widetilde{X}$ cannot be written as the union of two disjoint non empty sets S_A and T_A are ζ_S - $\mathcal{G}$ open sets. For (X, τ_S, ζ_S, A) contains a nonempty subset which is both ζ_S- $\mathcal{G}$ open and ζ_S- $\mathcal{G}$ closed. Now $\widetilde{X} = S_A \cup {S_A}^c$. Therefore S_A and ${S_A}^c$ are disjoint ζ_S - $\mathcal{G}$ open whose union is $\widetilde{X}$. This is the contradiction to our assumption. Hence, $\widetilde{X}$ contains no nonempty subset which is both ζ_S- $\mathcal{G}$ open and ζ_S- $\mathcal{G}$ closed.

(3)→(1): Assume $\widetilde{X}$ contains no nonempty subset which is both ζ_S- $\mathcal{G}$ open and ζ_S- $\mathcal{G}$ closed and $\widetilde{X}$ is ζ_S- $\mathcal{G}$ disconnected. Now $\widetilde{X}$ can be expressed as the union of two disjoint nonempty sets S_A and T_A such that [$S_A \cap \zeta_S$- $\mathcal{G}$ cl(T_A)] $\cup$[ζ_S- $\mathcal{G}$ cl(S_A) $\cap T_A$]=$\emptyset$. Since $S_A \cap T_A = \emptyset$, $S_A = \widetilde{X}/T_A$ and $T_A = \widetilde{X}/S_A$. Since ζ_S- $\mathcal{G}$ cl(S_A) $\cap T_A$=$\emptyset$, ζ_S- $\mathcal{G}$ cl(S_A) $\subset \widetilde{X}/T_A$. Therefore ζ_S- $\mathcal{G}$ cl(S_A)$\subset S_A$, which S_A is a ζ_S- $\mathcal{G}$ closed. Similarly T_A is a ζ_S- $\mathcal{G}$ closed. Since $S_A = \widetilde{X}/T_A$, S_A is a ζ_S- $\mathcal{G}$ open. Hence, there exist a nonempty subsets S_A is both ζ_S - $\mathcal{G}$ open and ζ_S - $\mathcal{G}$ closed. This is a contradiction to our assumptions. Therefore, $\widetilde{X}$ is ζ_S- $\mathcal{G}$ connected.

4.8 Theorem

If S_A and T_A are ζ_S - $\mathcal{G}$ connected spaces of (X, τ_S, ζ_S, A) and $S_A \cap T_A \neq \emptyset$, then $S_A \cup T_A$ is ζ_S- $\mathcal{G}$ connected spaces of (X, τ_S, ζ_S, A).

Proof

Assume (X, τ_S, ζ_S, A) be a soft grill topological spaces and S_A , $T_A \subseteq \widetilde{X}$, S_A and T_A is ζ_S- $\mathcal{G}$ connected spaces. Now, if $S_A \cup T_A$ is ζ_S- $\mathcal{G}$ disconnected spaces, so $S_A \cup T_A = P_A \cup Q_A \in \zeta_S$ - $\mathcal{G}$ O, $P_A \cap Q_A = \widetilde{\emptyset}$ and $P_A, Q_A \neq \widetilde{\emptyset}$, Then $S_A \subseteq S_A \cup T_A$, $S_A \subseteq P_A \cup Q_A$, $S_A \subseteq P_A$ or $S_A \subseteq Q_A$. Similarly, T_A leads to either $S_A \subseteq P_A$ and $T_A \subseteq P_A$ then $S_A \cup T_A \subseteq P_A$, so $Q_A = \widetilde{\emptyset}$ or $S_A \subseteq Q_A$ and $T_A \subseteq Q_A$ then $S_A \cup T_A \subseteq T_A$, so $P_A = \widetilde{\emptyset}$ or $T_A \subseteq Q_A$. Now $T_A \subseteq Q_A$ then $S_A \cap T_A \subseteq P_A \cap Q_A$,so $S_A \cap T_A = \emptyset$.Which is a contradiction, therefore $S_A \cup T_A$ is ζ_S- $\mathcal{G}$ connected spaces of (X, τ_S, ζ_S, A).

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